

## Magnetization Behavior of a Mixed Spin-3/2 and Spin-7/2 System on a Binary Square Lattice within the Blume-Capel Ising Model

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### Abstract

This study investigates the magnetization behavior of a mixed spin-3/2 and spin-7/2 system arranged on a binary square lattice using the Blume-Capel Ising ferrimagnetic model within the framework of mean-field theory. The primary focus is on the influence of crystal fields on critical temperatures and magnetization curves. It is observed that varying the crystal field ratio  $DB/|J|$  yields a spectrum of magnetic behaviors, classified as types Q, R, and S. These results provide further insight into the phase transition characteristics of mixed-spin systems.

**Keywords:** Ising model . Mixed spin . Ferrimagnetism . Mean-field theory . Anisotropy.

### 1 Introduction:

Ferrimagnetic materials are of significant technological interest, particularly in data storage devices [1] and other advanced applications [2–5]. The magnetic properties of mixed-spin Ising ferrimagnetic systems have been explored in various contexts [6–9]. Examples include heterocomplex compounds such as  $[\text{Cr}(\text{CN})_4(\mu\text{-CN})_2\text{Gd}(\text{H}_2\text{O})_4(\text{bpy})]_n \cdot 4n\text{H}_2\text{O} \cdot 1.5n\text{bpy}$  [10] and perovskite-type chromium oxides like  $\text{GdCrO}_3$  [11], where  $\text{Gd}(\text{III})$  has spin-7/2 and  $\text{Cr}(\text{III})$  has spin-3/2. In these systems, inequivalent magnetic moments interact antiferromagnetically, resulting in zero net magnetization at the Curie temperature. Prior studies employing mean-field theory and Monte Carlo simulations have provided foundational insights [9, 12, 16]. This paper aims to elucidate the role of crystal fields in determining the magnetization and critical behavior in such [15] mixed-spin systems.

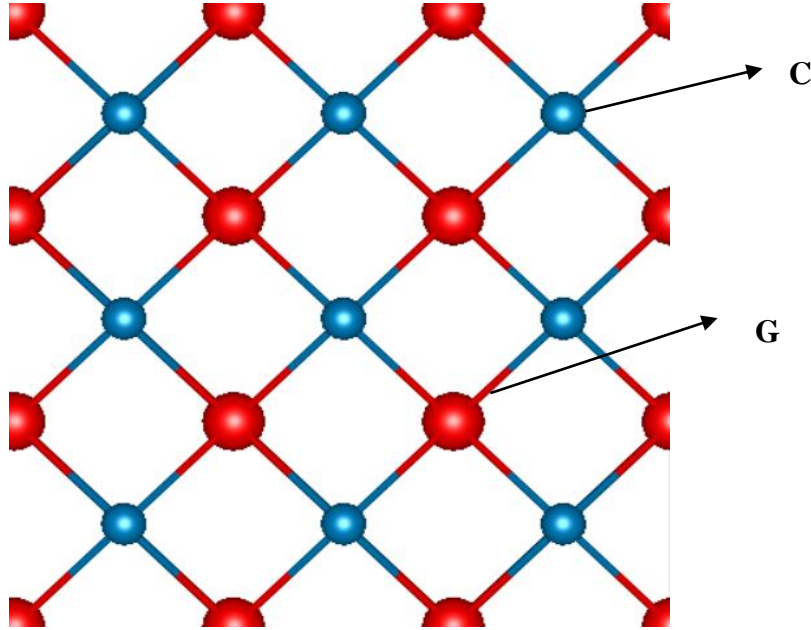
### 2. Methodology

We consider a two-dimensional Ising model on a square lattice with periodic boundary conditions. Spins  $S_A = \pm 3/2$  and  $\pm 1/2$  (Cr sites) and  $S_B = \pm 7/2, \pm 5/2, \pm 3/2, \text{ and } \pm 1/2$  (Gd sites) occupy alternating lattice sites (Fig. 1).

The Hamiltonian of the system in the absence of an external magnetic field is given by:

$$H = -J \sum \langle S_i, S_j \rangle S_i S_j + DA \sum_i (S_i)^2 + DB \sum_j (S_j)^2$$

where  $J$  is the exchange interaction between nearest neighbors, and  $D_A$  and  $D_B$  represent the single-ion anisotropy parameters (crystal fields) for Cr and Gd atoms



**Fig 1:** A section of the two-dimensional lattice systems. Gd (red ball) with spin  $S_B=7/2$ , and Cr (blue ball) with spin  $S_A=3/2$ .

The Hamiltonian of a mixed ferrimagnetic spin  $\pm \frac{3}{2}$  and spin  $\pm \frac{7}{2}$  square lattice Ising system in a zero magnetic field, can be written as [10],

$$H = -J \sum_{i,j} S_i^A S_j^B - D_A \sum_i (S_i^A)^2 - D_B \sum_j (S_j^B)^2 \quad (1)$$

Where  $[S_i^A, S_j^B]$  takes the values  $[\pm \frac{3}{2}, \pm \frac{7}{2}]$  respectively,  $J$  is a coupling constant between nearest neighbor,  $D_A$  is a magnetic anisotropy acting on Cr atoms, and  $D_B$  an acting on Gd atoms According to Maxwell-Boltzmann statistical the total magnetization of the system is obtained:

$$m_A = \frac{1}{2} \frac{3 \sinh(\frac{3}{2} \beta z J m_B) + e^{-2\beta D_A} \sinh(\frac{1}{2} \beta z J m_B)}{\cosh(\frac{3}{2} \beta z J m_B) + e^{-2\beta D_A} \cosh(\frac{1}{2} \beta z J m_B)} \quad (2)$$

$$m_B = \frac{1}{2} \frac{7 \sinh(\frac{7}{2} \beta z J m_A) + 5e^{-6\beta D_B} \sinh(\frac{5}{2} \beta z J m_A) + 3e^{-10\beta D_B} \sinh(\frac{3}{2} \beta z J m_A) + e^{-12\beta D_B} \sinh(\frac{1}{2} \beta z J m_A)}{\cosh(\frac{7}{2} \beta z J m_A) + e^{-6\beta D_B} \cosh(\frac{5}{2} \beta z J m_A) + e^{-10\beta D_B} \cosh(\frac{3}{2} \beta z J m_A) + e^{-12\beta D_B} \cosh(\frac{1}{2} \beta z J m_A)} \quad (3)$$

The mean-field approximation based on Bogoliubov inequality has been used to study the system's free energy [14]

### 3. Results and discussion:

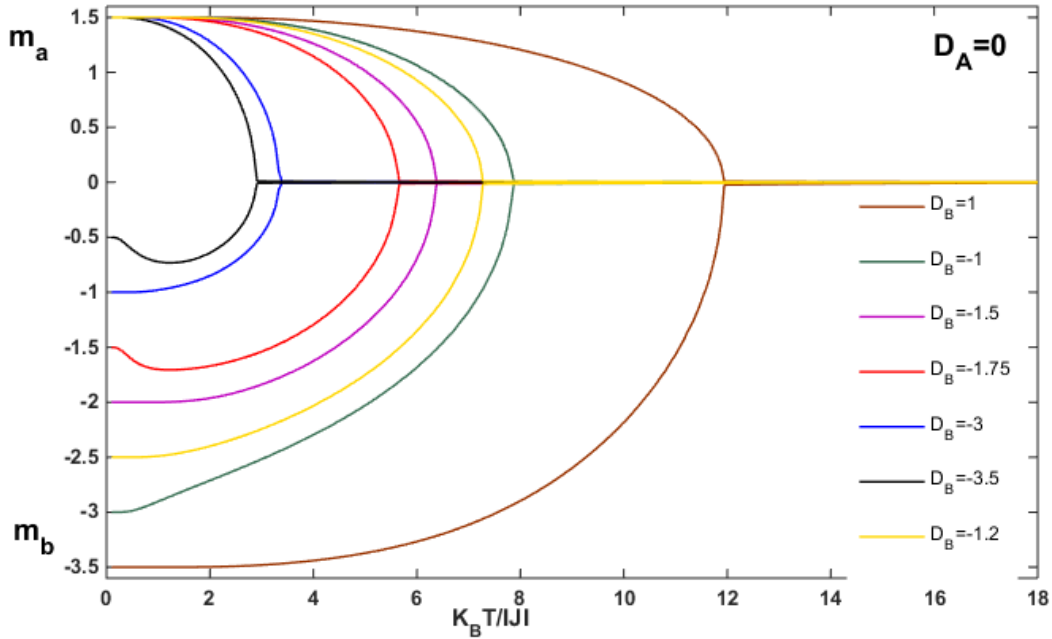


Fig. 2: Sublattice magnetizations  $m_a$  (Cr) and  $m_b$  (Gd) exhibit thermal change at  $D_A/|J|=0$  and at various  $D_B/|J|$  values.

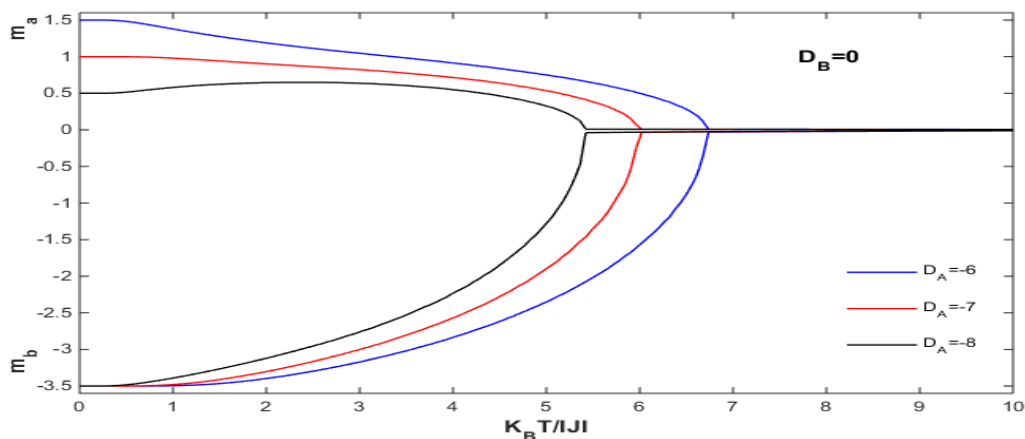
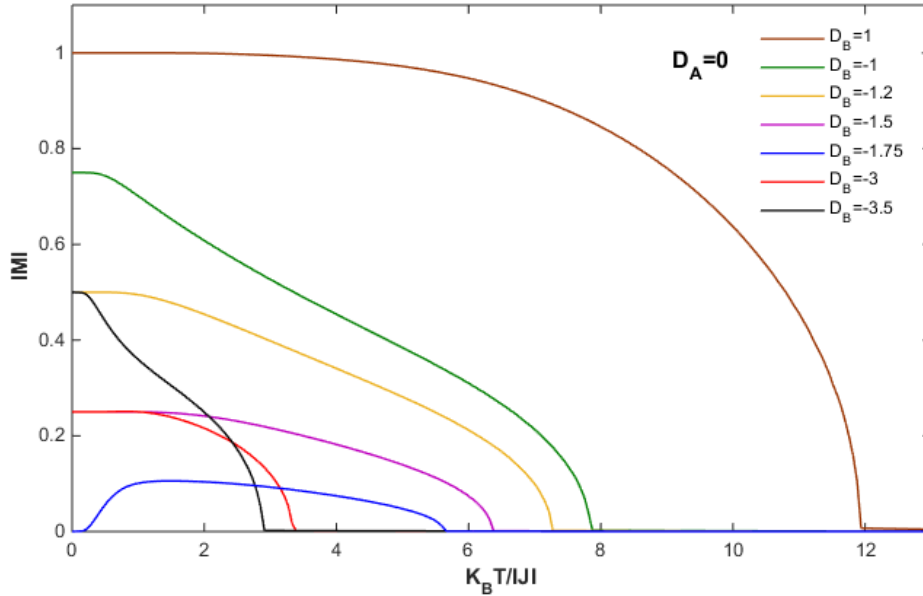
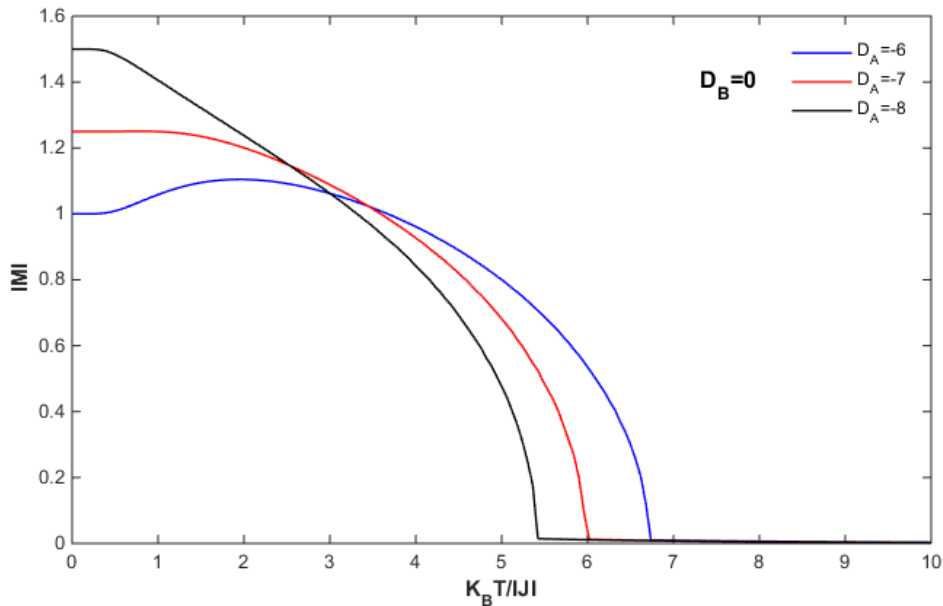


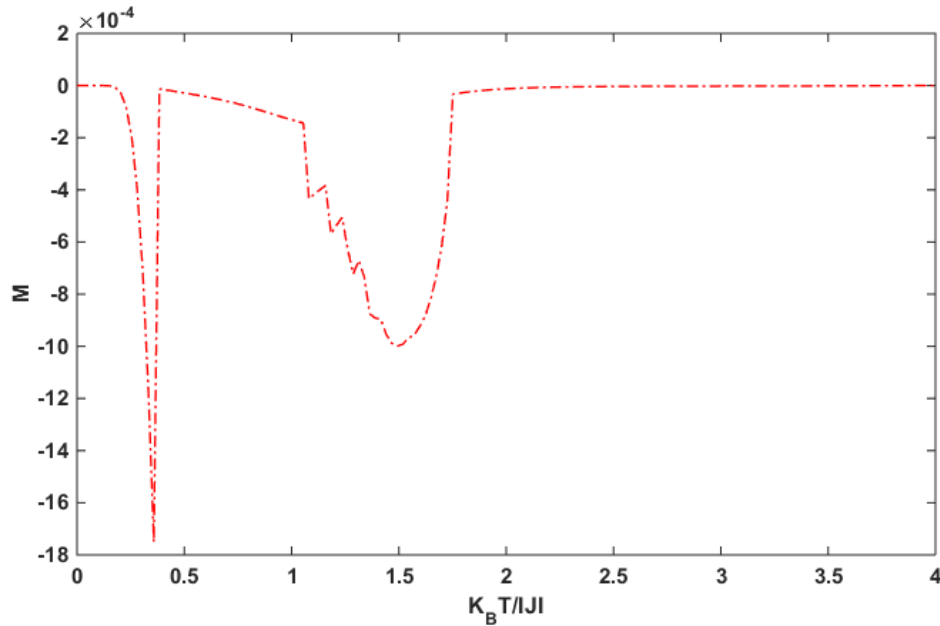
Fig. 3: Sublattice magnetizations' thermal variation  $m_a$  (Cr) and  $m_b$  (Gd) on  $D_B/|J|=0$  and different values of  $D_A/|J|$ .



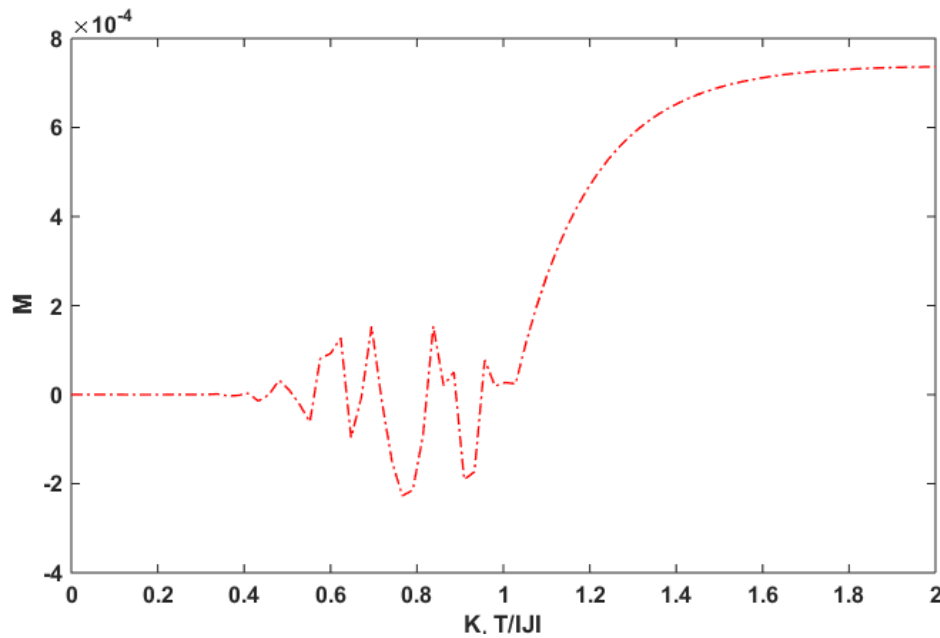
**Fig. 4:** Thermal behavior of the absolute value of the total magnetization  $|M|$  for various values of the crystal field  $D_B/|J|$  when  $D_A/|J| = 0$ .



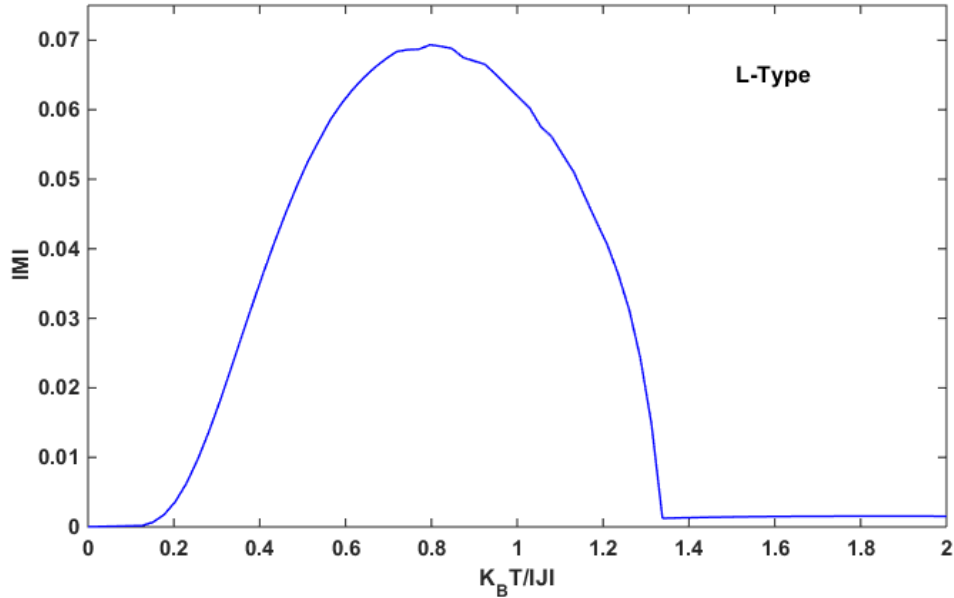
**Fig. 5:** Thermal behavior of the total magnetization's absolute value  $|M|$  for different crystal field  $D_A/|J|$  values. when  $D_B/|J|=0$



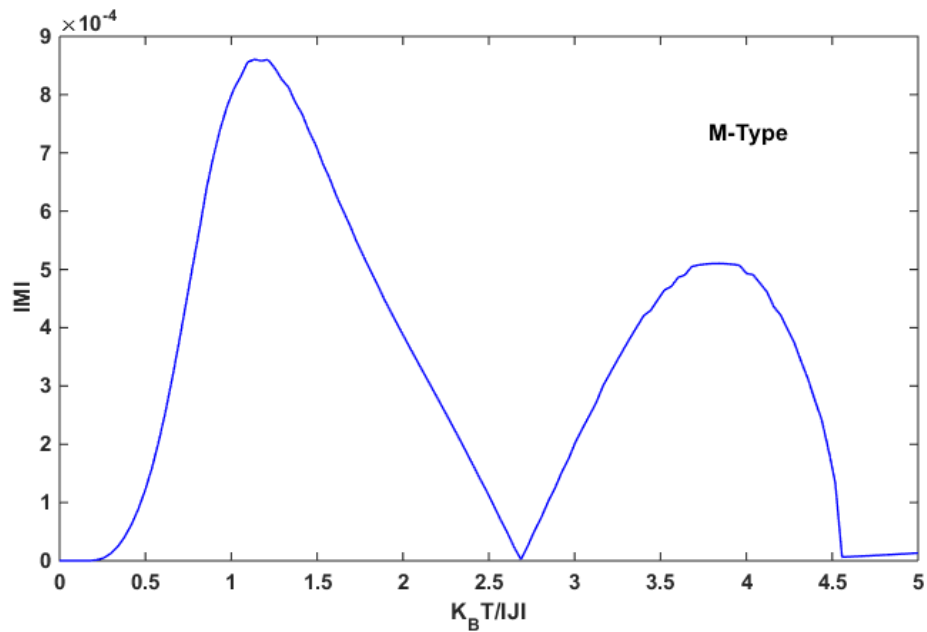
**Fig. 6:** Dependencies on temperature of the total magnetization  $M$  for the mixed ferrimagnet  $z = 4$ , when the value of  $D_A/|J| = -2.0$ , and  $D_B/|J| = -1.999$ .



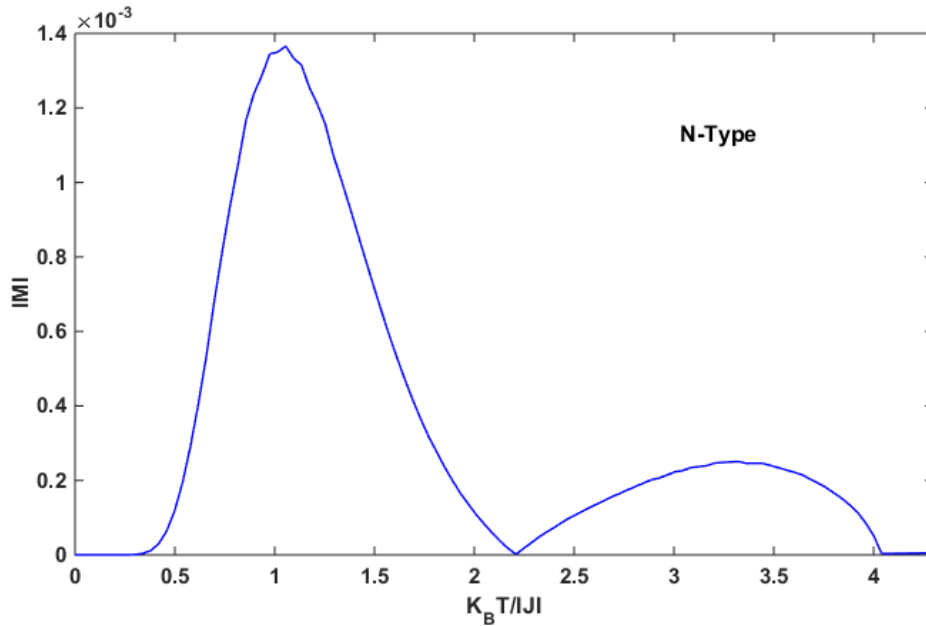
**Fig. 7:** Total magnetization  $M$  temperature dependences for the mixed ferrimagnet  $z = 4$  with  $D_A/|J| = -2.999$  and  $D_B/|J| = -3.0$ .



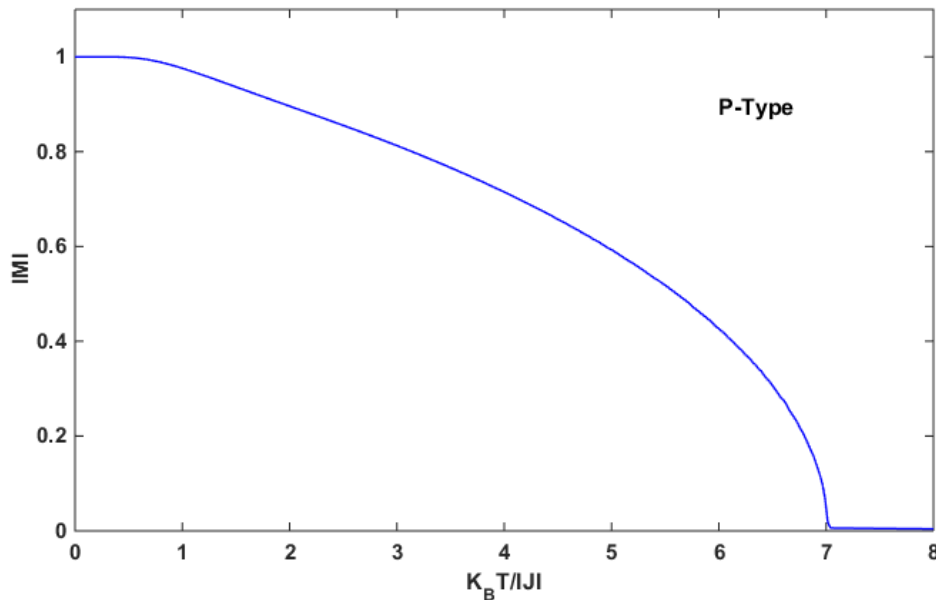
**Fig. 8:** Absolute magnetization value  $|M|$  temperature dependences for the mixed ferrimagnet  $z = 4$  with  $DA/|J| = 1.5$  and  $DB/|J| = -0.5$ .



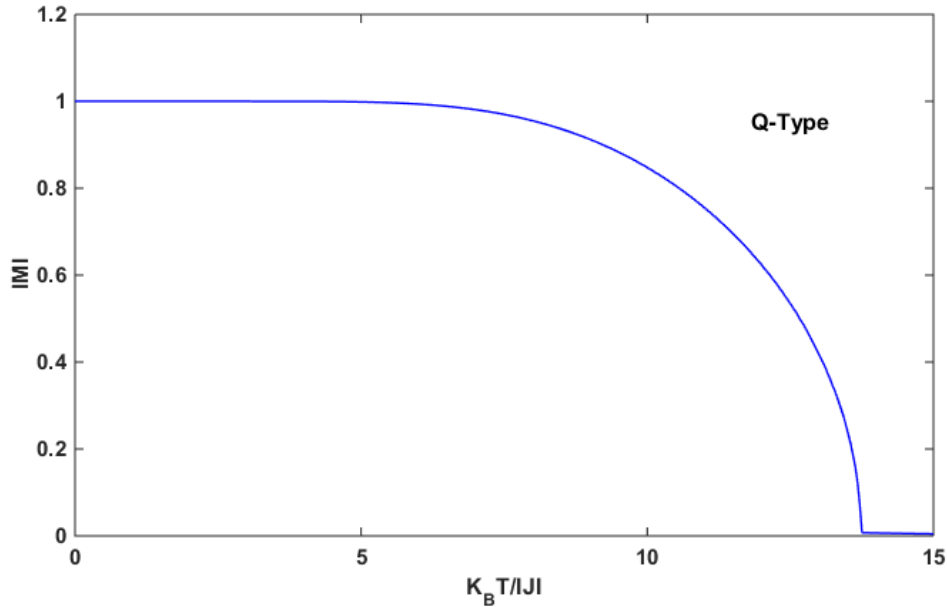
**Fig. 9:** Absolute magnetization value  $|M|$  temperature dependences for the mixed ferrimagnet  $z = 4$  with  $DA/|J| = -0.5$  and  $DB/|J| = -1.999$ .



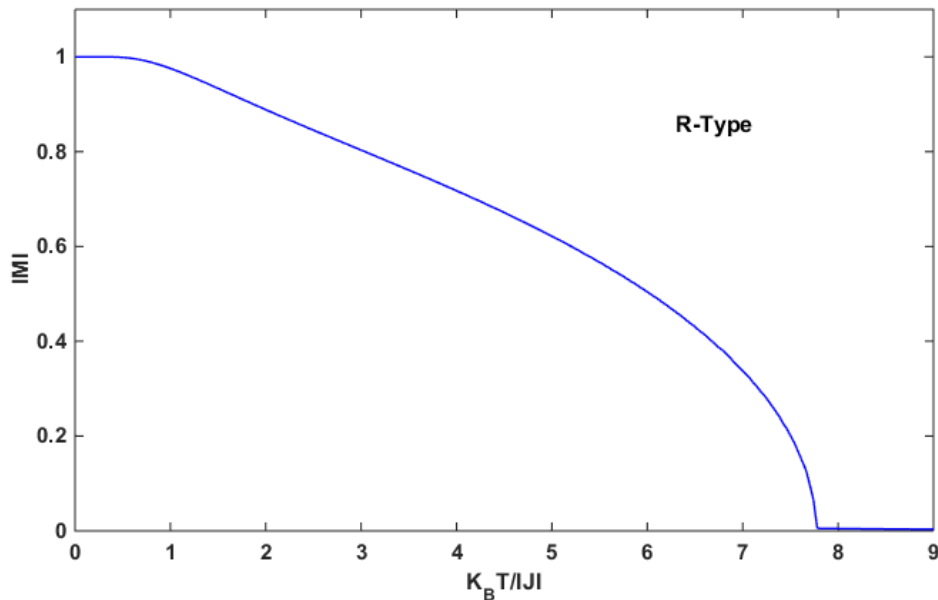
**Fig. 10:** Temperature dependences of the absolute magnetization value  $|M|$  for the mixed ferrimagnet  $z = 4$ , when the values of  $D_A/|J| = -1.0$ , and  $D_B/|J| = -2.0$ .



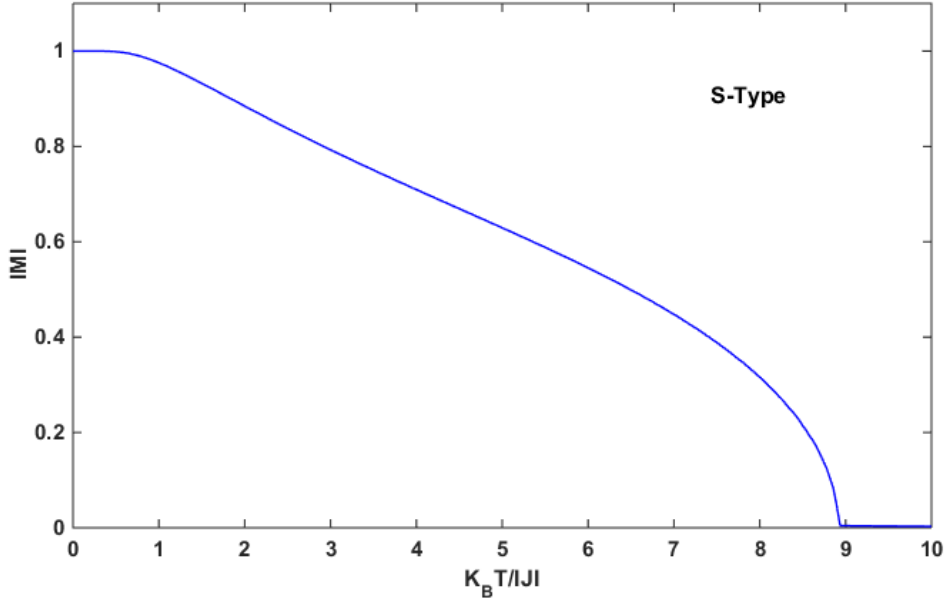
**Fig. 11:** Absolute magnetization value  $|M|$  temperature dependences for the mixed ferrimagnet  $z = 4$  with  $D_A/|J| = -3.5$  and  $D_B/|J| = -0.5$ .



**Fig. 12:** Temperature dependences of the absolute magnetization value  $|M|$  for the mixed ferrimagnet  $z = 4$ , when the values of  $D_A/|J| = 2.0$ , and  $D_B/|J| = 2.0$ .



**Fig. 13:** Dependencies on temperature of the total magnetization  $M$  for the mixed ferrimagnet  $z = 4$ , when the value of  $D_A/|j| = -2.5$ , and  $D_B/|J| = -0.5$ .



**Fig. 14:** Temperature dependences of the mixture of material's total magnetization  $M$  ferrimagnet  $z = 4$ , when the value of  $D_A/|J| = -0.5$ , and  $D_B/|J| = 0.5$ .

demonstrates the thermal evolution of sublattice magnetizations  $m_A$  (Cr) and  $m_B$  (Gd) at  $D_A/|J| = 0$  for varying  $D_B/|J|$  values, revealing multiple magnetic phases. The identified phases include  $(3/2, -1/2)$ ,  $(3/2, -1)$ ,  $(3/2, -3/2)$ ,  $(3/2, -2)$ ,  $(3/2, -5/2)$ ,  $(3/2, -3)$ , and  $(3/2, -7/2)$ . Three hybrid phase transitions  $(-1, -2, -3)$  are observed within the anisotropy range 1 to  $-3.5$ , consistent with second-order transitions.

In Fig. 3, the influence of varying  $D_A/|J|$  is examined at fixed  $D_B/|J| = 0$ , showcasing a transition from second- to first-order behavior. Figures 4 and 5 detail the thermal response of the absolute total magnetization  $|M|$  across different crystal field values. Notably, behaviors classified as types Q, R, and S emerge for  $D_B/|J|$  values between 1 and  $-3.5$ . At  $D_A/|J| = -6$ , a P-type response is observed.

Figures 6 through 14 provide a comprehensive view of the system's thermal magnetization response for various  $D_A$  and  $D_B$  combinations. The proximity of  $D_A/|J|$  and  $D_B/|J|$  in Fig. 6 suggests nearness to a critical point. Compensation points—where total magnetization vanishes at nonzero temperature—are evident in Fig. 7. Overall, the model exhibits seven distinct magnetic behavior types: L, M, N, P, Q, R, and S.

#### 4 Conclusion:

Using mean-field theory, we analyzed the thermodynamic behavior of a mixed spin-3/2 and spin-7/2 Blume-Capel Ising model on a binary square lattice (coordination number  $z = 4$ ). The study focused on the effects of crystal field anisotropies  $D_A$  and  $D_B$  on critical temperatures and magnetization profiles. A variety of phase transitions, including second- and first-order types, were observed. Specific combinations of anisotropies yielded complex magnetic responses, with notable behaviors such as types Q, R, S, and P. It has been observed the range of these crystal fields  $D_B/|J|$  (1 to -3.5), gave us interesting behaviors as types Q, R, and type S respectively. Besides, Fig.5 shows changes in absolute magnetization's temperature value  $|M|$  but in the case with  $D_B/|J|=0$ , when the value of  $D_A/|J|$  is changed in the range of (-6 to -8), one can see P-type at  $D_A/|J|=-6$ . The observed fluctuations are shown between temperature and total magnetization  $M$ . Interpretation of the phase transition, from a ferrimagnetic to a paramagnetic state.

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