مجلة كلية التربية الاساسية
كلية التربية الاساسية – الجامعة المستنصرية

Journal of the college of basic education Vol.29 (NO. 118) 2023, pp. 1-14

VARIATIONAL ITERATION METHOD FOR SOLVING FUZZY BOUNDARY VALUE PROBLEMS

Mohammed Ali Ahmed
Department of Mathematics, College of Education,
Ibn Al- Haitham, University of Baghdad, Baghdad, Iraq

mohammedali1975@yahoo.com

Abstract:

This paper will find the approximate solution to the linear and nonlinear fuzzy boundary value problems using the variational iteration method. The numerical scheme is based on analyzing the fuzzy problem into two crisp sub-problems. The first is for the upper solution and the second is for the lower solution of the fuzzy solution. Also, the convergence of the obtained variational iteration formula has been proved to converge to the exact solution of the problem under consideration in each illustrative example, since there is no general formula that may be obtained for the correction functional related to the problem under consideration which is due to the variation of the general Lagrange multiplier from one problem to another.

Keywords: Fuzzy boundary value problems, variational iteration method, solution of fuzzy differential equations using variational iteration method.

1. Introduction:

In the basic sciences, such as engineering, chemistry, or physics, we construct exact mathematical models of empirical phenomena, and then these models are used to make predictions. While some aspects of real-world problems always escape from such precise mathematical models and usually there is an elusive inexactness as a part of the original model. Also, the elements of real-world problems are perturbed by imperfection and thus, for example, there exist no elements that are perfectly round. Perfect notations or exact concepts correspond to the sort of things envisaged in pure mathematics, while inexact structures encounter us in real-life problems [Kandel A., 1986].

The fuzzy set had been introduced by Zadeh in 1965, in which, Zadeh's original definition of a fuzzy set is as follows "a fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one" [Pal S. K., 1986].

Pearson in 1997, introduced the analytical method for solving a linear system of fuzzy differential equations with the cooperation of complex numbers, while there is no such study for evaluating the analytical solution of fuzzy boundary value problems explicitly, except for the work for Al-Saedy A. J. in 2006 [Al-Saedy A. J., 2006] and Al-Adhami R. H. in 2007 [Al-Adhami R. H., 2007] therefore numerical and approximate methods seem to be necessary for solving such type of problems.

In this paper, we will use the variational iteration method for solving fuzzy boundary value problems. This method was proposed by Ji-Huan He in 1998 [He J-H., 1999] and has been recently and intensively studied by several scientists and engineers and favorably applied to various kinds of linear and nonlinear problems.

The variational iteration method has successfully been applied to many situations, for example, Ji-Huan proposed the variational iteration method to solve linear and nonlinear differential and integral equations. In 1998, He solved the classical Blasiu's equation using VIM. In 1999, He used the variational iteration method to give approximate solutions for some well-known non-linear problems. In 2000, He used the variational iteration method to solve autonomous ordinary differential systems. In 2006, Soliman applied the variational iteration method to solve the kdv-Burger's and Lax's seventh-order kdv equations. In 2006 the variational iteration method was applied to solving nonlinear coagulation problems with mass loss by Abulwafa and Momani. In 2006, the variational iteration method was applied to solving nonlinear differential equations of fractional order by Odibat et al. Also in 2006, the variational iteration method has been used to solve several types of problems, such as solving nonlinear PDEs by Bildiki et al., solving the Fokker-Plank equation by Dehghan and Tateri, solving quadratic Riccati differential equation with constant coefficients by Abbasbandy. In 2007 Wang and He, applied the variational iteration method to solve integro differential equations and also Sweilam used the variational iteration method to solve both linear and nonlinear boundary value problems for fourth-order integro differential equations. In 2009 Wen- Hua Wang used the variational iteration method to solve fractional integro differential equations [Batiha B., 2008].

2. The Variational Iteration Method for Solving Second Order Fuzzy Boundary Value Problems:

Consider the following fuzzy boundary value problem in operator form: $A(\tilde{y}(x)) = g(x),\ x \in [0, a]$

Where A is a second order differential operator and g is any given function called the inhomogeneous term. Suppose that the problem under consideration may be rewritten and decomposed as:

$$L(\tilde{y}(x)) + N(\tilde{y}(x)) = g(x), \quad x \in [0, b] \qquad \dots (1)$$

Where L is a second order linear differential operator, N is a nonlinear operator and $\tilde{y}$ is a fuzzy function that refers to the solution of eq (1).

Now, to use the variational iteration method, rewrite eq (1) in the form:

$$L(\tilde{y}(x)) + N(\tilde{y}(x)) - g(x) = 0 \qquad \dots (2)$$

and let $\tilde{y}_n$ be the approximate fuzzy solution of eq (2) which follows that:

$$L(\tilde{y}_n(x)) + N(\tilde{y}_n(x)) - g(x) \neq 0 \qquad \dots (3)$$

and according to the theory of the variational iteration method, the following correction functional for eq (1) may be constructed:

$$\tilde{y}_{n+1}(x) = \tilde{y}_n(x) + \int_0^x \lambda\left[L(\tilde{y}_n(s)) + N(\tilde{y}_n(s)) - g(s)\right]ds, \quad n = 0, 1, \qquad \dots (4)$$

where λ is the general Lagrange multiplier that can be identified optimally via the variational iteration method and $\tilde{y}_0$ is given initially. The subscript n denotes the *n*-th approximation and $\tilde{y}_n$ is considered as a restricted variation, i.e., it must satisfy the first variation $\delta\tilde{y}_n = 0$.

To find the approximate solution of eq (1) using the correction functional (4), we must first determine the value of the general Lagrange multiplier λ that will be identified optimally via the integration by parts and considering the first variation of (4) with respect to $\tilde{y}_n$, as follows:

Suppose first that the fuzzy function $\tilde{y}_n$ is a convex normalized fuzzy function which may be written in terms of its α-level sets as:

$$\tilde{y}_n(x) = [\underline{y}_n^{\alpha}, \overline{y}_n^{\alpha}],\ \alpha \in (0, 1].$$

Where $\underline{y}_n^{\alpha}$ is the lower solution and $\overline{y}_n^{\alpha}$ is the upper solution. Therefore, to find the fuzzy $\tilde{y}_n$, one must find separated the lower and upper nonfuzzy solutions, $\underline{y}_n^{\alpha}$ and $\overline{y}_n^{\alpha}$, respectively.

Therefore, for the lower solution $\underline{y}_n^{\alpha}$ and since eq (2), represent an equation in operator form in terms of the fuzzy function $\underline{y}_n^{\alpha}$. Hence, to find the optimal value of λ from which minimizes eq (4) using the methods of the calculus of variation. Hence, upon taking the first variation with respect to $\tilde{y}_n$ and noting that $\delta\, \underline{y}_n^{\alpha}(0) = 0$, which yield to:

$$\delta\, \underline{y}_{n+1}^{\alpha}(x) = \delta\, \underline{y}_n^{\alpha}(x) + \delta \int_0^x \lambda \left[L(\underline{y}_n^{\alpha}(s)) + N(\underline{y}_n^{\alpha}(s)) - g(s) \right] ds$$

$$= \delta\, \underline{y}_n^{\alpha}(x) + \delta \int_0^x \lambda \left[L(\underline{y}_n^{\alpha}(s)) + N(\underline{y}_n^{\alpha}(s)) \right] ds - \delta \int_0^x \lambda g(s)\, ds$$

, and since the first variation is taken with respect to $\underline{y}_n^{\alpha}$, therefore:

$$\delta \int_0^x \lambda g(s)\, ds = 0.$$

Hence:

$$\delta\, \underline{y}_{n+1}^{\alpha}(x) = \delta\, \underline{y}_n^{\alpha}(x) + \delta \int_0^x \lambda \left[L(\underline{y}_n^{\alpha}(s)) + N(\underline{y}_n^{\alpha}(s)) \right] ds \quad \ldots (5)$$

Now, from the theory of calculus of variation to find the critical value of $\underline{y}_n^{\alpha}$ which minimizes (5), set $\delta \underline{y}_{n+1}^{\alpha}(x)\Big|_{\text{linear parts in } \underline{y}_n^{\alpha}} = 0$ and since N ($\underline{y}_n^{\alpha}$) is a nonlinear operator, then eq (5) will be reduced to:

$$\underline{y}_{n+1}^{\alpha}(x) = \delta\, \underline{y}_n^{\alpha}(x) + \delta \int_0^x \lambda L(\underline{y}_n^{\alpha}(s))\, ds \quad \dots (6)$$

Therefore, the related Euler equation with its natural boundary conditions may be used to find the value of the general Lagrange multiplier λ, and thereafter eq (4) may be used to find the lower solution $\underline{y}_n^{\alpha}$ which may be proved to converge to the exact solution of the nonfuzzy problem in lower case.

Similarly, one can find the upper solution $\overline{y}_n^{\alpha}$ and therefore the fuzzy approximate solution of the nonlinear boundary value problem (1) is given in its α-level sets as $\tilde{y}_n(x) = [\underline{y}_n^{\alpha}, \overline{y}_n^{\alpha}]$, $\alpha \in (0, 1]$.

3. Numerical Illustrations:

In this section, we apply the variational iteration method to solve linear and nonlinear fuzzy boundary value problems.

Example (1):

Consider the second order linear fuzzy boundary value problem:

$$\tilde{y}''(x) + 2\tilde{y}'(x) - 8\tilde{y}(x) = 0,\ \tilde{y}(0) = \tilde{1},\ \tilde{y}(1) = \tilde{0},\ x \in [0, 1] \quad \dots (7)$$

where $\tilde{0}$ and $\tilde{1}$ are triangular normalized fuzzy numbers and thus, for $\alpha \in [0, 1]$, the boundary conditions may be written in terms of its α-level sets as:

$$[\tilde{y}(0)]_{\alpha} = [y_0]_{\alpha} = [\underline{y}_0(\alpha), \overline{y}_0(\alpha)] = [1 - \sqrt{1-\alpha}, 1 + \sqrt{1-\alpha}].$$

$$[\tilde{y}(1)]_{\alpha} = [y_1]_{\alpha} = [\underline{y}_1(\alpha), \overline{y}_1(\alpha)] = [-\sqrt{1-\alpha}, \sqrt{1-\alpha}].$$

Now, letting $\tilde{y}(x) = [\underline{y}(x), \overline{y}(x)]$ and by using the variational iteration method to find first the approximate lower solution $\underline{y}(x)$ of the nonfuzzy boundary value problem:

$$\underline{y}''(x) + 2\underline{y}'(x) - 8\underline{y}(x) = 0 \quad \dots (8)$$

with boundary conditions:

$$\underline{y}_0(\alpha) = 1 - \sqrt{1-\alpha},\ \underline{y}_1(\alpha) = -\sqrt{1-\alpha}$$

Then the correction functional related to eq (8) is:

$$\underline{y}_{n+1}(x) = \underline{y}_n(x) + \int_0^x \lambda\left[\underline{y}_n''(s) + 2\underline{y}_n'(s) - 8\underline{y}_n(x)\right]ds, \quad n = 0, 1, \ldots$$

… (9)

Taking the first variation of (9), with respect to $\underline{y}_n$ and noting that $\delta\,\underline{y}_n(0) = 0$, we get:

$$\delta\,\underline{y}_{n+1}(x) = \delta\,\underline{y}_n(x) + \delta\int_0^x \lambda\left[\underline{y}_n''(s) + 2\underline{y}_n'(s) - 8\underline{y}_n(x)\right]ds, \; n = 0, 1, \ldots$$

and consequently:

$$\delta\,\underline{y}_{n+1}(x) = \delta\,\underline{y}_n(x) + \delta\int_0^x \lambda\underline{y}_n''(s)\,ds + 2\delta\int_0^x \lambda\underline{y}_n'(s)\,ds - 8\delta\int_0^x \lambda\underline{y}_n(x)\,ds$$

… (10)

and carrying out integration by parts on eq (10), yields to:

$$\delta\,\underline{y}_{n+1}(x) = \delta\,\underline{y}_n(x) + \lambda\delta\,\underline{y}_n'(s)\Big|_{s=x} - \lambda'\delta\,\underline{y}_n(s)\Big|_{s=x} + \int_0^x \lambda''\delta\underline{y}_n(s)\,ds +$$

$$2\left[\lambda\delta\underline{y}_n(s)\Big|_{s=x} - \int_0^x \lambda'\delta\underline{y}_n(x)\,ds\right] - 8\int_0^x \lambda\delta\underline{y}_n(s)\,ds$$

$$= (1 - \lambda' + 2\lambda)\delta\,\underline{y}_n(s)\Big|_{s=x} + \lambda\delta\,\underline{y}_n'(s)\Big|_{s=x} + (\lambda'' - 2\lambda' - 8\lambda)\int_0^x \delta\underline{y}_n(s)\,ds$$

and since $\delta\underline{y}_{n+1}(x)\Big|_{\text{linear parts in } \underline{y}_n} = 0$ then as a result the following stationary conditions are obtained which represent the Euler equation with the natural boundary conditions (given in calculus of variation):

$$\left.\begin{array}{l}\lambda''(s,x) - 2\lambda'(s,x) - 8\lambda(s,x) = 0 \\ (1 + 2\lambda - \lambda')\Big|_{s=x} = 0,\; \lambda\Big|_{s=x} = 0\end{array}\right\} \quad \ldots$$

(11)

which has the solution:

$$\lambda(s, x) = c_1e^{4s} + c_2e^{-2s}$$

since $(1 + 2\lambda - \lambda')\Big|_{s=x} = 0,\; \lambda\Big|_{s=x} = 0$, which implies the following linear system:

$$c_1e^{4x} + c_2e^{-2x} = 0$$
$$1 + 2c_1e^{4x} + 2c_2e^{-2x} - 4c_1e^{4x} + 2c_2e^{-2x} = 0$$

which may be solved to get $c_1 = \dfrac{1}{6e^{4x}}$ and $c_2 = \dfrac{-1}{6e^{-2x}}$; and so,

$$\lambda(s, x) = \frac{1}{6e^{4x}}e^{4s} - \frac{1}{6e^{-2x}}e^{-2s}$$

Now, substituting the value of the general Lagrange multiplier $\lambda(s, x)$ back into eq (9) gives the following variational iteration formula:

$$\underline{y}_{n+1}(x) = \underline{y}_n(x) + \int_0^x \left(\frac{1}{6e^{4x}}e^{4s} - \frac{1}{6e^{-2x}}e^{-2s}\right)\left[\underline{y}''_n(s) + 2\underline{y}'_n(s) - 8\underline{y}_n(x)\right]ds,$$

$$n = 0, 1, \ldots, \ldots \quad (12).$$

Suppose the initial approximate solution is given by:

$$\underline{y}_0(x) = a + bx$$

where a and b are constants to be determined. From eq (12)

$$\underline{y}_1(x) = \frac{2ae^{2x}}{3} + \frac{ae^{-4x}}{3} + \frac{be^{2x}}{6} - \frac{be^{-4x}}{6}$$

$$\underline{y}_2(x) = \frac{2ae^{2x}}{3} + \frac{ae^{-4x}}{3} + \frac{be^{2x}}{6} - \frac{be^{-4x}}{6}$$

$$\vdots$$

$$\underline{y}_n(x) = \frac{2ae^{2x}}{3} + \frac{ae^{-4x}}{3} + \frac{be^{2x}}{6} - \frac{be^{-4x}}{6}$$

Applying the fuzzy boundary conditions, yields to:

$$a = 1 - \sqrt{1-\alpha}\ ;\ b = \frac{\dfrac{e^2(2\sqrt{1-\alpha}-2) + e^{-4}(\sqrt{1-\alpha}-1)}{3} - \sqrt{1-\alpha}}{\dfrac{e^2 - e^{-4}}{6}}$$

Thus:

$$\underline{y}_n(x) = \frac{e^{2x}\sqrt{1-\alpha} - e^{-4x}\sqrt{1-\alpha} - e^{2}e^{-4x} + e^{-4}e^{2x} + e^{2}e^{-4x}\sqrt{1-\alpha} - e^{-4}e^{2x}\sqrt{1-\alpha}}{e^{2}-e^{-4}}, \quad n = 1, 2, \ldots$$

Similarly, the upper solution is found to be:

$$\overline{y}_n(x) = \frac{e^{2x}\sqrt{1-\alpha} - e^{-4x}\sqrt{1-\alpha} + e^{2}e^{-4x} - e^{-4}e^{2x} + e^{2}e^{-4x}\sqrt{1-\alpha} - e^{-4}e^{2x}\sqrt{1-\alpha}}{e^{2}-e^{-4}}$$

, n =1, 2, …

As a comparison, one may see the accuracy of the results when $\alpha = 1$, which will produce that:

$$\underline{y}_n(x) = \overline{y}_n(x) = \frac{e^{2}e^{-4x} - e^{-4}e^{2x}}{e^{2}-e^{-4}} = \frac{e^{-4x}}{1-e^{-6}} + \frac{e^{2x}}{1-e^{6}}$$

which is the same of the exact solution of the crisp (nonfuzzy) boundary value problem:

$y''(x) + 2y'(x) - 8y(x) = 0$, $y(0) = 1$, $y(1) = 0$, $x \in [0, 1]$

Figure (1) presents the fuzzy solution for different values of $\alpha \in [0, 1]$.

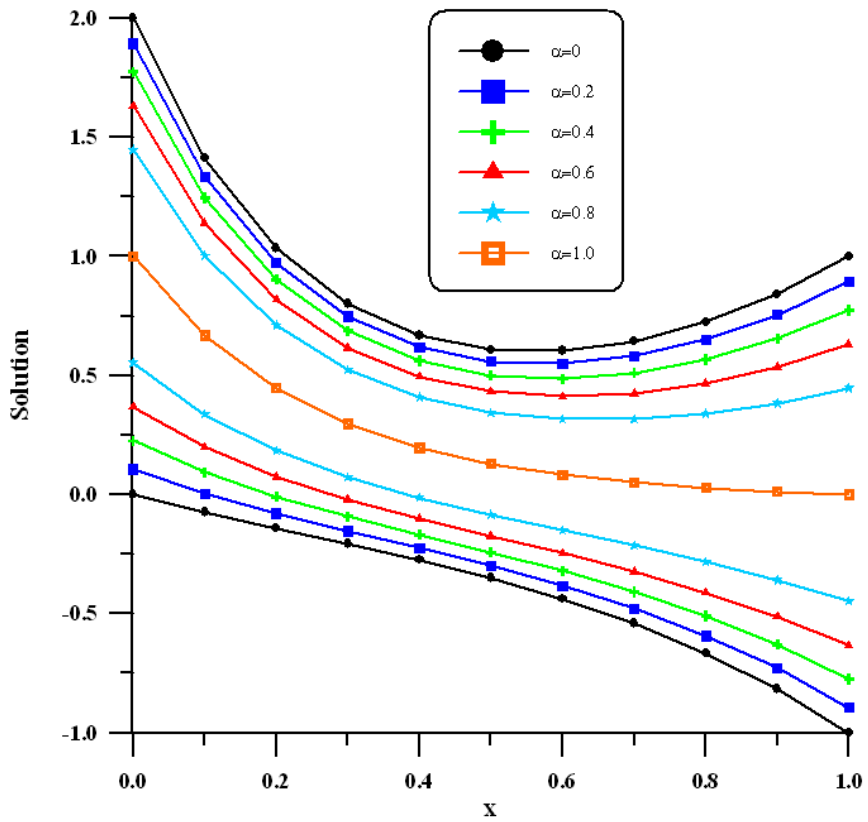

Fig (1) The approximate solution of example (1) for different values of α-levels.

It is remarkable from the obtained results, the sequence of the proximate solutions is the same in each case of the lower and upper solutions, therefore the convergence of these sequences to the exact solution is ensured.

Example (2):

Consider the second-order nonlinear fuzzy boundary value problem:

$$\tilde{y}''(x) = \frac{3}{2}\tilde{y}^2(x) - \frac{3}{2}x^4 + 2,\ \tilde{y}(0) = \tilde{0},\ \tilde{y}(1) = \tilde{1},\ x \in [0, 1] \quad \ldots (13)$$

where $\tilde{0}$ and $\tilde{1}$ are triangular normalized fuzzy numbers and thus, for $\alpha \in [0, 1]$, the boundary conditions may be written in terms of its α-level sets as:

$$[\,\tilde{y}(0)\,]_\alpha = [y_0]_\alpha = [\,\underline{y}_0(\alpha), \overline{y}_0(\alpha)\,] = [-\sqrt{1-\alpha}\,,\ \sqrt{1-\alpha}\,]$$

$$[\,\tilde{y}(1)\,]_\alpha = [y_1]_\alpha = [\,\underline{y}_1(\alpha), \overline{y}_1(\alpha)\,] = [1 - \sqrt{1-\alpha}\,,\ 1 + \sqrt{1-\alpha}\,]$$

Now, letting $\tilde{y}(x) = [\underline{y}(x), \overline{y}(x)]$ and by using the variational iteration method to find first the approximate lower solution $\underline{y}(x)$ of the nonfuzzy boundary value problem:

$$\overline{y}''(x) = \frac{3}{2}\overline{y}^2(x) - \frac{3}{2}x^4 + 2 \quad \ldots (14)$$

with boundary conditions:

$$\overline{y}_0(\alpha) = \sqrt{1-\alpha}, \ \overline{y}_1(\alpha) = 1 + \sqrt{1-\alpha}$$

The correction functional related to eq (14) is given by:

$$\overline{y}_{n+1}(x) = \overline{y}_n(x) + \int_0^x \lambda\left[\overline{y}_n''(s) - \frac{3}{2}\overline{y}_n^2(s) - g(s)\right]ds, \ n = 0, 1, \ldots \quad \ldots (15)$$

where $g(x) = \frac{3}{2}x^4 - 2$

Taking the first variation of (15) with respect to $\overline{y}_n$ and noting that $\delta\overline{y}_n(0) = 0$, we get:

$$\delta\overline{y}_{n+1}(x) = \delta\overline{y}_n(x) + \delta\int_0^x \lambda\left[\overline{y}_n''(s) - \frac{3}{2}\overline{y}_n^2(s) - g(s)\right]ds, \ n = 0, 1, \ldots$$

and consequently:

$$\delta\overline{y}_{n+1}(x) = \delta\overline{y}_n(x) + \delta\int_0^x \lambda\overline{y}_n''(s)\,ds \quad \ldots (16)$$

and carrying out integration by parts on eq (16), yields to:

$$\delta\overline{y}_{n+1}(x) = \delta\overline{y}_n(x) + \lambda\delta\overline{y}_n'(s)\big|_{s=x} - \lambda'\delta\overline{y}_n(s)\big|_{s=x} + \int_0^x \lambda''\delta\overline{y}_n(s)\,ds$$

$$= (1-\lambda')\delta\overline{y}_n(s)\big|_{s=x} + \lambda\delta\overline{y}_n'(s)\big|_{s=x} + \int_0^x \lambda''\delta\underline{y}_n(s)\,ds$$

and since $\delta\bar{y}_{n+1}(x)\Big|_{\text{linear parts in } \bar{y}_n} = 0$, then as a result the following stationary conditions are obtained which represent the Euler equation with the natural boundary conditions (given in calculus of variation):

$$\left.\begin{array}{l}\lambda''(s,x)=0\\ (1-\lambda')\big|_{s=x}=0,\ \lambda\big|_{s=x}=0\end{array}\right\} \qquad \ldots$$

(17)

which has the solution:

$$\lambda(s, x) = c_1 s + c_2$$

since $(1-\lambda')\big|_{s=x}=0,\ \lambda\big|_{s=x}=0$, which implies:

$$c_1 = 1 \text{ and } c_2 = -x$$

and so:

$$\lambda(s, x) = s - x$$

Now, substituting the value of the general Lagrange multiplier $\lambda(s, x)$ back into eq (15) gives the following variational iteration formula:

$$\bar{y}_{n+1}(x) = \bar{y}_n(x) + \int_0^x (s-x)\left[\bar{y}_n''(s) - \frac{3}{2}\bar{y}_n^2(s) - g(s)\right]ds,\ n = 0, 1, \ldots \ldots$$

(18)

Suppose the initial approximate solution is given by:

$$\bar{y}_0(x) = a + bx$$

where a and b are constants to be determined. From eq (18), we obtain the following results:

$$\bar{y}_1(x) = a + bx + \frac{x^2(30a^2+20abx+5b^2x^2-2x^4+40)}{40}$$

$$\bar{y}_2(x) = a + \frac{ax^4}{4} - \frac{3ax^8}{1120} + \frac{3bx^5}{20} - \frac{bx^9}{480} + \frac{x^6}{20} - \frac{x^{10}}{600} + \frac{3x^{14}}{145600} + \frac{3a^3x^4}{16} + \frac{3a^2x^6}{40} + \frac{9a^4x^6}{320} - \frac{a^2x^{10}}{800} + \frac{3b^2x^8}{448} + \frac{b^3x^7}{112} - \frac{b^2x^{12}}{7040} + \frac{b^4x^{10}}{3840} + bx + \frac{x^2(30a^2+20abx+5b^2x^2-2x^4+40)}{40} + \frac{3a^2bx^5}{16} + \frac{ab^2x^6}{16} + \frac{3a^3bx^7}{112} + \frac{ab^3x^9}{384} + \frac{3a^2b^2x^8}{256} + \frac{abx^7}{28} - \frac{3abx^{11}}{4400}$$

and because of the difficulty of the approximate solutions, the nonlinear system will be solved for each α = 0, 0.2, 0.4, 0.6, 0.8 and 1; with the application of the fuzzy boundary conditions, we get $a = \sqrt{1-\alpha}$, while:

$$b = \begin{cases} -0.7604, & \text{when } \alpha=0 \\ -0.6367, & \text{when } \alpha=0.2 \\ -0.5071, & \text{when } \alpha=0.4 \\ -0.3696, & \text{when } \alpha=0.6 \\ -0.2182, & \text{when } \alpha=0.8 \\ 2.3175\times10^{-7}, & \text{when } \alpha=1 \end{cases}$$

Similarly, the lower solution may be found in lower case, but it will not be presented here because of its difficulty.

Figure (2) presents the fuzzy solution for different values of $\alpha \in [0, 1]$ related to the above values of a and b.

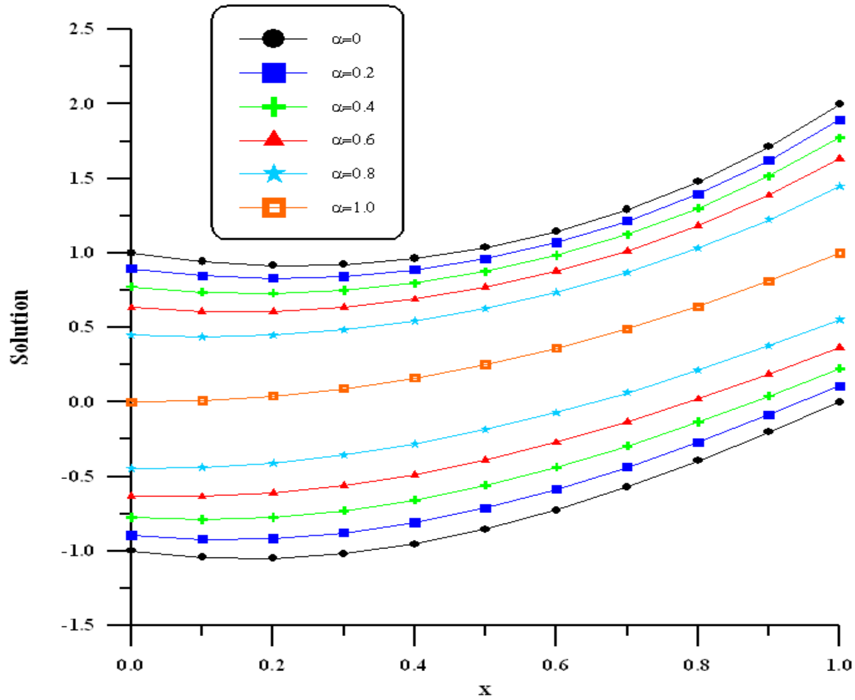

Fig (2) The approximate solution of example (2) for different values of α-levels.

The convergence of the numerical results to the exact solution is ensured from the convergence of the method for solving nonfuzzy (or crisp) ordinary differential equations, [He J-H., 1999].

Conclusion:

1. In the linear case, the method gives the exact solution in one iteration only, as is expected from the theory of the variational iteration method.
2. The variational iteration method is very powerful and efficient in solving approximately fuzzy boundary value problems, in which the sequence of solutions converges very rapidly to the exact solution of the problem in a crisp case.

References

The Journal of The College of Basic Education use the Harvard (author-date) reference system. A complete reference list in alphabetical order must be supplied.

Examples of how references should be cited are given below:

1. R. H. Al-Adhami (2007), "Numerical Solution of Fuzzy Boundary Value Problems", M.Sc. Thesis, College of Science, Al-Nahrain University.
2. A. J. M. Al-Saedy (2006), "Solution of Boundary Fuzzy Differential Equations", M. Sc. Thesis, College of Science, Al-Nahrain University.
3. B. Batiha, M. S. Noorani, and I. Hashim (2008), "Numerical Solutions of the Nonlinear Integro-Differential Equations", Journal of Open Problems Compt. Math., Vol.1, No.1, pp. 34-41.
4. J-H. He (1999), "Variational Iteration Method; A Kind of Non-Linear Analytical Technique; Some Examples", International Journal of Non-Linear Mechanics, 34, pp. 699-708.
5. A. Kandel (1986), “Fuzzy Mathematical Techniques with Applications”, Addison Wesley Publishing Company, Inc.
6. S. K. Pal and D.K Majumder (1986) “Fuzzy Mathematical Approach to Pattern Recognition”, John Wiley and Sons, Inc., New York.

مجلة كلية التربية الاساسية
كلية التربية الاساسية – الجامعة المستنصرية

Journal of the college of basic education Vol.29 (NO. 118) 2023, pp. 1-14

طريقة التكرار التغايرية لحل المعادلات التفاضلية الحدودية الضبابية

محمد علي احمد
جامعة بغداد / كلية التربية ابن الهيثم / قسم الرياضيات

mohammedali1975@yahoo.com

مستخلص البحث:

في هذا البحث سنجد الحل التقريبي للمعادلات التفاضلية الحدودية الضبابية الخطية وغير الخطية باستخدام طريقة التكرار التغايرية. يعتمد النهج العددي على تحليل المشكلة الضبابية إلى مشكلتين فرعيتين غير ضبابيتين. يمثل حل المشكلة الأولى الحل العلوي وحل المشكلة الثانية يمثل الحل السفلي لدالة الحل الضبابي. أيضاً، تم إثبات تقارب صيغة معادلة التكرار التغايرية التي تم الحصول عليها الى الحل الدقيق للمشكلة قيد الدراسة في كل مثال توضيحي، حيث لا توجد صيغة عامة يمكن الحصول عليها لدالي التصحيح والمتعلقة بالمشكلة قيد الدراسة حيث يرجع ذلك إلى اختلاف مضروب لاجرانج العام من مشكلة إلى أخرى.