Generalized Left Jordan ideals In Prime Rings

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Abstract

Let R be a prime ring and U be a (σ,τ)-left Jordan ideal .Then in this paper, we proved the following , if $aU \subset Z$ ($Ua \subset Z$), $a \in R$, then $a = 0$ or $U \subset Z$. If $aU \subset C_{\sigma,\tau}$ ($Ua \subset C_{\sigma,\tau}$), $a \in R$, then either $a = 0$ or $U \subset Z$. If $0 \neq [U,U]_{\sigma,\tau} \in Z$.Then $U \subset Z$. If $0 \neq [U,U]_{\sigma,\tau} \subset C_{\sigma,\tau}$, then $U \subset Z$ .Also, we checked the converse some of these theorems and showed that are not true, so we give an example for them.

Keywords: R be a prime ring, Z be a center of R, $C_{\sigma,\tau}$ be a (σ,τ)-centeralizer,U be (σ,τ)-left Jordan ideal of R , F be a field and d be a derivation of R.

(1)Introduction

Many authors studied Jordan ideals & Jordan ideals with derivation and proved many results when the ring is prime or semiprime .In the end of the twentieth century and the beginning of this century, Neset Aydin , H. Kandamar and K. Kaya Studied (σ,τ)-right Jordan ideals and proved that if R is a prime ring and U is a (σ,τ)-right Jordan ideal of R, then (i) if $(U,U)_{\sigma,\tau} \subset C_{\sigma,\tau}$, then R is commutative (ii) if U is commutative then R is commutative.(iii) $aU=0$ (or $Ua=0$) and $a \in R$, then $a=0$.(iv) if $U \subset C_{\sigma,\tau}$, then R is commutative ,see[2].

Also, Kassim A.Jassim proved when U is a (σ,τ)-left Jordan ideal of R that (i) if $aU=0$ (or $Ua=0$) and $a \in R$, then $a=0$ or $U \subset Z$.(ii) if characteristic of R not equal 2 and $U \subset C_{\sigma,\tau}$, then $\sigma(u)+\tau(u) \in Z(R)$ for all $u \in U$.(iii) if $d(U)=0$, $d\tau=\tau d$ and $d\sigma=\sigma d$,then $\sigma(u)+\tau(u) \in Z(R)$ for all $u \in U$,see[1].

In this paper we want to study the generalization some of above results in (σ,τ)-left Jordan ideal of R . So, we must recall the basic terms that we need them in this research, as the ring R is a prime ring if $aRb=0$,$a,b \in R$ implies that $a=0$ or $b=0$. Also, we must recall Z is the center of R if $r \in Z(R)$, then $rx=xr$ for all $x \in R$. Also ,we recall (σ,τ)-centeralizer $C_{\sigma,\tau}$ if $r \in C_{\sigma,\tau}$, then for all $x \in R$ $r\sigma(x)=\tau(x)r$ .Also we recall the product [,] on R as follows $[x,y]=xy-yx$, see [4]. Also, we used the identities in this paper as follows: For all $x ,y ,z \in R$. (i) $[xy,z]= x [y,z] + [x,z] y$. (ii)$[x,yz]= [x,y] z + y [x,z]$. (iii)$[xy,z]_{\sigma,\tau} = x [y, \sigma(z)]+ [x,z]_{\sigma,\tau}\, y = x[y,z]_{\sigma,\tau}$

$+[x,\ \tau(z)]y$ .see[5] . Also, the Jordan product is define as follows: (i)$(x,y)_{\sigma,\tau}=x\sigma(y)+\tau(y)x$.(ii)$(xy,z)_{\sigma,\tau}=x(y,z)_{\sigma,\tau}-[x,\tau(z)]y=x[y,\sigma(z)]+(x,z)_{\sigma,\tau}y$.

In this paper we considered U be an additive subgroup of R. $\sigma,\tau: R\rightarrow R$ be two mappings of R. Then we can defined U is a (σ,τ)-right Jordan ideal of R if $(U,R)_{\sigma,\tau}\subset U$. Also, U is (σ,τ)-Left Jordan Ideal of R if $(R,U)_{\sigma,\tau}\subset U$. So, a (σ,τ)-Jordan ideal of R, if U is a (σ,τ)-right and Left Jordan Ideal of R [3].

Also, every (σ,τ)-left Jordan ideal is a Jordan ideal but the converse is not true and the following example showed that

Example(1.1)[2]

Let $R=\left\{\begin{pmatrix} x & y \\ z & t \end{pmatrix},\ x,y,z,t\in F\right\}$, where F is a field of $ChF=2$ be a ring of 2x2 matrices with respect to the usual operations of addition and multiplication.

$U=\left\{\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix},\ a,b\in F\right\}$ be an additive subgroup of R .Let $\sigma,\tau : R\rightarrow R$ be two mappings, such that

$$\sigma\begin{pmatrix} x & y \\ z & t \end{pmatrix}=\begin{pmatrix} x & y \\ z & t \end{pmatrix},\ \tau\begin{pmatrix} x & y \\ z & t \end{pmatrix}=\begin{pmatrix} t & -z \\ -y & x \end{pmatrix}.\quad \text{Then}$$

$$\begin{pmatrix} x & y \\ z & t \end{pmatrix}\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}+\begin{pmatrix} b & 0 \\ 0 & a \end{pmatrix}\begin{pmatrix} x & y \\ z & t \end{pmatrix}=\begin{pmatrix} xa & yb \\ za & tb \end{pmatrix}+\begin{pmatrix} bx & by \\ az & at \end{pmatrix}=\begin{pmatrix} xa+bx & yb+by \\ za+az & tb+at \end{pmatrix}$$

$$=\begin{pmatrix} xa+bx & 0 \\ 0 & tb+at \end{pmatrix}\in U\ .$$

Thus, U is a (σ,τ)-left Jordan ideal of R, but U is not a Jordan ideal of R as follows

$$\begin{pmatrix} x & y \\ z & t \end{pmatrix}\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}+\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}\begin{pmatrix} x & y \\ z & t \end{pmatrix}=\begin{pmatrix} xa+ax & yb+ay \\ za+bz & tb+bt \end{pmatrix}=\begin{pmatrix} 0 & yb+ay \\ za+bz & 0 \end{pmatrix}\notin U$$

2.The Main Results

At first, we generalizing $aU=0(Ua=0)$ in [1] as below.

Theorem (2.1) :- Let U be a (σ,τ) – left Jordan ideal , $a\in R$, if $aU\subset Z$ ($Ua\subset Z$), then $a=0$ 0r $U\subset Z$.

Proof :- By the hypothesis $aU\subset Z$, so for all $u\in U$, $x\in R$, we have

$a(x\,\sigma(u)\,,u)_{\sigma,\tau} = a\,x[\sigma(u),\sigma(u)] + a(x,u)_{\sigma,\tau}\sigma(u)$ for all $u\in U$, $x\in R$

$= a\,(x,u)_{\sigma,\tau}\,\sigma(u)$ .Since $a(x\sigma(u),u)_{\sigma,\tau}\in Z$ for all $u\in U$, $x\in R$, then $a(x\sigma(u)\,,u)_{\sigma,\tau}= a\,(x,u)_{\sigma,\tau}\,\sigma(u)$ .So,we get $a(x,u)_{\sigma,\tau}\,\sigma(u)\in Z$ .This is implies that for all $r\in R$, we get $[a(x\,,u)_{\sigma,\tau}\,\sigma(u)\,,r\,]= 0$ for all $u\in U$, $x\in R$. Therefore, we have

$0= [a(x\,,u)_{\sigma,\tau}\,\sigma(u)\,,r\,]=a(x\,,u)_{\sigma,\tau}\,[\sigma(u)\,,r\,] + [a(x\,,u)_{\sigma,\tau}, r\,]\,\sigma(u)$, for all $u\in U$, $x,r\in R$. Since by hypothesis $aU\subset Z$, then $[a(x\,,u)_{\sigma,\tau}, r\,]\,\sigma(u)=0$, for all $u\in U$, $x,r\in R$. Thus

$a(x , u)_{\sigma,\tau} [\sigma(u) , r] =0$, for all $u \in U$, $x,r \in R$. Since $aU \subset Z$, $a(x , u)_{\sigma,\tau} R[\sigma (u) , r] = 0$, for all $u \in U$, $x,r \in R$. Also, we have R is a prime ring then either

$a(x , u)_{\sigma,\tau}=0$, for all $u \in U$, $x \in R$. or $[\sigma (u) , r] = 0$, for all $u \in U$, $r \in R$.

If $a(x , u)_{\sigma,\tau}=0$, for all $u \in U$, $x \in R$, then by the [1] , we get either $a=0$ or $U \subset Z$ .If $[\sigma (u) , r] = 0$, for all $u \in U$, $r \in R$, then $U \subset Z$

From the other hand if $Ua \subset Z$, then for all $u \in U$, $x \in R$.

$(\tau (u) x , u)_{\sigma,\tau} a = \tau (u) (x , u)_{\sigma,\tau} a- [\tau (u) , \tau (u)]x a = \tau (u) (x , u)_{\sigma,\tau} a$, for all $u \in U$, $x \in R$. So, we have

$(\tau (u) x , u)_{\sigma,\tau} a = \tau (u) (x , u)_{\sigma,\tau} a$, for all $u \in U$, $x \in R$ .Since $(\tau (u) x , u)_{\sigma,\tau} a \in Z$ (By hypothesis), then $\tau (u) (x , u)_{\sigma,\tau} a \in Z$, for all $u \in U$, $x \in R$. Therefore, for all $r \in R$

$0= [\tau (u) (x , u)_{\sigma,\tau} a, r]$, for all $r \in R$.

$=\tau (u) [(x , u)_{\sigma,\tau} a, r] + [\tau (u) , r] (x , u)_{\sigma,\tau} a$, for all $u \in U$, $x, r \in R$. Since $Ua \subset Z$, then $[(x , u)_{\sigma,\tau} a, r] =0$, for all $u \in U$, x, r $\in$ R .Thus , we have

$[\tau (u) , r] (x , u)_{\sigma,\tau}$ a =0 , for all $u \in U$, $x , r \in R$. Since $Ua \subset Z$, implies that

$[\tau (u) , r] R(x , u)_{\sigma,\tau} a = 0$, for all $u \in U$, $x, r \in R$. Since R is prime ring ,then either $[\tau (u) , r] = 0$ or $(x , u)_{\sigma,\tau} a = 0$, for all $u \in U$, $x, r \in R$.

If $[\tau (u) ,r] = 0$, for all $u \in U$, then implies that $U \subset Z$

If $(x , u)_{\sigma,\tau} a = 0$ for all $u \in U$, $x \in R$, then by [1] we get either $a=0$ or $U \subset Z$.

Also, we generalized the above Theorem as below.

<u>Theorem (2.2)</u> :- Let U be a (σ,τ) – left Jordan ideal , $a \in R$, if $aU \subset C_{\sigma,\tau}$ ($Ua \subset C_{\sigma,\tau}$), then either $a = 0$ or $U \subset Z$

Proof: - By hypothesis if $aU \subset C_{\sigma,\tau}$, we have

$a(x\sigma(u),u)_{\sigma,\tau}= a x[\sigma(u), \sigma(u)] + a(x , u)_{\sigma,\tau} \sigma(u) = a (x ,u)_{\sigma,\tau}\sigma (u)$, for all $u \in U$, $x \in R$. Therefore, we have

$a(x \sigma(u) , u)_{\sigma,\tau}= a (x , u)_{\sigma,\tau}\sigma (u)$, for all $u \in U$, $x \in R$. Since $aU \subset C_{\sigma,\tau}$, then $a(x \sigma(u) , u)_{\sigma,\tau} \in C_{\sigma,\tau}$, for all $u \in U$, $x \in R$ and also, we have $a (x , u)_{\sigma,\tau}\sigma (u) \in C_{\sigma,\tau}$, for all $u \in U$, $x \in R$. Then for all $r \in R$, we get $[a(x , u)_{\sigma,\tau}\sigma(u) , r]_{\sigma,\tau} = 0$

$0= a(x , u)_{\sigma,\tau} [\sigma(u) , \sigma(r)] + [a(x , u)_{\sigma,\tau}, r]_{\sigma,\tau}\sigma(u) =a(x , u)_{\sigma,\tau} [\sigma(u) , \sigma(r)] = 0$, for all $u \in U$, $x,r \in R$. Then , we have $a(x , u)_{\sigma,\tau} [\sigma(u) , \sigma(r)] = 0$, for all $u \in U$, $x,r \in R$. Thus,

implies that $\tau(y) a(x , u)_{\sigma,\tau} [\sigma(u) , \sigma(r)] = 0$, for all $u \in U$, $x,y,r \in R$. Since $aU \subset C_{\sigma,\tau}$, then $a(x , u)_{\sigma,\tau}\sigma(y) [\sigma(u) , \sigma(r)] = 0$, for all $u \in U$, $x,y,r \in R$. Therefore,

$a(x\,,u)_{\sigma,\tau}\,R\,[\sigma(u)\,,\sigma(r)\,]=0$, for all $u\in U$, $x,r\in R$. Since R is prime ring , then either $a(x\,,u)_{\sigma,\tau}=0$ or $[\sigma(u)\,,\sigma(r)\,]=0$, for all $u\in U$, $x,r\in R$.

If $a(x\,,u)_{\sigma,\tau}=0$, for all $u\in U$, $x\in R$.Then by [1] , $aU=0$ implies that eithr $a=0$ or $U\subset Z$ .If $[\sigma(u)\,,\sigma(r)\,]=0$, for all $u\in U$, $r\in R$. implies that $U\subset Z$. For the other hand , if $Ua\subset C_{\sigma,\tau}$, then , for all $u\in U$, $x\in R$, we get $(\tau(u)\,x\,,u\,)_{\sigma,\tau}\,a=\tau(u)\,(x\,,u\,)_{\sigma,\tau}\,a-[\tau(u)\,,\tau(u)]x\,a=\tau(u)\,(x\,,u\,)_{\sigma,\tau}\,a$. Since $Ua\subset C_{\sigma,\tau}$, then $(\tau(u)\,x\,,u\,)_{\sigma,\tau}\,a\in C_{\sigma,\tau}$ and also we have $\tau(u)\,(x\,,u\,)_{\sigma,\tau}a\in C_{\sigma,\tau}$.Therefore,

$[\tau(u)\,(x\,,u\,)_{\sigma,\tau}\,a,\,r\,]_{\sigma,\tau}=0$,$r\in R$, for all $u\in U$, $x,r\in R$. Thus

$0=[\tau(u)\,(x\,,u\,)_{\sigma,\tau}\,a,\,r\,]_{\sigma,\tau}=\tau(u)\,[\,(x\,,u\,)_{\sigma,\tau}a,\,r\,]_{\sigma,\tau}+[\tau(u)\,,\tau(r)\,]\,(x\,,u\,)_{\sigma,\tau}a$
$=[\tau(u)\,,\tau(r)\,]\,(x\,,u\,)_{\sigma,\tau}\,a$ .Therefore , $[\tau(u)\,,\tau(r)\,]\,(x\,,u\,)_{\sigma,\tau}\,a=0$, for all $u\in U$, $x,r\in R$. Also , we have $[\tau(u)\,,\tau(r)\,]\,(x\,,u\,)_{\sigma,\tau}\,a\,\sigma(y)=0$, for all $u\in U$, $x,y,r\in R$. So, by $Ua\subset C_{\sigma,\tau}$, we get that $[\tau(u)\,,\tau(r)\,]\,\tau(y)\,(x\,,u\,)_{\sigma,\tau}a=0$.So, we get for all $u\in U$, $x,y,r\in R$ $[\tau(u)\,,\tau(r)\,]\,R\,(x\,,u\,)_{\sigma,\tau}a=0$. Since R is a prime ring, then either

$[\tau(u),\tau(r)]=0$ or $(x,u)_{\sigma,\tau}a=0$, for all $u\in U$, $x,r\in R$.

If $[\tau(u),\tau(r)]=0$, for all $u\in U$, $r\in R$. Then we get $U\subset Z$. If $(x\,,u\,)_{\sigma,\tau}a=0$, for all $u\in U$, $x\in R$, then by [1] , we get $a=0$ or $U\subset Z$.

Now, the below theorem shows that if $0\neq[U,U]_{\sigma,\tau}\subset Z$,then $U\subset Z$.

<u>Theorem (2.3)</u> :- Let U be a (σ,τ) – left Jordan ideal ,if $0\neq[U,U]_{\sigma,\tau}\subset Z$, then $U\subset Z$.

Proof :- By the hypothesis , we get $[(x\,\sigma(u)\,,u)_{\sigma,\tau},u\,]_{\sigma,\tau}\in Z$, for all $u\in U$, $x\in R$ .So, we have $[(x\,\sigma(u)\,,u)_{\sigma,\tau},u\,]_{\sigma,\tau}=[\,x\,[\sigma(u)\,,\sigma(u)\,]+(x\,,u\,)_{\sigma,\tau}\sigma(u),\,u\,]_{\sigma,\tau}$
$=[\,(x\,,u\,)_{\sigma,\tau}\,\sigma(u),\,u\,]_{\sigma,\tau}$

Since $[U,U]_{\sigma,\tau}\subset Z$, we have $[(x\,\sigma(u)\,,u)_{\sigma,\tau},u\,]_{\sigma,\tau}\in Z$.

Therefore, $[\,(x,u\,)_{\sigma,\tau}\,\sigma(u),\,u\,]_{\sigma,\tau}\in Z$, for all $u\in U$, $x\in R$. Also, we have

$[\,(x,u\,)_{\sigma,\tau}\,\sigma(u),\,u\,]_{\sigma,\tau}=(x\,,u\,)_{\sigma,\tau}\,[\sigma(u),\sigma(u)\,]+[\,(x\,,u\,)_{\sigma,\tau},\,u\,]_{\sigma,\tau}\sigma(u)$
$=[\,(x\,,u\,)_{\sigma,\tau},\,u\,]_{\sigma,\tau}\sigma(u)\in Z$, for all $u\in U$, $x\in R$. Also ,we have

$0=[[\,(x\,,u\,)_{\sigma,\tau},\,u\,]_{\sigma,\tau}\sigma(u),\,r\,]=[(x\,,u\,)_{\sigma,\tau},\,u\,]_{\sigma,\tau}\,[\sigma(u),\,r\,]+[[(x\,,u\,)_{\sigma,\tau},\,u\,]_{\sigma,\tau}\,,\,r]\,\sigma(u)$,

for all $u\in U$, $x,r\in R$. By the hypothesis $[U,U]_{\sigma,\tau}\subset Z$, then $[(x\,,u\,)_{\sigma,\tau},\,u\,]_{\sigma,\tau}\,[\sigma(u),\,r\,]=0$. Since $[U,U]_{\sigma,\tau}\subset Z$, we get $[(x\,,u\,)_{\sigma,\tau},\,u\,]_{\sigma,\tau}\,R[\sigma(u),\,r\,]=0$, for all $u\in U$, $x,r\in R$. By the primeness of R we get either

$[\sigma(u),\,r\,]=0$, for all $u\in U$, $r\in R$ or $[(x\,,u\,)_{\sigma,\tau},\,u\,]_{\sigma,\tau}=0$, for all $u\in U$, $x\in R$ and this a contradiction with the hypothesis .So, we get $[\sigma(u),\,r\,]=0$, for all $u\in U$, $r\in R$. Therefore $U\subset Z$.

Remark(2.4):

The converse for the above Theorem is not necessary true all the time, and the following example shows

Example(2.5)[2]

Let $R=\left\{\begin{pmatrix} x & y \\ z & t \end{pmatrix}, x, y, z, t \in F\right\}$, where F is a field of ChF=2 be a ring of 2x2 matrices with respect to the usual operations of addition and multiplication.

$U=\left\{\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}, a \in F\right\}$ be an additive subgroup of R .Let $\sigma, \tau : R \to R$ be two mappings , such that

$$\sigma\begin{pmatrix} x & y \\ z & t \end{pmatrix}=\begin{pmatrix} x & y \\ z & t \end{pmatrix}, \tau\begin{pmatrix} x & y \\ z & t \end{pmatrix}=\begin{pmatrix} t & -z \\ -y & x \end{pmatrix}. \quad \text{Then}$$

$$\begin{pmatrix} a_1 & 0 \\ 0 & a_1 \end{pmatrix}\begin{pmatrix} a_2 & 0 \\ 0 & a_2 \end{pmatrix}+\begin{pmatrix} a_2 & 0 \\ 0 & a_2 \end{pmatrix}\begin{pmatrix} a_1 & 0 \\ 0 & a_1 \end{pmatrix}=\begin{pmatrix} a_1a_2 & 0 \\ 0 & a_1a_2 \end{pmatrix}+\begin{pmatrix} a_2a_1 & 0 \\ 0 & a_2a_1 \end{pmatrix}=\begin{pmatrix} 2(a_1a_2) & 0 \\ 0 & 2(a_1a_2) \end{pmatrix}$$

$$=\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \in U$$ .Then U be a (σ,τ) – left Jordan ideal

So, by the hypothesis $[U,U]_{\sigma,\tau} \subset Z$ we can show this condition

$$\begin{pmatrix} a_1 & 0 \\ 0 & a_1 \end{pmatrix}\begin{pmatrix} a_2 & 0 \\ 0 & a_2 \end{pmatrix}-\begin{pmatrix} a_2 & 0 \\ 0 & a_2 \end{pmatrix}\begin{pmatrix} a_1 & 0 \\ 0 & a_1 \end{pmatrix}=\begin{pmatrix} a_1a_2 & 0 \\ 0 & a_1a_2 \end{pmatrix}-\begin{pmatrix} a_2a_1 & 0 \\ 0 & a_2a_1 \end{pmatrix}=\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$ .But this result is

Contradict with the hypothesis $[U,U]_{\sigma,\tau} \neq 0$.

Also, we generalized the above Theorem as below.

Theorem (2.6) :- Let U be a (σ,τ) – left Jordan ideal ,if $0 \neq [U,U]_{\sigma,\tau} \subset C_{\sigma,\tau}$, then $U \subset Z$.

Proof: - By the hypothesis , we get $[(x\,\sigma(u), u)_{\sigma,\tau}, u]_{\sigma,\tau} \in C_{\sigma,\tau}$, for all $u \in U$, $x \in R$ .So, we have $[(x\,\sigma(u), u)_{\sigma,\tau}, u]_{\sigma,\tau} = [x[\sigma(u), \sigma(u)] + [(x, u)_{\sigma,\tau}\sigma(u), u]_{\sigma,\tau}$

$$=[(x, u)_{\sigma,\tau}\,\sigma(u), u]_{\sigma,\tau}$$

Since $[U,U]_{\sigma,\tau} \subset C_{\sigma,\tau}$, we have $[(x\,\sigma(u), u)_{\sigma,\tau}, u]_{\sigma,\tau} \in C_{\sigma,\tau}$ and therefore, $[(x, u)_{\sigma,\tau}\,\sigma(u), u]_{\sigma,\tau} \in C_{\sigma,\tau}$, for all $u \in U$, $x \in R$. Also, we have

$[(x, u)_{\sigma,\tau}\,\sigma(u), u]_{\sigma,\tau} = (x, u)_{\sigma,\tau}[\sigma(u), \sigma(u)] + [(x, u)_{\sigma,\tau}, u]_{\sigma,\tau}\sigma(u)$

$\qquad = [(x, u)_{\sigma,\tau}, u]_{\sigma,\tau}\sigma(u) \in C_{\sigma,\tau}$, for all $u \in U$, $x \in R$.

Therefore, $[[(x, u)_{\sigma,\tau}, u]_{\sigma,\tau}\sigma(u), r]_{\sigma,\tau} = 0$, for all $u \in U$, $x,r \in R$. So, $0=[[(x, u)_{\sigma,\tau}, u]_{\sigma,\tau}\sigma(u), r]_{\sigma,\tau}=[(x, u)_{\sigma,\tau}, u]_{\sigma,\tau}[\sigma(u), \sigma(r)] + [[(x, u)_{\sigma,\tau}, u]_{\sigma,\tau}, r]_{\sigma,\tau}\sigma(u) = 0$, for all $u \in U$, $x,r \in R$. Also , we have $[(x, u)_{\sigma,\tau}u]_{\sigma,\tau}[\sigma(u), \sigma(r)]_{\sigma,\tau} = 0$.

Also, $\tau(y)[(x, u)_{\sigma,\tau}u]_{\sigma,\tau}[\sigma(u), \sigma(r)]_{\sigma,\tau} = 0$, for all $u \in U$, $x,r,y \in R$. Also, $[(x, u)_{\sigma,\tau}u]_{\sigma,\tau}\sigma(y)[\sigma(u), \sigma(r)]_{\sigma,\tau} = 0$, for all $u \in U$, $x,r,y \in R$. Then

$[(x,u)_{\sigma,\tau} u]_{\sigma,\tau}\sigma(y)[\sigma(u),\sigma(r)]_{\sigma,\tau}=0$, for all $u\in U$, $x,r\in R$ .Also, we have $[(x,u)_{\sigma,\tau} u]_{\sigma,\tau} R[\sigma(u),\sigma(r)]_{\sigma,\tau}=0$, for all $u\in U$, $x,r\in R$. By the primeness of R , we get $[(x,u)_{\sigma,\tau}, u]_{\sigma,\tau}=0$ or $[\sigma(u),\sigma(r)]=0$ for all $u\in U$, $x,r\in R$.

If $[(x,u)_{\sigma,\tau}, u]_{\sigma,\tau}=0$ contradiction with the hypothesis. If $[\sigma(u),\sigma(r)]=0$ for all $u\in U$, $r\in R$, implies that $U\subset Z$.

Remark(2.7):

Also, by the previous example(2.5) we can show the above theorem is not necessary true all the time.

References

1.Abdul-Hameed, Kassim. , Some results on (σ,τ)-Left Jordan ideals in prime rings. Journal of science-Al-Nahrain Univ. (To appear)

2.Abdul-Ruhman hameed and Kassim Abdul-hameed ,Some results on (σ,τ)-Lie ideals in prime rings, Iraqi Journal Of Science,Vol.49,No.1,2008,pp.188-191.

3. Aydin Neset ,H. Kandamar and K. Kaya , Generalized Jordan structure of Prime rings , Doga Tr.J. of Mathematics Vol. 17 , 1993,pp 251-258.

4. Herstein,I.N. On the Lie structure of an associative ring, Journal of Algebra Vol.14 , 1970, pp.561-571.

5. Kaya,K. , Aydin,N. , Ogolbasi,O. , Some results for generalized Lie ideals with derivation II ,Applied Mathematics E-Notes ,Vol.1 ,2001, pp 24-30.