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Outcome for the partition (9,5)

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Abstract:
For the partition (9,5) in this studied we prove that the sequence of the Weyl (W.) resolution (R.) elements is exact by using letter place algebra and place polarization (p.p.) after we find the characteristic free (c.f.) resolution elements.

Keywords: Divided power algebra, characteristic free, resolution, Weyl resolution, polarization, letter place algebra.

Introduction:

Authors in [1] display the fundamental concepts in homological algebra the divided power algebra (d.p.a.) $\mathfrak{D}_i\mathcal{F}$, $\Lambda\mathcal{F}$ the exterior algebra of $\mathcal{F}$, of degree i, $\mathcal{K}_{\lambda/\mu}\mathcal{F} = \mathrm{Im}(d'_{\lambda/\mu})$ where $d'_{\lambda/\mu}$ is the W. map $d'_{\lambda/\mu}$: $\mathfrak{D}\mathcal{F} \longrightarrow \Lambda\mathcal{F}$ and d.p.a. of the p.p. $(\partial_{21}^{(k)}) = (\mathcal{Z}_{21}x)^{(k)}$.

In this article $\mathcal{R}$ has unity and commutative ring, $\mathcal{F}$ free $\mathcal{R}$-module. Authors in [2,3,4] studied the contracting homotopy for the partitions (3,3), (8,7) and (6,4) respectively. In this work we consider the partition (9,5) and find the c.f.R. and by using some of homological form we prove that the sequence of the W.R. elements is exact. We denote $\mathcal{P}_{\mathfrak{f}_2}^{|\mathfrak{f}|}$ for $\binom{|\mathfrak{f}|}{\mathfrak{f}_2}$.

Basic Concepts

Definition (1): [1]

A partition of length $n = \ell(\lambda)$ is a sequence $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_n)$ of non -negative integers in non-increasing order $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n > 0$.

Definition (2): [1]

The formula $w = \begin{pmatrix} w_1 & 1^{(\beta_1)} \\ w_2 & 2^{(\beta_2)} \\ \vdots & \vdots \\ w_n & n^{(\beta_n)} \end{pmatrix} \in \mathcal{D}_{\beta_1}\mathcal{F} \otimes \mathcal{D}_{\beta_2}\mathcal{F} \otimes \dots \otimes \mathcal{D}_{\beta_n}\mathcal{F}$

is the letter place algebra for the partition (p, q), where w is the tensor product of a basis element w_1 in degree β_1 in the first factor, w_2 in degree β_2 in the second factor and so on w_n of degree β_n in the last factor.

Theorem (3): [1]

For a partition $(p, q; t)$, if we take (t) graded strand of degree q, then the modules of the R. as $\mathcal{M}_i$ for $i = 0,1,\dots, q - t$, with $Z_{21}^{(k)}$ d.p.a. of degree k, $\mathcal{M}_0 = \mathcal{D}_p \otimes \mathcal{D}_q$, and

$\mathcal{M}_i = Z_{21}^{(t+k_1)} x Z_{21}^{(k_2)} x \dots Z_{21}^{(k_i)} x\, \mathcal{D}_{p+t+|k|} \otimes \mathcal{D}_{q-t-|k|}$, for $i \geq 1$.

$\mathcal{S}_0 : \mathcal{D}_p \otimes \mathcal{D}_q \longrightarrow \sum_{k>0} Z^{(t+k)} x \mathcal{D}_{p+t+k} \otimes \mathcal{D}_{q-t-k}$

$$\begin{pmatrix} w & 1^{(p)} \\ w' & 2^{(q-k)} \end{pmatrix} 2^{(k)} \longrightarrow \begin{cases} 0 & ; \text{ if } \quad k \leq t \\ Z_{21}^{(k)} x \begin{pmatrix} w & 1^{(p+k)} \\ w' & 2^{(q-k)} \end{pmatrix} & ; \text{ if } \quad k > t \end{cases}$$

And

$$\mathcal{S}_{\ell-1} : \sum_{k_i>0} Z_{21}^{(t+k_1)} x Z_{21}^{(k_2)} x \dots Z_{21}^{(k_{\ell-1})} x\, \mathcal{D}_{p+t+|k|} \otimes \mathcal{D}_{q-t-|k|}$$
$$\longrightarrow Z_{21}^{(t+k_1)} x Z_{21}^{(k_2)} x \dots Z_{21}^{(k_{\ell-1})} x\, Z_{21}^{(k_\ell)} x\, \mathcal{D}_{p+t+|k|} \otimes \mathcal{D}_{q-t-|k|}$$

$$Z_{21}^{(t+k_1)} x Z_{21}^{(k_2)} x \dots Z_{21}^{(k_{\ell-1})} x \begin{pmatrix} w & 1^{(p+t+|k|)} \\ w' & 2^{(q-t-|k|-m)} \end{pmatrix} 2^{(m)}$$

$$\longrightarrow \begin{cases} 0 & ;\text{if} \quad m=0 \\ Z_{21}^{(t+k_1)} x Z_{21}^{(k_2)} x \dots Z_{21}^{(k_{\ell-1})} x\, Z_{21}^{(m)} x \begin{pmatrix} w \\ w' \end{pmatrix} \Big| \begin{matrix} 1^{(p+t+|k|+m)} \\ 2^{(q-t-|k|-m)} \end{matrix} & ;\text{if} \quad m>0 \end{cases}$$

The W.R. Elements

The elements of c.f. W.R. for the partition (9,5) are

$\mathcal{M}_0 = \mathfrak{D}_9 \otimes \mathfrak{D}_5$,

$\mathcal{M}_1 = (Z_{21}x)^{(b)} \mathfrak{D}_{9+b} \otimes \mathfrak{D}_{5-b}$; where $1 \le b \le 5$

$\mathcal{M}_2 = (Z_{21}x)^{(b_1)} (Z_{21}x)^{(b_2)} \mathfrak{D}_{9+|b|} \otimes \mathfrak{D}_{5-|b|}$; where $2 \le |b| = b_1 + b_2 \le 5$

$\mathcal{M}_3 = (Z_{21}x)^{(b_1)} (Z_{21}x)^{(b_2)} (Z_{21}x)^{(b_3)} \mathfrak{D}_{9+|b|} \otimes \mathfrak{D}_{5-|b|}$; where $3 \le |b| = \sum_{i=1}^{3} b_i \le 5$

$\mathcal{M}_3 = (Z_{21}x)^{(b_1)} (Z_{21}x)^{(b_2)} (Z_{21}x)^{(b_3)} (Z_{21}x)^{(b_4)} \mathfrak{D}_{9+|b|} \otimes \mathfrak{D}_{5-|b|}$; where $4 \le |b| = \sum_{i=1}^{3} b_i \le 5$

$\mathcal{M}_5 = Z_{21}x Z_{21}x\, Z_{21}x Z_{21}x Z_{21}x \mathfrak{D}_{14} \otimes \mathfrak{D}_0$

The homotopies $\{\mathcal{S}_i\}$; where $i = 0,1,2,3,4$ below with prove that they are contracting

$$\mathcal{S}_0 \left(\begin{pmatrix} w \\ w' \end{pmatrix} \Big| \begin{matrix} 1^{(9)} & 2^{(\mathfrak{v})} \\ & 2^{(5-\mathfrak{v})} \end{matrix} \right) = \begin{cases} 0 & ;\text{if} \quad \mathfrak{v} \le 0 \\ (Z_{21}x)(\mathfrak{v}) \begin{pmatrix} w \\ w' \end{pmatrix} \Big| \begin{matrix} 1^{(9+|\mathfrak{v}|)} \\ 2^{(5-\mathfrak{v})} \end{matrix} & ;\text{if} \quad \mathfrak{v} = 1,2,3,4,5 \end{cases}$$

$$\mathcal{S}_1 \left((Z_{21}x)(\mathfrak{v}) \begin{pmatrix} w \\ w' \end{pmatrix} \Big| \begin{matrix} 1^{(9+|\mathfrak{v}|)} & 2^{(\mathfrak{g})} \\ & 2^{(5-|\mathfrak{v}|-\mathfrak{g})} \end{matrix} \right) =$$

$$\begin{cases} 0 & ;\text{if} \quad \mathfrak{g}=0 \\ (Z_{21}x)\big((\mathfrak{v})(\mathfrak{g})\big) \begin{pmatrix} w \\ w' \end{pmatrix} \Big| \begin{matrix} 1^{(9+|\mathfrak{v}|+\mathfrak{g})} \\ 2^{(5-|\mathfrak{v}|-\mathfrak{g})} \end{matrix} & ;\text{if} \quad \mathfrak{g} = 1,2,3,4 \end{cases}$$

For $|\mathfrak{v}| = \sum_{e=1}^{2} \mathfrak{v}_e$,

$$\mathcal{S}_2 \left((Z_{21}x)\big((\mathfrak{v}_1)(\mathfrak{v}_2)\big) \begin{pmatrix} w \\ w' \end{pmatrix} \Big| \begin{matrix} 1^{(9+|\mathfrak{v}|)} & 2^{(\mathfrak{g})} \\ & 2^{(5-|\mathfrak{v}|-\mathfrak{g})} \end{matrix} \right)$$

$$= \begin{cases} 0 & ; \text{ if } \mathfrak{g} = 0 \\ (\mathcal{Z}_{21}x)\big((\mathfrak{v}_1)(\mathfrak{v}_2)(\mathfrak{g})\big)\begin{pmatrix} w \\ w' \end{vmatrix} 1^{(9+|\mathfrak{v}|+\mathfrak{g})} \\ 2^{(5-|\mathfrak{v}|-\mathfrak{g})} \end{pmatrix} & ; \text{ if } \mathfrak{g} = 1,2,3 \end{cases}$$

For $|\mathfrak{v}| = \sum_{e=1}^{3} \mathfrak{v}_e$,

$$\mathcal{S}_3\left((\mathcal{Z}_{21}x)((\mathfrak{v}_1)(\mathfrak{v}_2)(\mathfrak{v}_3))\begin{pmatrix} w \\ w' \end{vmatrix} 1^{(9+|\mathfrak{v}|)} \quad 2^{(\mathfrak{g})} \\ 2^{(5-|\mathfrak{v}|-\mathfrak{g})} \end{pmatrix}\right)$$

$$= \begin{cases} 0 & ; \text{ if } \mathfrak{g} = 0 \\ (\mathcal{Z}_{21}x)\big((\mathfrak{v}_1)(\mathfrak{v}_2)(\mathfrak{v}_3)(\mathfrak{g})\big)\begin{pmatrix} w \\ w' \end{vmatrix} 1^{(9+|\mathfrak{v}|+\mathfrak{g})} \\ 2^{(5-|\mathfrak{v}|-\mathfrak{g})} \end{pmatrix}; \text{ if } \mathfrak{g} = 1,2 \end{cases}$$

For $|\mathfrak{v}| = \sum_{e=1}^{4} \mathfrak{v}_e$,

$$\mathcal{S}_4\left((\mathcal{Z}_{21}x)((\mathfrak{v}_1)(\mathfrak{v}_2)(\mathfrak{v}_3)(\mathfrak{v}_4))\begin{pmatrix} w \\ w' \end{vmatrix} 1^{(9+|\mathfrak{v}|)} \quad 2^{(\mathfrak{g})} \\ 2^{(5-|\mathfrak{v}|-\mathfrak{g})} \end{pmatrix}\right)$$

$$= \begin{cases} 0 & ; \text{ if } \mathfrak{g} = 0 \\ (\mathcal{Z}_{21}x)\big((\mathfrak{v}_1)(\mathfrak{v}_2)(\mathfrak{v}_3)(\mathfrak{v}_4)(\mathfrak{g})\big)\begin{pmatrix} w \\ w' \end{vmatrix} 1^{(9+|\mathfrak{v}|+\mathfrak{g})} \\ 2^{(5-|\mathfrak{v}|-\mathfrak{g})} \end{pmatrix} & ; \text{ if } \mathfrak{g} = 1 \end{cases}$$

Now we prove that, $\mathcal{S}_{i-1}\partial_x + \partial_x\mathcal{S}_i = \mathrm{id}_{\mathcal{M}_i}$, $i = 1, 2, 3, 4$.

$$\mathcal{S}_0\partial_x\left((\mathcal{Z}_{21}x)(\mathfrak{v})\begin{pmatrix} w \\ w' \end{vmatrix} 1^{(9+\mathfrak{v})} \quad 2^{(\mathfrak{g})} \\ 2^{(5-\mathfrak{v}-\mathfrak{g})} \end{pmatrix}\right) = \binom{\mathfrak{v}+\mathfrak{g}}{\mathfrak{g}}(\mathcal{Z}_{21}x)(\mathfrak{v}+\mathfrak{g})\begin{pmatrix} w \\ w' \end{vmatrix} 1^{(9+\mathfrak{v}+\mathfrak{g})} \\ 2^{(5-\mathfrak{v}-\mathfrak{g})} \end{pmatrix},$$ and

$$\partial_x\mathcal{S}_1\left((\mathcal{Z}_{21}x)(\mathfrak{v})\begin{pmatrix} w \\ w' \end{vmatrix} 1^{(9+\mathfrak{v})} \quad 2^{(\mathfrak{g})} \\ 2^{(5-\mathfrak{v}-\mathfrak{g})} \end{pmatrix}\right)$$

$$= -\binom{\mathfrak{v}+\mathfrak{g}}{\mathfrak{g}}(\mathcal{Z}_{21}x)(\mathfrak{v}+\mathfrak{g})\begin{pmatrix} w \\ w' \end{vmatrix} 1^{(9+\mathfrak{v}+\mathfrak{g})} \\ 2^{(5-\mathfrak{v}-\mathfrak{g})} \end{pmatrix} +$$

$$(\mathcal{Z}_{21}x)(\mathfrak{v})\begin{pmatrix} w \\ w' \end{vmatrix} 1^{(9+\mathfrak{v})} \quad 2^{(\mathfrak{g})} \\ 2^{(5-\mathfrak{v}-\mathfrak{g})} \end{pmatrix}.$$

Hence, $\mathcal{S}_0\partial_x + \partial_x\mathcal{S}_1 = \mathrm{id}_{\mathcal{M}_1}$.

For $|\mathfrak{f}| = \sum_{e=1}^{2} \mathfrak{f}_e$,

$$\mathcal{S}_1\partial_x\left((\mathcal{Z}_{21}x)((\mathfrak{f}_1)(\mathfrak{f}_2))\begin{pmatrix} w \\ w' \end{vmatrix} 1^{(9+|\mathfrak{f}|)} \quad 2^{(\mathfrak{g})} \\ 2^{(5-|\mathfrak{f}|-\mathfrak{g})} \end{pmatrix}\right)$$

$$= -\mathcal{P}_{\mathfrak{f}_2}^{|\mathfrak{f}|}(\mathcal{Z}_{21}x)^{(|\mathfrak{f}|)(\mathfrak{g})}\begin{pmatrix} w \\ w' \end{vert} \begin{matrix} 1^{(9+|\mathfrak{f}|+\mathfrak{g})} \\ 2^{(5-|\mathfrak{f}|-\mathfrak{g})} \end{matrix}\Bigg) +$$

$$\mathcal{P}_{\mathfrak{g}}^{\mathfrak{f}_2+\mathfrak{g}}(\mathcal{Z}_{21}x)^{((\mathfrak{f}_1)(\mathfrak{f}_2+\mathfrak{g}))}\begin{pmatrix} w \\ w' \end{vert} \begin{matrix} 1^{(9+|\mathfrak{v}|+\mathfrak{g})} \\ 2^{(5-|\mathfrak{v}|-\mathfrak{g})} \end{matrix}\Bigg), \text{ and}$$

$$\partial_x \mathcal{S}_2\left((\mathcal{Z}_{21}x)^{((\mathfrak{f}_1)(\mathfrak{f}_2))}\begin{pmatrix} w \\ w' \end{vert} \begin{matrix} 1^{(9+|\mathfrak{f}|)} & 2^{(\mathfrak{g})} \\ 2^{(5-|\mathfrak{f}|-\mathfrak{g})} & \end{matrix}\Bigg)\right)$$

$$= \mathcal{P}_{\mathfrak{f}_2}^{|\mathfrak{f}|}(\mathcal{Z}_{21}x)^{((|\mathfrak{f}|)(\mathfrak{g}))}\begin{pmatrix} w \\ w' \end{vert} \begin{matrix} 1^{(9+|\mathfrak{f}|+\mathfrak{g})} \\ 2^{(5-|\mathfrak{f}|-\mathfrak{g})} \end{matrix}\Bigg) -$$

$$\mathcal{P}_{\mathfrak{g}}^{\mathfrak{f}_2+\mathfrak{g}}(\mathcal{Z}_{21}x)^{((\mathfrak{f}_1)(\mathfrak{f}_2+\mathfrak{g}))}\begin{pmatrix} w \\ w' \end{vert} \begin{matrix} 1^{(9+|\mathfrak{f}|+\mathfrak{g})} \\ 2^{(5-|\mathfrak{f}|-\mathfrak{g})} \end{matrix}\Bigg) +$$

$$(\mathcal{Z}_{21}x)^{((\mathfrak{f}_1)(\mathfrak{f}_2))}\begin{pmatrix} w \\ w' \end{vert} \begin{matrix} 1^{(9+|\mathfrak{f}|)} & 2^{(\mathfrak{g})} \\ 2^{(5-|\mathfrak{f}|-\mathfrak{g})} & \end{matrix}\Bigg).$$

Hence $\mathcal{S}_1\partial_x + \partial_x\mathcal{S}_2 = \mathrm{id}_{\mathcal{M}_2}$

For $|\mathfrak{f}| = \sum_{e=1}^{3}\mathfrak{f}_e$, $\mathcal{S}_2\partial_x\left((\mathcal{Z}_{21}x)^{((\mathfrak{f}_1)(\mathfrak{f}_2)(\mathfrak{f}_3))}\begin{pmatrix} w \\ w' \end{vert} \begin{matrix} 1^{(9+|\mathfrak{f}|)} & 2^{(\mathfrak{g})} \\ 2^{(5-|\mathfrak{f}|-\mathfrak{g})} & \end{matrix}\Bigg)\right)$

$$= \mathcal{P}_{\mathfrak{f}_2}^{\mathfrak{f}_1+\mathfrak{f}_2}(\mathcal{Z}_{21}x)^{((\mathfrak{f}_1+\mathfrak{f}_2)(\mathfrak{f}_3)(\mathfrak{g}))}\begin{pmatrix} w \\ w' \end{vert} \begin{matrix} 1^{(9+|\mathfrak{f}|+\mathfrak{g})} \\ 2^{(5-|\mathfrak{f}|-\mathfrak{g})} \end{matrix}\Bigg) -$$

$$\mathcal{P}_{\mathfrak{f}_3}^{\mathfrak{f}_2+\mathfrak{f}_3}(\mathcal{Z}_{21}x)^{((\mathfrak{f}_1)(\mathfrak{f}_2+\mathfrak{f}_3)(\mathfrak{g}))}\begin{pmatrix} w \\ w' \end{vert} \begin{matrix} 1^{(9+|\mathfrak{v}|+\mathfrak{g})} \\ 2^{(5-|\mathfrak{v}|-\mathfrak{g})} \end{matrix}\Bigg) +$$

$$\mathcal{P}_{\mathfrak{g}}^{\mathfrak{f}_3+\mathfrak{g}}(\mathcal{Z}_{21}x)^{((\mathfrak{f}_1)(\mathfrak{f}_2)(\mathfrak{f}_3+\mathfrak{g}))}\begin{pmatrix} w \\ w' \end{vert} \begin{matrix} 1^{(9+|\mathfrak{v}|+\mathfrak{g})} \\ 2^{(5-|\mathfrak{v}|-\mathfrak{g})} \end{matrix}\Bigg), \text{ and}$$

$$\partial_x \mathcal{S}_3\left((\mathcal{Z}_{21}x)^{((\mathfrak{f}_1)(\mathfrak{f}_2)(\mathfrak{f}_3))}\begin{pmatrix} w \\ w' \end{vert} \begin{matrix} 1^{(9+|\mathfrak{v}|)} & 2^{(\mathfrak{g})} \\ 2^{(5-|\mathfrak{v}|-\mathfrak{g})} & \end{matrix}\Bigg)\right)$$

$$= -\mathcal{P}_{\mathfrak{f}_2}^{\mathfrak{f}_1+\mathfrak{f}_2}(\mathcal{Z}_{21}x)^{((\mathfrak{f}_1+\mathfrak{f}_2)(\mathfrak{f}_3)(\mathfrak{g}))}\begin{pmatrix} w \\ w' \end{vert} \begin{matrix} 1^{(9+|\mathfrak{f}|+\mathfrak{g})} \\ 2^{(5-|\mathfrak{f}|-\mathfrak{g})} \end{matrix}\Bigg) +$$

$$\mathcal{P}_{\mathfrak{f}_3}^{\mathfrak{f}_2+\mathfrak{f}_3}(\mathcal{Z}_{21}x)^{((\mathfrak{f}_1)(\mathfrak{f}_2+\mathfrak{f}_3)(\mathfrak{g}))}\begin{pmatrix} w \\ w' \end{vert} \begin{matrix} 1^{(9+|\mathfrak{f}|+\mathfrak{g})} \\ 2^{(5-|\mathfrak{f}|-\mathfrak{g})} \end{matrix}\Bigg) -$$

$$\mathcal{P}_{\mathfrak{g}}^{\mathfrak{f}_3+\mathfrak{g}}(\mathcal{Z}_{21}x)^{((\mathfrak{f}_1)(\mathfrak{f}_2)(\mathfrak{f}_3+\mathfrak{g}))}\begin{pmatrix} w \\ w' \end{vert} \begin{matrix} 1^{(9+|\mathfrak{f}|+\mathfrak{g})} \\ 2^{(5-|\mathfrak{f}|-\mathfrak{g})} \end{matrix}\Bigg) +$$

$$(\mathcal{Z}_{21}x)^{((\mathfrak{f}_1)(\mathfrak{f}_2)(\mathfrak{f}_3))}\begin{pmatrix} w \\ w' \end{vert} \begin{matrix} 1^{(9+|\mathfrak{f}|)} & 2^{(\mathfrak{g})} \\ 2^{(5-|\mathfrak{f}|-\mathfrak{g})} & \end{matrix}\Bigg).$$

Hence, $\mathcal{S}_2\partial_x + \partial_x\mathcal{S}_3 = \mathrm{id}_{\mathcal{M}_3}$.

For $|\mathfrak{f}| = \sum_{e=1}^{4} \mathfrak{f}_e$,

$$\mathcal{S}_3\partial_x\left((\mathcal{Z}_{21}x)^{((\mathfrak{f}_1)(\mathfrak{f}_2)(\mathfrak{f}_3)(\mathfrak{f}_4))}\begin{pmatrix}\mathcal{w}\\ \mathcal{w}'\end{vmatrix}1^{(9+|\mathfrak{f}|)} \quad 2^{(\mathfrak{g})}\\ \quad 2^{(5-|\mathfrak{f}|-\mathfrak{g})}\end{pmatrix}\right)$$

$$= -\mathcal{P}_{\mathfrak{f}_2}^{\mathfrak{f}_1+\mathfrak{f}_2}(\mathcal{Z}_{21}x)^{((\mathfrak{f}_1+\mathfrak{f}_2)(\mathfrak{f}_3)(\mathfrak{f}_4)(\mathfrak{g}))}\begin{pmatrix}\mathcal{w}\\ \mathcal{w}'\end{vmatrix}1^{(9+|\mathfrak{f}|+\mathfrak{g})}\\ 2^{(5-|\mathfrak{f}|-\mathfrak{g})}\end{pmatrix} +$$

$$\mathcal{P}_{\mathfrak{f}_3}^{\mathfrak{f}_2+\mathfrak{f}_3}(\mathcal{Z}_{21}x)^{((\mathfrak{f}_1)(\mathfrak{f}_2+\mathfrak{f}_3)(\mathfrak{f}_4)(\mathfrak{g}))}\begin{pmatrix}\mathcal{w}\\ \mathcal{w}'\end{vmatrix}1^{(9+|\mathfrak{f}|+\mathfrak{g})}\\ 2^{(5-|\mathfrak{f}|-\mathfrak{g})}\end{pmatrix} -$$

$$\mathcal{P}_{\mathfrak{f}_4}^{\mathfrak{f}_3+\mathfrak{f}_4}(\mathcal{Z}_{21}x)^{((\mathfrak{f}_1)(\mathfrak{f}_2)(\mathfrak{f}_3+\mathfrak{f}_4)(\mathfrak{g}))}\begin{pmatrix}\mathcal{w}\\ \mathcal{w}'\end{vmatrix}1^{(9+|\mathfrak{f}|+\mathfrak{g})}\\ 2^{(5-|\mathfrak{f}|-\mathfrak{g})}\end{pmatrix} +$$

$$\mathcal{P}_{\mathfrak{g}}^{\mathfrak{f}_4+\mathfrak{g}}(\mathcal{Z}_{21}x)^{((\mathfrak{f}_1)(\mathfrak{f}_2)(\mathfrak{f}_3)(\mathfrak{f}_4+\mathfrak{g}))}\begin{pmatrix}\mathcal{w}\\ \mathcal{w}'\end{vmatrix}1^{(9+|\mathfrak{f}|+\mathfrak{g})}\\ 2^{(5-|\mathfrak{f}|-\mathfrak{g})}\end{pmatrix},$$

and

$$\partial_x\mathcal{S}_4\left((\mathcal{Z}_{21}x)^{((\mathfrak{f}_1)(\mathfrak{f}_2)(\mathfrak{f}_3)(\mathfrak{f}_4))}\begin{pmatrix}\mathcal{w}\\ \mathcal{w}'\end{vmatrix}1^{(9+|\mathfrak{f}|)} \quad 2^{(\mathfrak{g})}\\ \quad 2^{(5-|\mathfrak{f}|-\mathfrak{g})}\end{pmatrix}\right)$$

$$= \mathcal{P}_{\mathfrak{f}_2}^{\mathfrak{f}_1+\mathfrak{f}_2}(\mathcal{Z}_{21}x)^{((\mathfrak{f}_1+\mathfrak{f}_2)(\mathfrak{f}_3)(\mathfrak{f}_4)(\mathfrak{g}))}\begin{pmatrix}\mathcal{w}\\ \mathcal{w}'\end{vmatrix}1^{(9+|\mathfrak{f}|+\mathfrak{g})}\\ 2^{(5-|\mathfrak{f}|-\mathfrak{g})}\end{pmatrix} -$$

$$\mathcal{P}_{\mathfrak{f}_3}^{\mathfrak{f}_2+\mathfrak{f}_3}(\mathcal{Z}_{21}x)^{((\mathfrak{f}_1)(\mathfrak{f}_2+\mathfrak{f}_3)(\mathfrak{f}_4)(\mathfrak{g}))}\begin{pmatrix}\mathcal{w}\\ \mathcal{w}'\end{vmatrix}1^{(9+|\mathfrak{f}|+\mathfrak{g})}\\ 2^{(5-|\mathfrak{f}|-\mathfrak{g})}\end{pmatrix} +$$

$$\mathcal{P}_{\mathfrak{f}_4}^{\mathfrak{f}_3+\mathfrak{f}_4}(\mathcal{Z}_{21}x)^{((\mathfrak{f}_1)(\mathfrak{f}_2)(\mathfrak{f}_3+\mathfrak{f}_4)(\mathfrak{g}))}\begin{pmatrix}\mathcal{w}\\ \mathcal{w}'\end{vmatrix}1^{(9+|\mathfrak{f}|+\mathfrak{g})}\\ 2^{(5-|\mathfrak{f}|-\mathfrak{g})}\end{pmatrix} -$$

$$\mathcal{P}_{\mathfrak{g}}^{\mathfrak{f}_4+\mathfrak{g}}(\mathcal{Z}_{21}x)^{((\mathfrak{f}_1)(\mathfrak{f}_2)(\mathfrak{f}_3)(\mathfrak{f}_4+\mathfrak{g}))}\begin{pmatrix}\mathcal{w}\\ \mathcal{w}'\end{vmatrix}1^{(9+|\mathfrak{f}|+\mathfrak{g})}\\ 2^{(5-|\mathfrak{f}|-\mathfrak{g})}\end{pmatrix} +$$

$$(\mathcal{Z}_{21}x)^{((\mathfrak{f}_1)(\mathfrak{f}_2)(\mathfrak{f}_3)(\mathfrak{f}_4))}\begin{pmatrix}\mathcal{w}\\ \mathcal{w}'\end{vmatrix}1^{(9+|\mathfrak{f}|)} \quad 2^{(\mathfrak{g})}\\ \quad 2^{(5-|\mathfrak{f}|-\mathfrak{g})}\end{pmatrix}.$$

Hence, $\mathcal{S}_3\partial_x + \partial_x\mathcal{S}_4 = \mathrm{id}_{\mathcal{M}_4}$.

Thus for i = 0,1,2,3,4 we gain that $\{\mathcal{S}_i\}$; is a contracting homotopy [5], which implies the exactness of the complex $\mathcal{M}_i$.

Conclusion:

In this work we obtain the exactness of the sequence of the c.f.W.R. elements for the partition (9,5).

References

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نتيجة للتجزئة (9,5)

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ملاحظة : هل البحث مستل من رسالة ماجستير او اطروحة دكتوراه ؟ نعم : ✓ كلا :

مستخلص البحث:
للتجزئة (9,5) في هذه الدراسة نبرهن ان سلسلة عناصر تحلل وايل هي تامة باستخدام جبر مكان الحروف و مكان الاستقطاب بعد إيجاد عناصر تحلل المميز الحر.

الكلمات المفتاحية : جبر قوى القسمة، المميز الحر، تحلل، تحلل وايل، الاستقطاب، جبر مكان الحروف.