On the exact sequences of FW_6- pair of hooks representation modules over a field of characteristic 0

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Abstract:

The intention of this paper is to find the exact and split sequences of the sub modules of FW_6-second and third pair of hooks representation modules, when F be a field of characteristic 0 . This intention is revealing by presenting the main results theorems *(3-1)* and *(3-2)*, where we gave two of this kind of sequences.

$$0 \longrightarrow U_F^1 \xrightarrow{\delta} H_F^1 \xrightarrow{\varphi} G_F^1 \longrightarrow 0$$

$$0 \longrightarrow U_F^2 \xrightarrow{\eta} H_F^2 \xrightarrow{\theta} G_F^2 \longrightarrow 0$$

Here we refer that depending on the structural constructing of the elements (modules) of these sequences or it‘s sub modules by counting the dimensions was essential for proving these theorems.

Index: Second and third pair of hooks representation modules , Group algebra , Weyl group , Field of characteristic 0 , Homomophisms.

1-Introduction:-

Let F be a field of characteristic 0 , and W_6 is the Weyl group of type B_n of the set $\{\mp x_1 , \mp x_2 , \mp x_3 , \mp x_4 , \mp x_5 , \mp x_6\}$, where $x_1 , x_2 , x_3 , x_4 , x_5 , x_6$ are independent indeterminate over F.

The set FW_6 of all liner combination of the form $\sum_{\tau \in W_6} c_i \ \tau_i$, where $c_i \in F$ is the group algebra of Weyl group of type B_n .

The hook representation modules of the symmetric groups have been given first in 1971 by M. H. Peel in [6], and later in 1975 in [5]. In 1977 E.M.A. Al-Aamily presents in [1] the analogues of some results in [4] and [5] for the Weyl groups of type B_n. in 1981 E.M.A. Al-Aamily and F.A. Al-Tayar in [2] present the analogues of more result in the symmetric groups for the Weyl groups of type B_n.

The purpose of this paper is to find exact and split sequences of the sub modules of FW_6 – second and third pair of hooks representation modules. When F be a field of characteristic 0 .

Finally it be useful to refer that μ_r denotes to the monomial $x_i\, x_j\, x_k\, x_p\, x_q$, where $1 \le i < j < k < p < q \le 6$,and $\{x_r\} \cup \{x_i , x_j , x_k , x_p , x_q\} = \{x_1 , x_2 , x_3 , x_4 , x_5 , x_6\}$.

In another word μ_r denotes to the monomial $x_1x_2x_3x_4x_5x_6$ in which x_r is omit, where $r \in \{1, 2, 3, 4, 5, 6\}$.

2-Preliminaries:

2-1 :- Partitions and tableaux:-

The pairs of hook partitions of the number 6 are :

((6) , ( ))	(( ) , (6))
((5,1) , ( ))	(( ) , (5,1))
((4,1,1) , ( ))	(( ) , (4,1,1))
((3,1,1,1) , ( ))	(( ) , (3,1,1,1))
((2,1,1,1,1) , ( ))	(( ) , (2,1,1,1,1))
((1,1,1,1,1) , ( ))	(( ) , (1,1,1,1,1))
((5) , (1))	((1) , (5))
((4,1) , (1))	((1) , (4,1))
((3,1,1) , (1))	((1) , (3,1,1))
((2,1,1,1) , (1))	((1) , (2,1,1,1))
((1,1,1,1) , (1))	((1) , (1,1,1,1))
((4) , (2))	((2) , (4))
((3,1) , (2))	((2) , (3,1))
((2,1,1) , (2))	((2) , (2,1,1))
((1,1,1,1) , (2))	((2) , (1,1,1,1))
((4) , (1,1))	((1,1) , (4))
((3,1) , (1,1))	((1,1) , (3,1))
((2,1,1) , (1,1))	((1,1) , (2,1,1))
((1,1,1,1) , (1,1))	((1,1) , (1,1,1,1))
((3) , (3))	((2,1) , (2,1))
((3) , (2))	((2,1) , (3))
((3) , (1,1,1))	((1,1,1) , (3))
((2,1) , (1,1,1))	((1,1,1) , (2,1))
((1,1,1) , (1,1,1))	

* There are exactly 30 row standard tableaux corresponds to the pair of hook partition ((4,1) , (1)).

$x_1\, x_2\, x_3\, x_4$ x_6 x_5	$x_1\, x_2\, x_3\, x_5$ x_6 x_4	$x_1\, x_2\, x_3\, x_6$ x_4 x_5
$x_1\, x_2\, x_3\, x_4$ x_5 x_6	$x_1\, x_2\, x_3\, x_5$ x_4 x_6	$x_1\, x_2\, x_3\, x_6$ x_5 x_4
$x_1\, x_2\, x_4\, x_5$ x_3 x_6	$x_1\, x_2\, x_4\, x_6$ x_5 x_3	$x_1\, x_2\, x_5\, x_6$ x_4 x_3
$x_1\, x_3\, x_4\, x_5$ x_6 x_2	$x_1\, x_3\, x_4\, x_6$ x_5 x_2	$x_1\, x_3\, x_5\, x_6$ x_4 x_2

$x_1\ x_3\ x_4\ x_5 \quad x_2$ x_6	$x_1\ x_3\ x_4\ x_6 \quad x_2$ x_5	$x_1\ x_3\ x_5\ x_6 \quad x_2$ x_4
$x_1\ x_4\ x_5\ x_6 \quad x_3$ x_2	$x_2\ x_3\ x_4\ x_5 \quad x_6$ x_1	$x_2\ x_3\ x_4\ x_6 \quad x_5$ x_1
$x_1\ x_4\ x_5\ x_6 \quad x_2$ x_3	$x_2\ x_3\ x_4\ x_5 \quad x_1$ x_6	$x_2\ x_3\ x_4\ x_6 \quad x_1$ x_5
$x_2\ x_3\ x_5\ x_6 \quad x_4$ x_1	$x_3\ x_4\ x_5\ x_6 \quad x_2$ x_1	$x_2\ x_4\ x_5\ x_6 \quad x_3$ x_1
$x_2\ x_3\ x_5\ x_6 \quad x_1$ x_4	$x_3\ x_4\ x_5\ x_6 \quad x_1$ x_2	$x_2\ x_4\ x_5\ x_6 \quad x_1$ x_3
$x_1\ x_2\ x_4\ x_5 \quad x_6$ x_3	$x_1\ x_2\ x_4\ x_6 \quad x_3$ x_5	$x_1\ x_2\ x_5\ x_6 \quad x_3$ x_4

2-2 :- Some FW_6 – modules :

We are interesting in the following FW_6 – modules which we are dealing with in this paper.

i- M_F^1 and M_F^2 are the second pair of hooks representation modules corresponding to the pairs of partition ((4,1),(1)) and ((1),(4,1)) respectively.

M_F^1 is generated over FW_6 by $\mu_6 x_5^2$, and consists of all polynomials in x_1 , x_2 , x_3 , x_4 , x_5 , x_6 of the form :

$$\sum_{1 \le i < j < k < p < q \le 6} \mu_r \sum_{s=i}^{q} c_{i,j,k,p,q,s}\, x_s^2 \quad \text{, where } c_{i,j,k,p,q,s} \in F.$$

M_F^2 is generated over FW_6 by $x_6\, x_5^2$, and consists of all polynomials in x_1 , x_2 , x_3 , x_4 , x_5 , x_6 of the form:

$$\sum_{1 \le i < j < k < p < q \le 6} x_r \sum_{s=i}^{q} c_{i,j,k,p,q,s}\, x_s^2 \quad \text{, where } c_{i,j,k,p,q,s} \in F$$

and $\{x_r\} \cup \{x_i , x_j , x_k , x_p , x_q\} = \{x_1 , x_2 , x_3 , x_4 , x_5 , x_6\}$. *see [1]*

ii- N_F^1 and N_F^2 are the third pair of hooks representation modules corresponding to the pairs of partition ((3,1,1) , (1)) and ((1) , (3,1,1)) respectively.

N_F^1 is generated over FW_6 by $\mu_6 x_4^2\ x_5^4$ and consists of all polynomials in x_1 , x_2 , x_3 , x_4 , x_5 , x_6 of the form:

$$\sum_{1 \le i < j < k < p < q \le 6} \mu_r \sum_{s=i}^{q} \sum_{\substack{t=i \\ t \ne s}}^{q} c_{i,j,k,p,q,s,t}\ x_s^2\ x_t^4 \text{ , where } c_{i,j,k,p,q,s,t} \in F$$

N_F^2 is generated over FW_6 by $x_6x_4^2\ x_5^4$ and consists of all polynomials in x_1 , x_2 , x_3 , x_4 , x_5 , x_6 of the form:

$$\sum_{1 \le i < j < k < p < q \le 6} x_r \sum_{s=i}^{q} \sum_{\substack{t=i \\ t \ne s}}^{q} c_{i,j,k,p,q,s}\ x_s^2\ x_t^4 \text{ , where } c_{i,j,k,p,q,s,t} \in F$$

and $\{x_r\} \cup \{x_i , x_j , x_k , x_p , x_q\} = \{x_1 , x_2 , x_3 , x_4 , x_5 , x_6\}$. *see [1]*

iii- U_F^1 and U_F^2 are the FW_6 - sub modules of the modules N_F^1, N_F^2 respectively.

U_F^1 generated over FW_6 by $\mu_6(x^2_1x^4_3 - x^2_1x^4_2)$, and consists of all polynomials in x_1 , x_2 , x_3 , x_4 , x_5 , x_6 of the form:

$$\sum_{1 \le i < j < k < p < q \le 6} \mu_r \sum_{s=i}^{q} \sum_{\substack{t=i \\ t \ne s}}^{q} c_{i,j,k,p,q,s,t}\ x_s^2\ x_t^4 \text{ , and } \sum_{\substack{t=i \\ t \ne s}}^{q} c_{i,j,k,p,q,s,t} = 0 .$$

U_F^2 generated over FW_6 by $x_6\ (x^2_1x^4_3 - x^2_1x^4_2)$, and consists of all polynomials in x_1 , x_2 , x_3 , x_4 , x_5 , x_6 of the form:

$$\sum_{1 \le i < j < k < p < q \le 6} x_r \sum_{s=i}^{q} \sum_{\substack{t=i \\ t \ne s}}^{q} c_{i,j,k,p,q,s,t}\ x_s^2\ x_t^4 \text{ , and } \sum_{\substack{t=i \\ t \ne s}}^{q} c_{i,j,k,p,q,s,t} = 0$$

where $c_{i,j,k,p,q,s,t} \in F$, $\{x_r\} \cup \{x_i , x_j , x_k , x_p , x_q\}=\{x_1 , x_2 , x_3 , x_4 , x_5 , x_6\}$.

see [3]

iv- G_F^1 and G_F^2 are the FW_6 - sub modules of the modules M_F^1, M_F^2 respectively.

G_F^1 consists of all polynomials in x_1 , x_2 , x_3 , x_4 , x_5 , x_6 of the form:

$$\sum_{1 \le i < j < k < p < q \le 6} \mu_r \sum_{s=i}^{q} c_{i,j,k,p,q,s}\ x_s^2 \text{ , and } \sum_{s=i}^{q} c_{i,j,k,p,q,s} = 0.$$

G_F^2 consists of all polynomials in x_1 , x_2 , x_3 , x_4 , x_5 , x_6 of the form :

$$\sum_{1 \le i < j < k < p < q \le 6} x_r \sum_{s=i}^{q} c_{i,j,k,p,q,s}\ x_s^2 \text{ , and } \sum_{s=i}^{q} c_{i,j,k,p,q,s} = 0.$$

where $c_{i,j,k,p,q,s} \in F$, and $\{x_r\} \cup \{x_i , x_j , x_k , x_p , x_q\}=\{x_1 , x_2 , x_3 , x_4 , x_5 , x_6\}$.

see [3]

v- H_F^1 and H_F^2 are the FW_6 - sub modules of the modules N_F^1, N_F^2 respectively.

H_F^1 consists of all polynomials in x_1 , x_2 , x_3 , x_4 , x_5 , x_6 of the form :

$$\sum_{1 \le i < j < k < p < q \le 6} \mu_r \sum_{s=i}^{q} \sum_{\substack{t=i \\ t \ne s}}^{q} c_{i,j,k,p,q,s,t} \; x_s^2 \, x_t^4 \quad \text{, and} \quad \sum_{s=i}^{q} \sum_{\substack{t=i \\ t \ne s}}^{q} c_{i,j,k,p,q,s,t} = 0.$$

H_F^2 consists of all polynomials in x_1 , x_2 , x_3 , x_4 , x_5 , x_6 of the form :

$$\sum_{1 \le i < j < k < p < q \le 6} x_r \sum_{s=i}^{q} \sum_{\substack{t=i \\ t \ne s}}^{q} c_{i,j,k,p,q,s,t} \; x_s^2 \, x_t^4 \quad \text{, and} \quad \sum_{s=i}^{q} \sum_{\substack{t=i \\ t \ne s}}^{q} c_{i,j,k,p,q,s,t} = 0.$$

where $c_{i,j,k,p,q,s} \in F$,and $\{x_r\} \cup \{x_i , x_j , x_k , x_p , x_q\}=\{x_1 , x_2 , x_3 , x_4 , x_5 , x_6\}$.

2-3 :- Basis and dimensions of FW_6 – modules:-

i- The set:-

$A^1 = \{\mu_1 (x_3^2 - x_2^2)$, $\mu_1 (x_4^2 - x_2^2)$, $\mu_1 (x_5^2 - x_2^2)$,
$\mu_1 (x_6^2 - x_2^2)$, $\mu_2 (x_3^2 - x_1^2)$, $\mu_2 (x_4^2 - x_1^2)$,
$\mu_2 (x_5^2 - x_1^2)$, $\mu_2 (x_6^2 - x_1^2)$, $\mu_3 (x_2^2 - x_1^2)$,
$\mu_3 (x_4^2 - x_1^2)$, $\mu_3 (x_5^2 - x_1^2)$, $\mu_3 (x_6^2 - x_1^2)$,
$\mu_4 (x_2^2 - x_1^2)$, $\mu_4 (x_3^2 - x_1^2)$, $\mu_4 (x_5^2 - x_1^2)$,
$\mu_4 (x_6^2 - x_1^2)$, $\mu_5 (x_2^2 - x_1^2)$, $\mu_5 (x_3^2 - x_1^2)$,
$\mu_5 (x_4^2 - x_1^2)$, $\mu_5 (x_6^2 - x_1^2)$, $\mu_6 (x_2^2 - x_1^2)$,
$\mu_6 (x_3^2 - x_1^2)$, $\mu_6 (x_4^2 - x_1^2)$, $\mu_6 (x_5^2 - x_1^2) \}$

is a F – basis of the module G_F^1 , and $\dim_F \; G_F^1 = 24$.

ii- The set:-

$A^2 = \{ x_1 (x_3^2 - x_2^2)$, $x_1 (x_4^2 - x_2^2)$, $x_1 (x_5^2 - x_2^2)$,
$x_1 (x_6^4 - x_2^2)$, $x_2 (x_3^2 - x_1^2)$, $x_2 (x_4^2 - x_1^2)$,
$x_2 (x_5^2 - x_1^2)$, $x_2 (x_6^2 - x_1^2)$, $x_3 (x_2^2 - x_1^2)$,
$x_3 (x_4^2 - x_1^2)$, $x_3 (x_5^2 - x_1^2)$, $x_3 (x_6^2 - x_1^2)$,
$x_4 (x_2^2 - x_1^2)$, $x_4 (x_3^2 - x_1^2)$, $x_4 (x_5^2 - x_1^2)$,
$x_4 (x_6^2 - x_1^2)$, $x_5 (x_2^2 - x_1^2)$, $x_5 (x_3^2 - x_1^2)$,
$x_5 (x_4^2 - x_1^2)$, $x_5 (x_6^2 - x_1^2)$, $x_6 (x_2^2 - x_1^2)$,
$x_6 (x_3^2 - x_1^2)$, $x_6 (x_4^2 - x_1^2)$, $x_6 (x_5^2 - x_1^2) \}$.

is a F – basis of the module G_F^2 , and $\dim_F \; G_F^2 = 24$.

iii- The set:-

$B^1 = \{\mu_1 (x_2^2 x_4^4 - x_2^2 x_3^4)$, $\mu_1 (x_4^2 x_2^4 - x_2^2 x_3^4)$, $\mu_1 (x_2^2 x_5^4 - x_2^2 x_3^4)$,
$\mu_1 (x_5^2 x_2^4 - x_2^2 x_3^4)$, $\mu_1 (x_2^2 x_6^4 - x_2^2 x_3^4)$, $\mu_1 (x_6^2 x_2^4 - x_2^2 x_3^4)$,
$\mu_1 (x_3^2 x_2^4 - x_2^2 x_3^4)$, $\mu_1 (x_3^2 x_4^4 - x_2^2 x_3^4)$ $\mu_1 (x_4^2 x_3^4 - x_2^2 x_3^4)$,
$\mu_1 (x_3^2 x_5^4 - x_2^2 x_3^4)$, $\mu_1 (x_5^2 x_3^4 - x_2^2 x_3^4)$, $\mu_1 (x_3^2 x_6^4 - x_2^2 x_3^4)$,
$\mu_1 (x_6^2 x_3^4 - x_2^2 x_3^4)$, $\mu_1 (x_4^2 x_5^4 - x_2^2 x_3^4)$, $\mu_1 (x_5^2 x_4^4 - x_2^2 x_3^4)$,
$\mu_1 (x_4^2 x_6^4 - x_2^2 x_3^4)$, $\mu_1 (x_6^2 x_4^4 - x_2^2 x_3^4)$, $\mu_1 (x_5^2 x_6^4 - x_2^2 x_3^4)$,
$\mu_1 (x_6^2 x_5^4 - x_2^2 x_3^4)$, $\mu_2 (x_1^2 x_4^4 - x_1^2 x_3^4)$, $\mu_2 (x_1^2 x_5^4 - x_1^2 x_3^4)$,
$\mu_2 (x_5^2 x_1^4 - x_1^2 x_3^4)$, $\mu_2 (x_1^2 x_6^4 - x_1^2 x_3^4)$, $\mu_2 (x_6^2 x_1^4 - x_1^2 x_3^4)$,
$\mu_2 (x_3^2 x_1^4 - x_1^2 x_3^4)$, $\mu_2 (x_3^2 x_4^4 - x_1^2 x_3^4)$, $\mu_2 (x_4^2 x_3^4 - x_1^2 x_3^4)$,
$\mu_2 (x_3^2 x_5^4 - x_1^2 x_3^4)$, $\mu_2 (x_5^2 x_3^4 - x_1^2 x_3^4)$, $\mu_2 (x_3^2 x_6^4 - x_1^2 x_3^4)$,
$\mu_2 (x_6^2 x_3^4 - x_1^2 x_3^4)$, $\mu_2 (x_4^2 x_5^4 - x_1^2 x_3^4)$, $\mu_2 (x_5^2 x_4^4 - x_1^2 x_3^4)$,
$\mu_2 (x_4^2 x_6^4 - x_1^2 x_3^4)$, $\mu_2 (x_6^2 x_4^4 - x_1^2 x_3^4)$, $\mu_2 (x_5^2 x_6^4 - x_1^2 x_3^4)$,

$\mu_2 (x_6^2x_5^4 - x_1^2x_3^4)$, $\mu_2 (x_4^2x_1^4 - x_1^2x_3^4)$, $\mu_3 (x_1^2x_4^4 - x_1^2x_2^4)$,
$\mu_3 (x_4^2x_1^4 - x_1^2x_2^4)$, $\mu_3 (x_1^2x_5^4 - x_1^2x_2^4)$, $\mu_3 (x_5^2x_1^4 - x_1^2x_2^4)$,
$\mu_3 (x_1^2x_6^4 - x_1^2x_2^4)$, $\mu_3 (x_6^2x_1^4 - x_1^2x_2^4)$, $\mu_3 (x_2^2x_1^4 - x_1^2x_2^4)$,
$\mu_3 (x_2^2x_4^4 - x_1^2x_2^4)$, $\mu_3 (x_4^2x_2^4 - x_1^2x_2^4)$, $\mu_3 (x_2^2x_5^4 - x_1^2x_2^4)$,
$\mu_3 (x_5^2x_2^4 - x_1^2x_2^4)$, $\mu_3 (x_2^2x_6^4 - x_1^2x_2^4)$, $\mu_3 (x_6^2x_2^4 - x_1^2x_2^4)$,
$\mu_3 (x_4^2x_5^4 - x_1^2x_2^4)$, $\mu_3 (x_5^2x_4^4 - x_1^2x_2^4)$, $\mu_3 (x_4^2x_6^4 - x_1^2x_2^4)$,
$\mu_3 (x_6^2x_4^4 - x_1^2x_2^4)$, $\mu_3 (x_5^2x_6^4 - x_1^2x_2^4)$, $\mu_3 (x_6^2x_5^4 - x_1^2x_2^4)$,
$\mu_4 (x_1^2x_3^4 - x_1^2x_2^4)$, $\mu_4 (x_3^2x_1^4 - x_1^2x_2^4)$, $\mu_4 (x_1^2x_5^4 - x_1^2x_2^4)$,
$\mu_4 (x_5^2x_1^4 - x_1^2x_2^4)$, $\mu_4 (x_1^2x_6^4 - x_1^2x_2^4)$, $\mu_4 (x_6^2x_1^4 - x_1^2x_2^4)$,
$\mu_4 (x_2^2x_3^4 - x_1^2x_2^4)$, $\mu_4 (x_3^2x_2^4 - x_1^2x_2^4)$, $\mu_4 (x_2^2x_5^4 - x_1^2x_2^4)$,
$\mu_4 (x_5^2x_2^4 - x_1^2x_2^4)$, $\mu_4 (x_2^2x_6^4 - x_1^2x_2^4)$, $\mu_4 (x_6^2x_2^4 - x_1^2x_2^4)$,
$\mu_4 (x_3^2x_5^4 - x_1^2x_2^4)$, $\mu_4 (x_5^2x_3^4 - x_1^2x_2^4)$, $\mu_4 (x_3^2x_6^4 - x_1^2x_2^4)$,
$\mu_4 (x_6^2x_3^4 - x_1^2x_2^4)$, $\mu_4 (x_5^2x_6^4 - x_1^2x_2^4)$, $\mu_4 (x_6^2x_5^4 - x_1^2x_2^4)$,
$\mu_4 (x_2^2x_1^4 - x_1^2x_2^4)$, $\mu_5 (x_1^2x_3^4 - x_1^2x_2^4)$, $\mu_5 (x_3^2x_1^4 - x_1^2x_2^4)$,
$\mu_5 (x_1^2x_4^4 - x_1^2x_2^4)$, $\mu_5 (x_4^2x_1^4 - x_1^2x_2^4)$, $\mu_5 (x_1^2x_6^4 - x_1^2x_2^4)$,
$\mu_5 (x_6^2x_1^4 - x_1^2x_2^4)$, $\mu_5 (x_2^2x_1^4 - x_1^2x_2^4)$, $\mu_5 (x_2^2x_3^4 - x_1^2x_2^4)$,
$\mu_5 (x_3^2x_2^4 - x_1^2x_2^4)$, $\mu_5 (x_2^2x_4^4 - x_1^2x_2^4)$, $\mu_5 (x_4^2x_2^4 - x_1^2x_2^4)$,
$\mu_5 (x_2^2x_6^4 - x_1^2x_2^4)$, $\mu_5 (x_6^2x_2^4 - x_1^2x_2^4)$, $\mu_5 (x_3^2x_4^4 - x_1^2x_2^4)$,
$\mu_5 (x_4^2x_3^4 - x_1^2x_2^4)$, $\mu_5 (x_3^2x_6^4 - x_1^2x_2^4)$, $\mu_5 (x_6^2x_3^4 - x_1^2x_2^4)$,
$\mu_5 (x_4^2x_6^4 - x_1^2x_2^4)$, $\mu_5 (x_6^2x_4^4 - x_1^2x_2^4)$, $\mu_6 (x_2^2x_1^4 - x_1^2x_2^4)$,
$\mu_6 (x_1^2x_3^4 - x_1^2x_2^4)$, $\mu_6 (x_3^2x_1^4 - x_1^2x_2^4)$, $\mu_6 (x_1^2x_4^4 - x_1^2x_2^4)$,
$\mu_6 (x_4^2x_1^4 - x_1^2x_2^4)$, $\mu_6 (x_1^2x_5^4 - x_1^2x_2^4)$, $\mu_6 (x_5^2x_1^4 - x_1^2x_2^4)$,
$\mu_6 (x_2^2x_3^4 - x_1^2x_2^4)$, $\mu_6 (x_3^2x_2^4 - x_1^2x_2^4)$, $\mu_6 (x_2^2x_4^4 - x_1^2x_2^4)$,
$\mu_6 (x_3^2x_4^4 - x_1^2x_2^4)$, $\mu_6 (x_4^2x_3^4 - x_1^2x_2^4)$, $\mu_6 (x_3^2x_5^4 - x_1^2x_2^4)\}$,
$\mu_6 (x_3^2x_4^4 - x_1^2x_2^4)$, $\mu_6 (x_3^2x_4^4 - x_1^2x_2^4)$, $\mu_6 (x_4^2x_3^4 - x_1^2x_2^4)$,
$\mu_6 (x_5^2x_3^4 - x_1^2x_2^4)$, $\mu_6 (x_3^2x_6^4 - x_1^2x_2^4)$, $\mu_6 (x_6^2x_3^4 - x_1^2x_2^4)$ }

is a F – basis of the module H_F^1 , and $\dim_F$ $H_F^1 = 114$.

iv- The set :

$B^2 =$ $\{x_1 (x_2^2x_4^4 - x_2^2x_3^4)$, $x_1 (x_4^2x_2^4 - x_2^2x_3^4)$, $x_1 (x_2^2x_5^4 - x_2^2x_3^4)$,
$x_1 (x_5^2x_2^4 - x_2^2x_3^4)$, $x_1 (x_2^2x_6^4 - x_2^2x_3^4)$, $x_1 (x_6^2x_2^4 - x_2^2x_3^4)$,
$x_1 (x_3^2x_2^4 - x_2^2x_3^4)$, $x_1 (x_3^2x_4^4 - x_2^2x_3^4)$ $x_1 (x_4^2x_3^4 - x_2^2x_3^4)$,
$x_1 (x_3^2x_5^4 - x_2^2x_3^4)$, $x_1 (x_5^2x_3^4 - x_2^2x_3^4)$, $x_1 (x_3^2x_6^4 - x_2^2x_3^4)$,
$x_1 (x_6^2x_3^4 - x_2^2x_3^4)$, $x_1 (x_4^2x_5^4 - x_2^2x_3^4)$, $x_1 (x_5^2x_4^4 - x_2^2x_3^4)$,
$x_1 (x_4^2x_6^4 - x_2^2x_3^4)$, $x_1 (x_6^2x_4^4 - x_2^2x_3^4)$, $x_1 (x_5^2x_6^4 - x_2^2x_3^4)$,
$x_1 (x_6^2x_5^4 - x_2^2x_3^4)$, $x_2 (x_1^2x_4^4 - x_1^2x_3^4)$, $x_2 (x_1^2x_5^4 - x_1^2x_3^4)$,
$x_2 (x_5^2x_1^4 - x_1^2x_3^4)$, $x_2 (x_1^2x_6^4 - x_1^2x_3^4)$, $x_2 (x_6^2x_1^4 - x_1^2x_3^4)$,
$x_2 (x_3^2x_1^4 - x_1^2x_3^4)$, $x_2 (x_3^2x_4^4 - x_1^2x_3^4)$, $x_2 (x_4^2x_3^4 - x_1^2x_3^4)$,
$x_2 (x_3^2x_5^4 - x_1^2x_3^4)$, $x_2 (x_5^2x_3^4 - x_1^2x_3^4)$, $x_2 (x_3^2x_6^4 - x_1^2x_3^4)$,
$x_2 (x_6^2x_3^4 - x_1^2x_3^4)$, $x_2 (x_4^2x_5^4 - x_1^2x_3^4)$, $x_2 (x_5^2x_4^4 - x_1^2x_3^4)$,
$x_2 (x_4^2x_6^4 - x_1^2x_3^4)$, $x_2 (x_6^2x_4^4 - x_1^2x_3^4)$, $x_2 (x_5^2x_6^4 - x_1^2x_3^4)$,
$x_2 (x_6^2x_5^4 - x_1^2x_3^4)$, $x_2 (x_4^2x_1^4 - x_1^2x_3^4)$, $x_3 (x_1^2x_4^4 - x_1^2x_2^4)$,
$x_3 (x_4^2x_1^4 - x_1^2x_2^4)$, $x_3 (x_1^2x_5^4 - x_1^2x_2^4)$, $x_3 (x_5^2x_1^4 - x_1^2x_2^4)$,
$x_3 (x_1^2x_6^4 - x_1^2x_2^4)$, $x_3 (x_6^2x_1^4 - x_1^2x_2^4)$, $x_3 (x_2^2x_1^4 - x_1^2x_2^4)$,
$x_3 (x_2^2x_4^4 - x_1^2x_2^4)$, $x_3 (x_4^2x_2^4 - x_1^2x_2^4)$, $x_3 (x_2^2x_5^4 - x_1^2x_2^4)$,

$x_3(x_5^2x_2^4 - x_1^2x_2^4)$, $x_3(x_2^2x_6^4 - x_1^2x_2^4)$, $x_3(x_6^2x_2^4 - x_1^2x_2^4)$,
$x_3(x_4^2x_5^4 - x_1^2x_2^4)$, $x_3(x_5^2x_4^4 - x_1^2x_2^4)$, $x_3(x_4^2x_6^4 - x_1^2x_2^4)$,
$x_3(x_6^2x_4^4 - x_1^2x_2^4)$, $x_3(x_5^2x_6^4 - x_1^2x_2^4)$, $x_3(x_6^2x_5^4 - x_1^2x_2^4)$,
$x_4(x_1^2x_3^4 - x_1^2x_2^4)$, $x_4(x_3^2x_1^4 - x_1^2x_2^4)$, $x_4(x_1^2x_5^4 - x_1^2x_2^4)$,
$x_4(x_5^2x_1^4 - x_1^2x_2^4)$, $x_4(x_1^2x_6^4 - x_1^2x_2^4)$, $x_4(x_6^2x_1^4 - x_1^2x_2^4)$,
$x_4(x_2^2x_3^4 - x_1^2x_2^4)$, $x_4(x_3^2x_2^4 - x_1^2x_2^4)$, $x_4(x_2^2x_5^4 - x_1^2x_2^4)$,
$x_4(x_5^2x_2^4 - x_1^2x_2^4)$, $x_4(x_2^2x_6^4 - x_1^2x_2^4)$, $x_4(x_6^2x_2^4 - x_1^2x_2^4)$,
$x_4(x_3^2x_5^4 - x_1^2x_2^4)$, $x_4(x_5^2x_3^4 - x_1^2x_2^4)$, $x_4(x_3^2x_6^4 - x_1^2x_2^4)$,
$x_4(x_6^2x_3^4 - x_1^2x_2^4)$, $x_4(x_5^2x_6^4 - x_1^2x_2^4)$, $x_4(x_6^2x_5^4 - x_1^2x_2^4)$,
$x_4(x_2^2x_1^4 - x_1^2x_2^4)$, $x_5(x_1^2x_3^4 - x_1^2x_2^4)$, $x_5(x_3^2x_1^4 - x_1^2x_2^4)$,
$x_5(x_1^2x_4^4 - x_1^2x_2^4)$, $x_5(x_4^2x_1^4 - x_1^2x_2^4)$, $x_5(x_1^2x_6^4 - x_1^2x_2^4)$,
$x_5(x_6^2x_1^4 - x_1^2x_2^4)$, $x_5(x_2^2x_1^4 - x_1^2x_2^4)$, $x_5(x_2^2x_3^4 - x_1^2x_2^4)$,
$x_5(x_3^2x_2^4 - x_1^2x_2^4)$, $x_5(x_2^2x_4^4 - x_1^2x_2^4)$, $x_5(x_4^2x_2^4 - x_1^2x_2^4)$,
$x_5(x_2^2x_6^4 - x_1^2x_2^4)$, $x_5(x_6^2x_2^4 - x_1^2x_2^4)$, $x_5(x_3^2x_4^4 - x_1^2x_2^4)$,
$x_5(x_4^2x_3^4 - x_1^2x_2^4)$, $x_5(x_3^2x_6^4 - x_1^2x_2^4)$, $x_5(x_6^2x_3^4 - x_1^2x_2^4)$,
$x_5(x_4^2x_6^4 - x_1^2x_2^4)$, $x_5(x_6^2x_4^4 - x_1^2x_2^4)$, $x_6(x_2^2x_1^4 - x_1^2x_2^4)$,
$x_6(x_1^2x_3^4 - x_1^2x_2^4)$, $x_6(x_3^2x_1^4 - x_1^2x_2^4)$, $x_6(x_1^2x_4^4 - x_1^2x_2^4)$,
$x_6(x_4^2x_1^4 - x_1^2x_2^4)$, $x_6(x_1^2x_5^4 - x_1^2x_2^4)$, $x_6(x_5^2x_1^4 - x_1^2x_2^4)$,
$x_6(x_2^2x_3^4 - x_1^2x_2^4)$, $x_6(x_3^2x_2^4 - x_1^2x_2^4)$, $x_6(x_2^2x_4^4 - x_1^2x_2^4)$,
$x_6(x_3^2x_4^4 - x_1^2x_2^4)$, $x_6(x_4^2x_3^4 - x_1^2x_2^4)$, $x_6(x_3^2x_5^4 - x_1^2x_2^4)\}$,
$x_6(x_3^2x_4^4 - x_1^2x_2^4)$, $x_6(x_3^2x_4^4 - x_1^2x_2^4)$, $x_6(x_4^2x_3^4 - x_1^2x_2^4)$,
$x_6(x_5^2x_3^4 - x_1^2x_2^4)$, $x_6(x_3^2x_6^4 - x_1^2x_2^4)$, $x_6(x_6^2x_3^4 - x_1^2x_2^4)\}$

is a F – basis of the module H_F^2 , and $\dim_F H_F^2 = 114$.

iii- The set:-

$D^1 = \{\mu_4(x_1^2x_3^4 - x_1^2x_2^4)$, $\mu_5(x_1^2x_3^4 - x_1^2x_2^4)$, $\mu_6(x_1^2x_3^4 - x_1^2x_2^4)$,
$\mu_3(x_1^2x_4^4 - x_1^2x_2^4)$, $\mu_5(x_1^2x_4^4 - x_1^2x_2^4)$, $\mu_6(x_1^2x_4^4 - x_1^2x_2^4)$,
$\mu_3(x_1^2x_5^4 - x_1^2x_2^4)$, $\mu_4(x_1^2x_5^4 - x_1^2x_2^4)$, $\mu_6(x_1^2x_5^4 - x_1^2x_2^4)$,
$\mu_3(x_1^2x_6^4 - x_1^2x_2^4)$, $\mu_4(x_1^2x_6^4 - x_1^2x_2^4)$, $\mu_5(x_1^2x_6^4 - x_1^2x_2^4)$,
$\mu_4(x_2^2x_3^4 - x_2^2x_1^4)$, $\mu_5(x_2^2x_3^4 - x_2^2x_1^4)$, $\mu_6(x_2^2x_3^4 - x_2^2x_1^4)$,
$\mu_3(x_2^2x_4^4 - x_2^2x_1^4)$, $\mu_5(x_2^2x_4^4 - x_2^2x_1^4)$, $\mu_6(x_2^2x_4^4 - x_2^2x_1^4)$,
$\mu_3(x_2^2x_5^4 - x_2^2x_1^4)$, $\mu_4(x_2^2x_5^4 - x_2^2x_1^4)$, $\mu_6(x_2^2x_5^4 - x_2^2x_1^4)$,
$\mu_3(x_2^2x_6^4 - x_2^2x_1^4)$, $\mu_4(x_2^2x_6^4 - x_2^2x_1^4)$, $\mu_5(x_2^2x_6^4 - x_2^2x_1^4)$,
$\mu_4(x_3^2x_2^4 - x_3^2x_1^4)$, $\mu_5(x_3^2x_2^4 - x_3^2x_1^4)$, $\mu_6(x_3^2x_2^4 - x_3^2x_1^4)$,
$\mu_2(x_3^2x_4^4 - x_3^2x_1^4)$, $\mu_5(x_3^2x_4^4 - x_3^2x_1^4)$, $\mu_6(x_3^2x_4^4 - x_3^2x_1^4)$,
$\mu_2(x_3^2x_5^4 - x_3^2x_1^4)$, $\mu_4(x_3^2x_5^4 - x_3^2x_1^4)$, $\mu_6(x_3^2x_5^4 - x_3^2x_1^4)$,
$\mu_2(x_3^2x_6^4 - x_3^2x_1^4)$, $\mu_4(x_3^2x_6^4 - x_3^2x_1^4)$, $\mu_5(x_3^2x_6^4 - x_3^2x_1^4)$,
$\mu_3(x_4^2x_2^4 - x_4^2x_1^4)$, $\mu_5(x_4^2x_2^4 - x_4^2x_1^4)$, $\mu_6(x_4^2x_2^4 - x_4^2x_1^4)$,
$\mu_2(x_4^2x_3^4 - x_4^2x_1^4)$, $\mu_5(x_4^2x_3^4 - x_4^2x_1^4)$, $\mu_6(x_4^2x_3^4 - x_4^2x_1^4)$,
$\mu_2(x_4^2x_5^4 - x_4^2x_1^4)$, $\mu_3(x_4^2x_5^4 - x_4^2x_1^4)$, $\mu_6(x_4^2x_5^4 - x_4^2x_1^4)$,
$\mu_2(x_4^2x_6^4 - x_4^2x_1^4)$, $\mu_3(x_4^2x_6^4 - x_4^2x_1^4)$, $\mu_5(x_4^2x_6^4 - x_4^2x_1^4)$,
$\mu_3(x_5^2x_2^4 - x_5^2x_1^4)$, $\mu_4(x_5^2x_2^4 - x_5^2x_1^4)$, $\mu_6(x_5^2x_2^4 - x_5^2x_1^4)$,
$\mu_2(x_5^2x_3^4 - x_5^2x_1^4)$, $\mu_4(x_5^2x_3^4 - x_5^2x_1^4)$, $\mu_6(x_5^2x_3^4 - x_5^2x_1^4)$,
$\mu_2(x_5^2x_4^4 - x_5^2x_1^4)$, $\mu_3(x_5^2x_4^4 - x_5^2x_1^4)$, $\mu_6(x_5^2x_4^4 - x_5^2x_1^4)$,
$\mu_2(x_5^2x_6^4 - x_5^2x_1^4)$, $\mu_3(x_5^2x_6^4 - x_5^2x_1^4)$, $\mu_4(x_5^2x_6^4 - x_5^2x_1^4)$,

$$\begin{array}{lll}
\mu_3 (x_6^2x_2^4 - x_6^2x_1^4) , & \mu_4 (x_6^2x_2^4 - x_6^2x_1^4) , & \mu_5 (x_6^2x_2^4 - x_6^2x_1^4) , \\
\mu_2 (x_6^2x_3^4 - x_6^2x_1^4) , & \mu_4 (x_6^2x_3^4 - x_6^2x_1^4) , & \mu_5 (x_6^2x_3^4 - x_6^2x_1^4) , \\
\mu_2 (x_6^2x_4^4 - x_6^2x_1^4) , & \mu_3 (x_6^2x_4^4 - x_6^2x_1^4) , & \mu_5 (x_6^2x_4^4 - x_6^2x_1^4) , \\
\mu_2 (x_6^2x_5^4 - x_6^2x_1^4) , & \mu_3 (x_6^2x_5^4 - x_6^2x_1^4) , & \mu_4 (x_6^2x_5^4 - x_6^2x_1^4) , \\
\mu_2 (x_1^2x_4^4 - x_1^2x_3^4) , & \mu_2 (x_1^2x_5^4 - x_1^2x_3^4) , & \mu_2 (x_1^2x_6^4 - x_1^2x_3^4) , \\
\mu_1 (x_2^2x_4^4 - x_2^2x_3^4) , & \mu_1 (x_2^2x_5^4 - x_2^2x_3^4) , & \mu_1 (x_2^2x_6^4 - x_2^2x_3^4) , \\
\mu_1 (x_3^2x_4^4 - x_3^2x_2^4) , & \mu_1 (x_3^2x_5^4 - x_3^2x_2^4) , & \mu_1 (x_3^2x_6^4 - x_3^2x_2^4) , \\
\mu_1 (x_4^2x_3^4 - x_4^2x_2^4) , & \mu_1 (x_4^2x_5^4 - x_4^2x_2^4) , & \mu_1 (x_4^2x_6^4 - x_4^2x_2^4) , \\
\mu_1 (x_5^2x_3^4 - x_5^2x_2^4) , & \mu_1 (x_5^2x_4^4 - x_5^2x_2^4) , & \mu_1 (x_5^2x_6^4 - x_5^2x_2^4) , \\
\mu_1 (x_6^2x_3^4 - x_6^2x_2^4) , & \mu_1 (x_6^2x_4^4 - x_6^2x_2^4) , & \mu_1 (x_6^2x_5^4 - x_6^2x_2^4)\}
\end{array}$$

is a F – basis of the module U_F^1 , and $\dim_F U_F^1 = 90$.

iv- The set:-

$$D^2 = \begin{array}{lll}
\{x_4 (x_1^2x_3^4 - x_1^2x_2^4) , & x_5 (x_1^2x_3^4 - x_1^2x_2^4) , & x_6 (x_1^2x_3^4 - x_1^2x_2^4) , \\
x_3 (x_1^2x_4^4 - x_1^2x_2^4) , & x_5 (x_1^2x_4^4 - x_1^2x_2^4) , & x_6 (x_1^2x_4^4 - x_1^2x_2^4) , \\
x_3 (x_1^2x_5^4 - x_1^2x_2^4) , & x_4 (x_1^2x_5^4 - x_1^2x_2^4) , & x_6 (x_1^2x_5^4 - x_1^2x_2^4) , \\
x_3 (x_1^2x_6^4 - x_1^2x_2^4) , & x_4 (x_1^2x_6^4 - x_1^2x_2^4) , & x_5 (x_1^2x_6^4 - x_1^2x_2^4) , \\
x_4 (x_2^2x_3^4 - x_2^2x_1^4) , & x_5 (x_2^2x_3^4 - x_2^2x_1^4) , & x_6 (x_2^2x_3^4 - x_2^2x_1^4) , \\
x_3 (x_2^2x_4^4 - x_2^2x_1^4) , & x_5 (x_2^2x_4^4 - x_2^2x_1^4) , & x_6 (x_2^2x_4^4 - x_2^2x_1^4) , \\
x_3 (x_2^2x_5^4 - x_2^2x_1^4) , & x_4 (x_2^2x_5^4 - x_2^2x_1^4) , & x_6 (x_2^2x_5^4 - x_2^2x_1^4) , \\
x_3 (x_2^2x_6^4 - x_2^2x_1^4) , & x_4 (x_2^2x_6^4 - x_2^2x_1^4) , & x_5 (x_2^2x_6^4 - x_2^2x_1^4) , \\
x_4 (x_3^2x_2^4 - x_3^2x_1^4) , & x_5 (x_3^2x_2^4 - x_3^2x_1^4) , & x_6 (x_3^2x_2^4 - x_3^2x_1^4) , \\
x_2 (x_3^2x_4^4 - x_3^2x_1^4) , & x_5 (x_3^2x_4^4 - x_3^2x_1^4) , & x_6 (x_3^2x_4^4 - x_3^2x_1^4) , \\
x_2 (x_3^2x_5^4 - x_3^2x_1^4) , & x_4 (x_3^2x_5^4 - x_3^2x_1^4) , & x_6 (x_3^2x_5^4 - x_3^2x_1^4) , \\
x_2 (x_3^2x_6^4 - x_3^2x_1^4) , & x_4 (x_3^2x_6^4 - x_3^2x_1^4) , & x_5 (x_3^2x_6^4 - x_3^2x_1^4) , \\
x_3 (x_4^2x_2^4 - x_4^2x_1^4) , & x_5 (x_4^2x_2^4 - x_4^2x_1^4) , & x_6 (x_4^2x_2^4 - x_4^2x_1^4) , \\
x_2 (x_4^2x_3^4 - x_4^2x_1^4) , & x_5 (x_4^2x_3^4 - x_4^2x_1^4) , & x_6 (x_4^2x_3^4 - x_4^2x_1^4) , \\
x_2 (x_4^2x_5^4 - x_4^2x_1^4) , & x_3 (x_4^2x_5^4 - x_4^2x_1^4) , & x_6 (x_4^2x_5^4 - x_4^2x_1^4) , \\
x_2 (x_4^2x_6^4 - x_4^2x_1^4) , & x_3 (x_4^2x_6^4 - x_4^2x_1^4) , & x_5 (x_4^2x_6^4 - x_4^2x_1^4) , \\
x_3 (x_5^2x_2^4 - x_5^2x_1^4) , & x_4 (x_5^2x_2^4 - x_5^2x_1^4) , & x_6 (x_5^2x_2^4 - x_5^2x_1^4) , \\
x_2 (x_5^2x_3^4 - x_5^2x_1^4) , & x_4 (x_5^2x_3^4 - x_5^2x_1^4) , & x_6 (x_5^2x_3^4 - x_5^2x_1^4) , \\
x_2 (x_5^2x_4^4 - x_5^2x_1^4) , & x_3 (x_5^2x_4^4 - x_5^2x_1^4) , & x_6 (x_5^2x_4^4 - x_5^2x_1^4) , \\
x_2 (x_5^2x_6^4 - x_5^2x_1^4) , & x_3 (x_5^2x_6^4 - x_5^2x_1^4) , & x_4 (x_5^2x_6^4 - x_5^2x_1^4) , \\
x_3 (x_6^2x_2^4 - x_6^2x_1^4) , & x_4 (x_6^2x_2^4 - x_6^2x_1^4) , & x_5 (x_6^2x_2^4 - x_6^2x_1^4) , \\
x_2 (x_6^2x_3^4 - x_6^2x_1^4) , & x_4 (x_6^2x_3^4 - x_6^2x_1^4) , & x_5 (x_6^2x_3^4 - x_6^2x_1^4) , \\
x_2 (x_6^2x_4^4 - x_6^2x_1^4) , & x_3 (x_6^2x_4^4 - x_6^2x_1^4) , & x_5 (x_6^2x_4^4 - x_6^2x_1^4) , \\
x_2 (x_6^2x_5^4 - x_6^2x_1^4) , & x_3 (x_6^2x_5^4 - x_6^2x_1^4) , & x_4 (x_6^2x_5^4 - x_6^2x_1^4) , \\
x_2 (x_1^2x_4^4 - x_1^2x_3^4) , & x_2 (x_1^2x_5^4 - x_1^2x_3^4) , & x_2 (x_1^2x_6^4 - x_1^2x_3^4) , \\
x_1 (x_2^2x_4^4 - x_2^2x_3^4) , & x_1 (x_2^2x_5^4 - x_2^2x_3^4) , & x_1 (x_2^2x_6^4 - x_2^2x_3^4) , \\
x_1 (x_3^2x_4^4 - x_3^2x_2^4) , & x_1 (x_3^2x_5^4 - x_3^2x_2^4) , & x_1 (x_3^2x_6^4 - x_3^2x_2^4) , \\
x_1 (x_4^2x_3^4 - x_4^2x_2^4) , & x_1 (x_4^2x_5^4 - x_4^2x_2^4) , & x_1 (x_4^2x_6^4 - x_4^2x_2^4) , \\
x_1 (x_5^2x_3^4 - x_5^2x_2^4) , & x_1 (x_5^2x_4^4 - x_5^2x_2^4) , & x_1 (x_5^2x_6^4 - x_5^2x_2^4) , \\
x_1 (x_6^2x_3^4 - x_6^2x_2^4) , & x_1 (x_6^2x_4^4 - x_6^2x_2^4) , & x_1 (x_6^2x_5^4 - x_6^2x_2^4)\}
\end{array}$$

is a F – basis of the module U_F^2 , and $\dim_F U_F^2 = 90$.

2-4:- FW_6 – homomorphisms:

We are interesting in the following FW_6 – homomorphisms throughout this paper.

i- $\varphi : H_F^1 \longrightarrow G_F^1$, defined by:

$$\varphi(\mu_r(x_s^2x_t^4 - x_i^2x_j^4)) = \frac{1}{5!}\sum_{k=1}^{6}\frac{\partial^4}{\partial x_k^4}(\mu_r(x_s^2x_t^4 - x_i^2x_j^4))$$

$$= \mu_r x_s^2 - \mu_r x_i^2$$

$$= \mu_r(x_s^2 - x_i^2) \ , \ \text{for all } s,t \in \{i,j,k,p,q\}, \text{ and } (s,t)\neq(i,j) .$$

Where $\{x_r\} \cup \{x_i , x_j , x_k , x_p , x_q\} = \{x_1 , x_2 , x_3 , x_4 , x_5 , x_6\}$. *see [4]*

ii- θ: $H_F^2 \longrightarrow G_F^2$, defined by:-

$$\theta(x_r(x_s^2x_t^4 - x_i^2x_j^4)) = \frac{1}{4!}\sum_{k=1}^{6}\frac{\partial^4}{\partial x_k^4}(x_r(x_s^2x_t^4 - x_i^2x_j^4))$$

$$= x_r^2x_s^4 - x_r^2x_i^4$$

$$= x_r(x_s^2 - x_i^2) \ , \text{for all } s,t \in \{i,j,k,p,q\}, \text{ and } (s,t)\neq(i,j) .$$

Where $\{x_r\} \cup \{x_i , x_j , x_k , x_p , x_q\} = \{x_1 , x_2 , x_3 , x_4 , x_5 , x_6\}$. *see [4]*

3-Exact sequences:

Theorem (3-1): The following sequence of FW_6 – modules is exact and split.

$$0 \longrightarrow U_F^1 \xrightarrow{\delta} H_F^1 \xrightarrow{\varphi} G_F^1 \longrightarrow 0$$

proof **:** δ is one – to – one map since it's inclusion map .

Since $Im\ \varphi = \varphi(H_F^1)$

$$= \varphi(FW_6\ \mu_r(x_s^2x_t^4 - x_i^2x_j^4))$$

$$= FW_6\ \varphi(\mu_r(x_s^2x_t^4 - x_i^2x_j^4))$$

$$= FW_6\ \mu_r(x_s^2 - x_i^2) = G_F^1$$

Thus φ is onto .

Now $\varphi(\mu_6(x_1^2x_3^4 - x_1^2x_2^4)) = \mu_6(x_1^2 - x_1^2)$

$$= 0$$

Furthermore $\mu_6(x_1^2x_3^4 - x_1^2x_2^4)$ generates U_F^1 , so that $U_F^1 \subset ker\varphi$.

We prove the reverse inclusion by counting dimensions .

$$dim_F\ ker\varphi = dim_F\ H_F^1 - dim_F\ G_F^1$$

$$= 114 - 24 = 90$$

$$= dim_F\ U_F^1$$

Hence $ker\varphi = U_F^1$

But $Im\ \delta = U_F^1$ (*δ is inclusion map*)

Therefore $Im\ \delta = ker\varphi$, and the sequence is exact.

Second , define a FW_6 - homomorphism $g : G_F^1 \longrightarrow H_F^1$ by:-

$$g\left(\mu_r(x_s^2 - x_i^2)\right) = (-\mu_r \sum_{\substack{t=i \\ t\neq s}}^{q} x_t^2 x_s^4 + \mu_r \sum_{t=j}^{q} x_t^2 x_i^4) \ ,\forall \mu_r \left(x_s^2 - x_i^2\right) \in G_F^1$$

Where $s \in \{i,j,k,p,q\}$, $s>i$, and $1\leq i< j< k< p< q\leq 6$.

$$\text{Then } \varphi\, g \left(\mu_r\left(x_s^2 - x_i^2\right)\right) = \varphi\, (-\mu_r \sum_{\substack{t=i \\ t\neq s}}^{q} x_t^2 x_s^4 + \mu_r \sum_{t=j}^{q} x_t^2 x_i^4)$$

$$= - \sum_{\substack{t=i \\ t\neq s}}^{q} \mu_r x_t^2 + \sum_{t=j}^{q} \mu_r x_t^2$$

$$= - \sum_{\substack{t=j \\ t\neq s}}^{q} \mu_r x_t^2 - \mu_r x_i^2 + \sum_{\substack{t=j \\ t\neq s}}^{q} \mu_r x_t^2 + \mu_r x_s^2$$

$$= \mu_r x_s^2 - \mu_r x_i^2 = \mu_r\left(x_s^2 - x_i^2\right)$$

That is $\varphi\, g$ is the identity on G_F^1 and consequently the sequence split .

Theorem (3-2): The following sequence of FW_6 – modules is exact and split.

$$0 \longrightarrow U_F^2 \xrightarrow{\ \eta\ } H_F^2 \xrightarrow{\ \theta\ } G_F^2 \longrightarrow 0$$

proof : η is one - to - one map, and $Im\, \eta = U_F^2$. (*inclusion map*)

also θ is onto since :

$$Im\, \theta = \theta\, (H_F^2)$$
$$= \theta\, (FW_6\ x_r\, (x_s^2 x_t^4 - x_i^2 x_j^4)\,)$$
$$= FW_6\ \theta\, (x_r\, (x_s^2 x_t^4 - x_i^2 x_j^4)\,)$$
$$= FW_6\, x_r \left(x_s^2 - x_i^2\right)$$
$$= G_F^2$$

Thus θ is onto ,and we need only to prove the exactness at N_F^2.

First $\theta\, (x_6\, (x_1^2 x_3^4 - x_1^2 x_2^4\,)) = x_6\, (x_1^2 - x_1^2)$
$= 0$

Furthermore $x_6\, (x_1^2 x_3^4 - x_1^2 x_2^4\,)$ generates U_F^2 , that is:-

$$U_F^2 \subset ker\, \theta$$

By counting the dimension of $ker\, \theta$ we get :

$$dim_F\ ker\, \theta = dim_F\ H_F^2 - dim_F\ G_F^2$$
$$= 114 - 24 = 90$$
$$= dim_F\ U_F^2$$

Then $ker\, \theta = U_F^2$.

Hence $Im\,\eta = ker\,\theta$, and the sequence is exact.
Finally to prove that the sequence split, define a FW_6 – homomorphism
$f: G_F^2 \longrightarrow H_F^2$ by:-

$$f\left(x_r(x_s^2 - x_i^2)\right) = (-x_r \sum_{\substack{t=i \\ t\neq s}}^{q} x_t^2\, x_s^4 + x_r \sum_{t=j}^{q} x_t^2 x_i^4)\ ,\forall x_r\,(x_s^2 - x_i^2) \in G_F^2$$

Where $s \in \{i,j,k,p,q\}$, $s>i$, and $1\leq i< j< k< p< q\leq 6$.

$$\text{Then } \theta f\ \left(x_r\left(x_s^2 - x_i^2\right)\right) = \theta\,(-x_r \sum_{\substack{t=i \\ t\neq s}}^{q} x_t^2\, x_s^4 + x_r \sum_{t=j}^{q} x_t^2 x_i^4)$$

$$= -\sum_{\substack{t=i \\ t\neq s}}^{q} x_r\, x_t^2 + \sum_{t=j}^{q} x_r\, x_t^2$$

$$= -\sum_{\substack{t=j \\ t\neq s}}^{q} x_r\, x_t^2 - x_r\, x_i^2 + \sum_{\substack{t=j \\ t\neq s}}^{q} x_r\, x_t^2 + x_r\, x_s^2$$

$$= x_r\, x_s^2 - x_r\, x_i^2 = x_r\left(x_s^2 - x_i^2\right)$$

Which mean that $\theta\, f$ is the identity on G_F^2 ,therefore the sequence split.

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المستخلص:

يهدف هذا البحث الى إيجاد متسلسلات مضبوطة (exact) ومنفلقة (split) لموديولات جزئية من موديولات التمثيل للزوج الثاني من السسناريات وموديولات التمثيل للزوج الثالث من السسناريات ومن النوعين الأول والثاني على جبر الزمرة FW_6 ، عندما يكون F حقلا ذا مميز 0 .وتجلى هذا الهدف بتقديم النتائج الأساسية من خلال المبرهنتين (3-1) و (3-2) حيث قدمنا اثنتين لهذا النوع من المتسلسلات. وهنا نشير إلى أن الاعتماد على البناء التركيبي لعناصر(موديولات) هذه المتسلسلات او الموديولات الجزئية لها من خلال حساب البعد (dimension) لها كان أساسيا في إثبات هذه المبرهنات.