Applications of Collocation Method for solving IDE and Signal Processing

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Abstract:

In this work, the display of Chebyshev Wavelets (CW) functions and their Operational Matrix of Integration (OMI) are provided. The method is based on the approximation of the first Chebysheve wavelets. It has been applied to solve signal processing such as electricity consumption signal in addition to medical applications and some Integro-Differential Equations (IDE). In the end, some of numerical examples and applications of signal processing are given, they are solved by using the presented method and we found that this method is the most efficient way, The signal was processed using the proposed method and denoise from it and the Matlab program was used after processing the proposed theory to be used to solve all the above problems.

Keywords: First Chebyshev Wavelets, Integro-Differential Equations, Operational Matrix of Integration , Signal processing.

1. Introduction

(IDE) arise in many engineering and scientific disciplines, often as approximations to partial equations, many from these equations are possible. The use of the wavelet transform to analyze the behavior of the complex systems from various fields started to be widely recognized and applied successfully during the last few decades. Particularly in last 10 years, great progress has been made in the theory and applications of wavelets and many publications have been seen in the field of fault diagnosis [1]. The method of Hermite wavelets for solving nonlinear Variational problems [2], First kind CW method for solving linear Fredholm IDE [3], Legender wavelets method is applied to Fredholm integral equation of the second kind [4], the method of Hermite wavelets is used to solve nth-order Volterra IDE [5], the method of collocation wavelet to solve IDE [6], wavelets of Haar are used to find the solution of IDE [7]. Applications are presented in signal processing, electrical systems, fault diagnosis and monitoring and image processing [1]. Many applications of wavelet transform in fault diagnosis of rotary machines are discussed in [8]. This paper will construct the first Chebyshev wavelets on the interval [0,1], the wavelets basis are suitable for numerical solutions of the differential equation. The OMP of the first CW is presented and applied for obtaining approximate solution of the following nth-order VIDE.

$$u^{(n)}(y)=g(y)+\int_0^y f(y,t)u^{(s)}(t)dt \qquad (1)$$

where f(y,t) and g(y) are known functions, and u(y) is an unknown function.

2. First kind Chebysheve Polynomials (FKCP)

These polynomials defined in [9]:

$$T_a(x) = \cos(a \arccos x) \qquad (2)$$

where a =0,1,2,…

A recurrence formula for $T_a(x)$ is given by

$$T_{a+1}(x) = 2xT_a(x) - T_{a-1}(x) \qquad (3)$$

And the generating function is

$$\frac{1-xt}{1-2xt+t^2} = \sum_{a=0}^{\infty} T_a(x)t^a \qquad (4)$$

Here $T_a(x)$ are the FKCP of order a, they are orthogonal with the function of weight $V(x) = \frac{1}{\sqrt{1-x^2}}, x \neq \pm 1$, ,

$$T_0(x) = 1, T_1(x) = x \qquad (5)$$

We also have

$$\int_{-1}^{1} \frac{T_a(x)\,T_b(x)}{\sqrt{1-x^2}} = 0 \;\; , a \neq b, x \neq \pm 1 \qquad (6)$$

$$\int_{-1}^{1} \frac{(T_a(x))^2}{\sqrt{1-x^2}} = \begin{cases} \pi & a = 0 \\ \frac{\pi}{2} & a = 1,2,\ldots \end{cases} , x \neq \pm 1 \qquad (7)$$

3. Wavelets

Mother wavelet is a family of functions constructed from translation and dilation of a single function. Several methods of wavelets used to approximate the solution of the differential and integral equations. The mother wavelet $\mu(x)$ is a family of functions constructed from dilation and translation of a single function. If the dilation parameter c and the translation parameter d disparity continuously, we have the following family of continuous wavelet [2-6].

$$\mu_{c,d}(x) = |c|^{\frac{-1}{2}} \mu\left(\frac{x-d}{c}\right), \; d \in R, \; c \in R^+. \qquad (8)$$

4. Chebyshev wavelets

4.1 First kind of Chebyshev wavelets (FKC) [9]

Chebyshev wavelets : $\mu_{a,f}(x) = \mu(x, a, f, h)$ have four arguments; $h = 1,2,3, \dots$, $f = 1,2,3, \dots, 2^h$, a is the order of CP and x is adjusted time. They are defined on the interval [0,1) [3,8]:

$$\mu_{a,f}(x) = \begin{cases} \frac{\alpha_a 2^{h/2}}{\sqrt{\pi}} T_a(2^{h+1}x - 2f + 1) & \frac{f-1}{2^h} \le x < \frac{f}{2^h} \\ 0 & otherewise \end{cases} \qquad (9)$$

$$\alpha_a = \begin{cases} \sqrt{2} & a = 0 \\ 2 & a = 1,2,3, \dots \end{cases}$$

(10)

The set of CP are orthogonal with the function of weight

$V_f(x) = V(2^{h+1}x - 2f + 1)$ (11)

4.2 The OMI for CW

In this section, first P will be found the OMI which are 6×6, the following six functions are [10]:

$$\mu_{1,0}(x) = \begin{cases} \frac{2}{\sqrt{\pi}} & 0 \le x < \frac{1}{2} \\ 0 & otherwise \end{cases} \qquad (12)$$

$$\mu_{1,1}(x) = \begin{cases} \frac{2\sqrt{2}}{\sqrt{\pi}}(4x-1) & 0 \le x < \frac{1}{2} \\ 0 & otherwise \end{cases} \qquad (13)$$

$$\mu_{1,2}(x) = \begin{cases} \frac{2\sqrt{2}}{\sqrt{\pi}}(2(4x-1)^2-1) & 0 \le x < \frac{1}{2} \\ 0 & otherwise \end{cases} \qquad (14)$$

$$\mu_{2,0}(x) = \begin{cases} \frac{2}{\sqrt{\pi}} & \frac{1}{2} \le x < 1 \\ 0 & otherwise \end{cases} \qquad (15)$$

$$\mu_{2,1}(x) = \begin{cases} \frac{2\sqrt{2}}{\sqrt{\pi}}(4x-3) & \frac{1}{2} \le x < 1 \\ 0 & otherwise \end{cases} \qquad (16)$$

$$\mu_{2,2}(x) = \begin{cases} \frac{2\sqrt{2}}{\sqrt{\pi}}(2(4x-3)^2-1) & \frac{1}{2} \le x < 1 \\ 0 & otherwise \end{cases} \qquad (17)$$

By using the first type of CW and integrating (13-17) we get:

$$\int_0^x \mu_{1,0}(x)dx = \frac{1}{4}\mu_{1,0}(x) + \frac{1}{8}\mu_{1,1}(x) + \frac{1}{2}\mu_{2,0}(x)$$

$$\int_0^x \mu_{1,1}(x)dx = \frac{-3}{16}\mu_{1,0}(x) + \frac{1}{16}\mu_{1,2}(x)$$

$$\int_0^x \mu_{1,2}(x)dx = \frac{1}{12}\mu_{1,0}(x) - \frac{1}{24}\mu_{1,1}(x)$$

$$\int_0^x \mu_{2,0}(x)dx = \frac{1}{4}\mu_{2,0}(x) + \frac{1}{8}\mu_{2,1}(x)$$

$$\int_0^x \mu_{2,1}(x)dx = \frac{-3}{16}\mu_{2,0}(x) + \frac{1}{16}\mu_{2,2}(x)$$

$$\int_0^x \mu_{2,2}(x)dx = \frac{1}{12}\mu_{2,0}(x) - \frac{1}{24}\mu_{2,1}(x)$$

$$\int_0^x \mu_6(x)dx = P_{6\times6}\mu_6(x) \qquad (18)$$

$$P_{6\times6} = \left(\begin{array}{ccc|ccc} 1/4 & 1/8 & 0 & 1/2 & 0 & 0 \\ -3/16 & 0 & 1/16 & 0 & 0 & 0 \\ 1/12 & -1/24 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 1/8 & 0 \\ 0 & 0 & 0 & -3/16 & 0 & 1/16 \\ 0 & 0 & 0 & 1/12 & -1/24 & 0 \end{array}\right)$$

where $\mu_6(x)$ is the basis of the wavelet function, in equation (18), we can write $P_{6\times6}$ as:

$$P_{6\times6} = \begin{bmatrix} F_{3\times3} & Q_{3\times3} \\ O_{3\times3} & F_{3\times3} \end{bmatrix}$$

where

$$F_{3\times3} = \begin{bmatrix} \frac{1}{4} & \frac{1}{8} & 0 \\ -\frac{3}{16} & 0 & \frac{1}{16} \\ \frac{1}{12} & -\frac{1}{24} & 0 \end{bmatrix},$$

$$Q_{3\times3} = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$O_{3\times3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

In general we have

$$\int_0^x \mu(x)dx = P\mu(x)\,, \qquad (19)$$

where

$$P = P_{2^{h-1}N\times2^{h-1}N} = \frac{1}{2^h}\begin{bmatrix} F & Q & Q & \cdots & Q & Q \\ O & F & Q & & Q & Q \\ & \vdots & & \ddots & & \vdots \\ O & O & O & \cdots & F & Q \\ O & O & O & & O & F \end{bmatrix},$$

where

$$Q = Q_{N\times N} = \begin{pmatrix} 2 & 0 & \cdots & 0 \\ 0 & 0 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \text{ and}$$

$$F = F_{N\times N} = \begin{bmatrix} 1 & 1/2 & & 0 & \cdots & 0 \\ -3/4 & 0 & \cdots & 1/4 & \cdots & 0 \\ 1/3 & -1/6 & & 0 & \cdots & 0 \\ -1/4 & 0 & & -1/8 & & 0 \\ \vdots & & \ddots & & \vdots & \\ (-1)^{N-2}\frac{1}{N-1} & 0 & \cdots & 0 & \cdots & \frac{1}{2(N-1)} \\ (-1)^{N-1}\frac{1}{N} & 0 & & 0 & \cdots & 0 \end{bmatrix}$$

5. Numerical solutions

In this section FCW is used to solve Volterra IDE and Fredholm IDE

<u>5.1 Study the following Volterra IDE:</u>

$Z'' = e^{2x} - \int_0^x e^{2(x-t)} Z'(t)dt$ (20)

The primary conditions $Z(0) = Z'(0) = 0.$

The exact solution $Z(x) = xe^x - e^x + 1$ (21)

Table1 shows the numerical results compared with the exact solution

Table 1: the numerical results

x	First CW	exact solution
0	0.00000000	0.00000000
0.2	0.02087779	0.02087779
0.4	0.10940518	0.10940518
0.6	0.27115248	0.27115248
0.8	0.55489181	0.55489181
1	1.00000000	1.00000000

<u>5.2 Study the following Fredholm IDE:</u>

$S^{(3)}(y) = \frac{55}{12} - 2y + \int_0^1 (y+t)\, S'(y)dy$ (22)

The initial conditions $S^{(2)}(0) = 2,\ S'(0) = S(0) = 0.$

The exact solution is $S(y) = y^3 + y^2$ (23)

Table 2 shows our numerical results compared with the exact solution

Table 2: the numerical results

y	First CW	exact solution

0	0	0
0.2	0.048000000000000	0.048000000000000
0.4	0.224000000000000	0.224000000000000
0.6	0.576000000000000	0.576000000000000
0.8	1.152000000000000	1.152000000000000
1	2	2

6. Signal Processing Using FCW

In this section FCW is used in signal processing such as electricity consumption signal in addition to medical applications to prove the efficiency of wavelet is processed the original signal with denoising it by using Matlab program.

The following figures show that:

6.1 Original signal with histogram

In this section the original signal with it's histogram with noising Fig.1 shows that

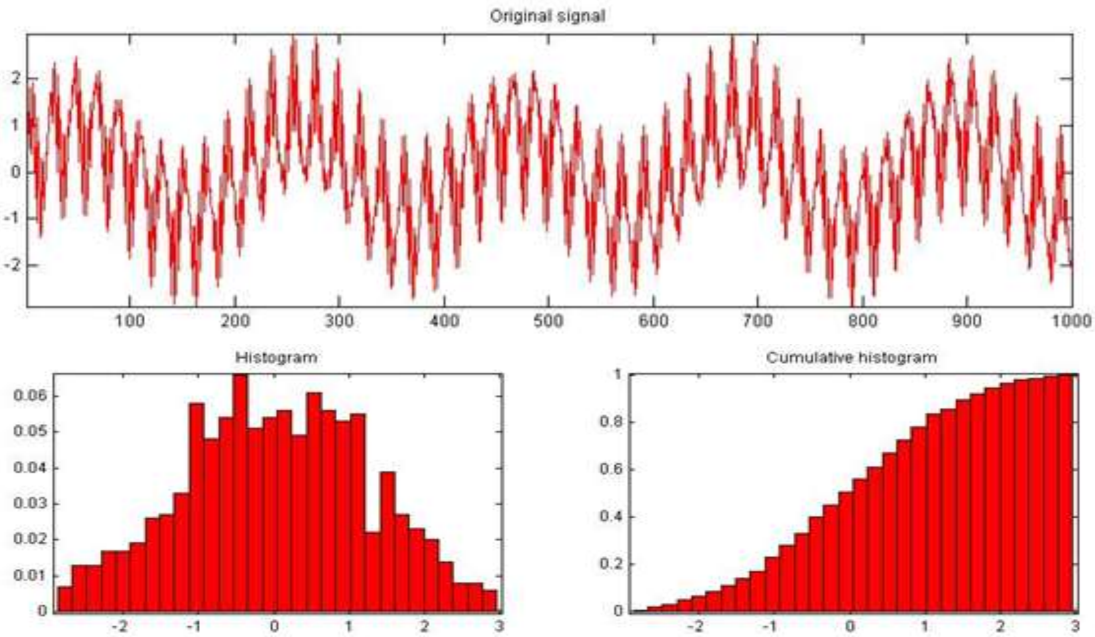

Fig.1 the original signal

and Fig.2 shows approximation coefficients after using wavelet to processed the original signal with denoising it.

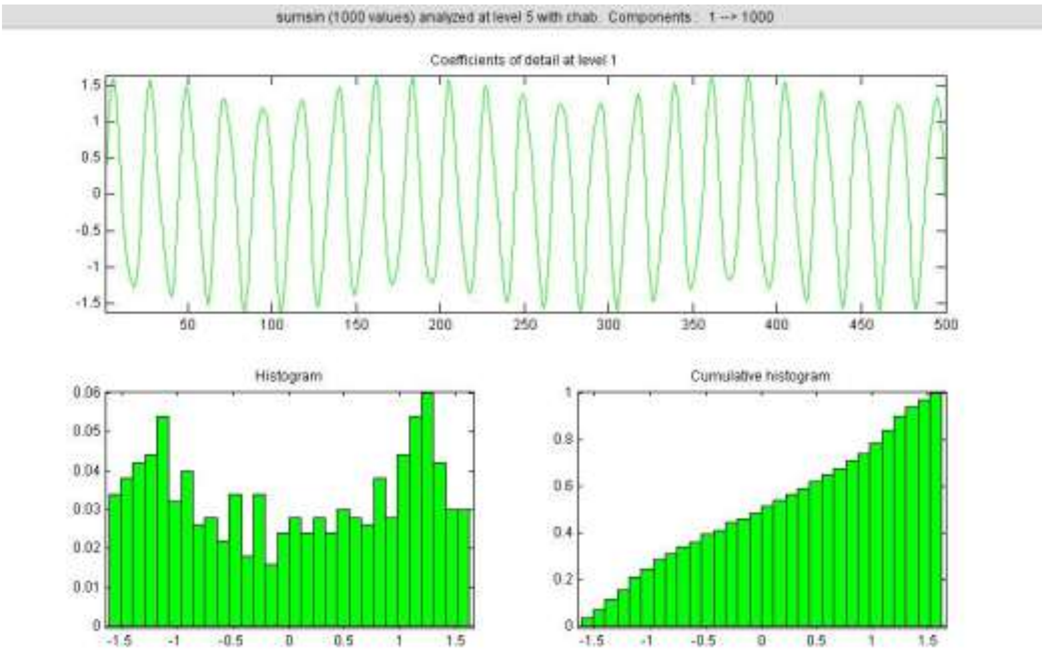

Fig.2 approximation coefficients

and Fig.3 shows detail coefficients after using wavelet is processed the original signal with denoising it.

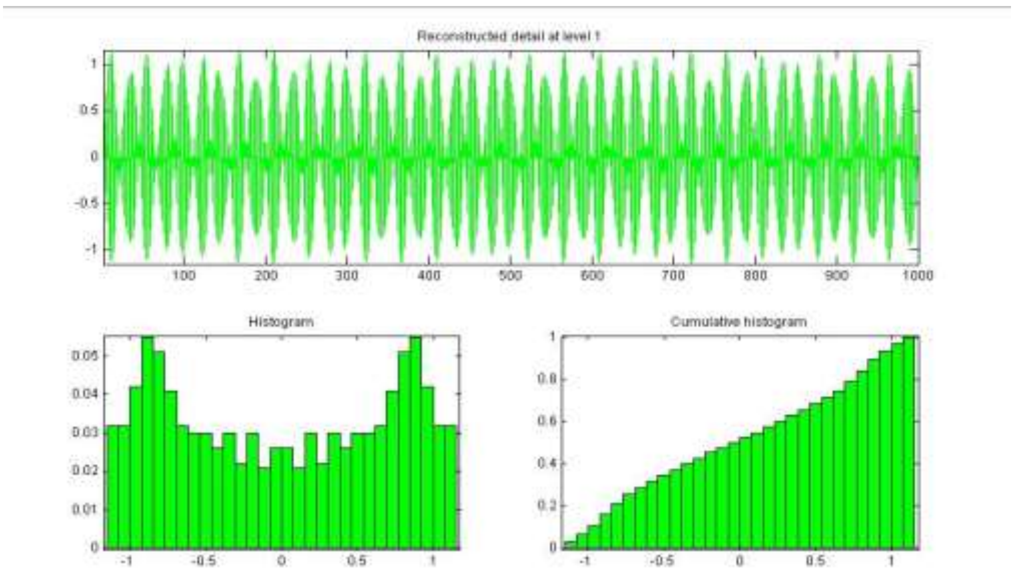

Fig.3 detail coefficients

and Fig.4 shows Synthesized signal after using wavelet to processed the original signal with denoising it.

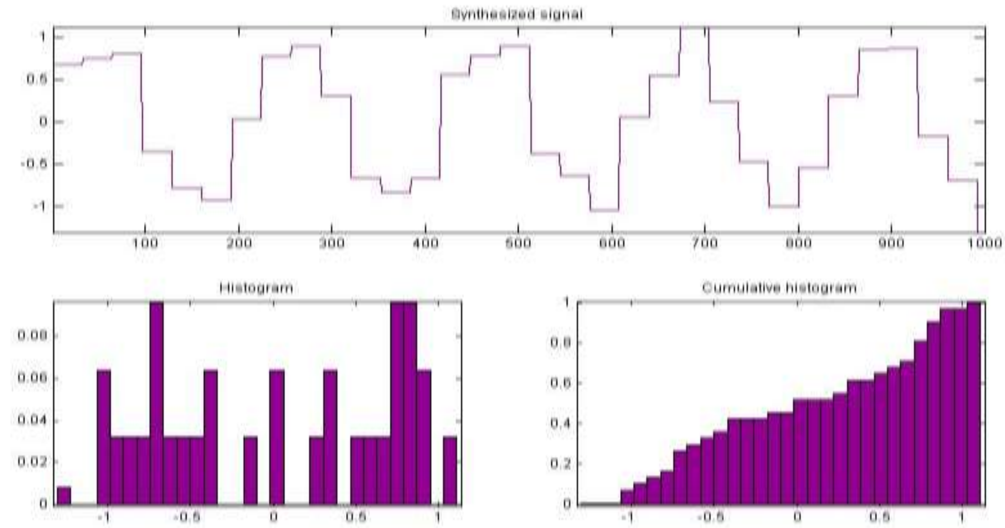

Fig.4 Synthesized signal

6.2 Signal Processing by using inverse wavelet

In this section the signal processing by using inverse wavelet we back to the original signal without losing the signal Fig.5 shows that

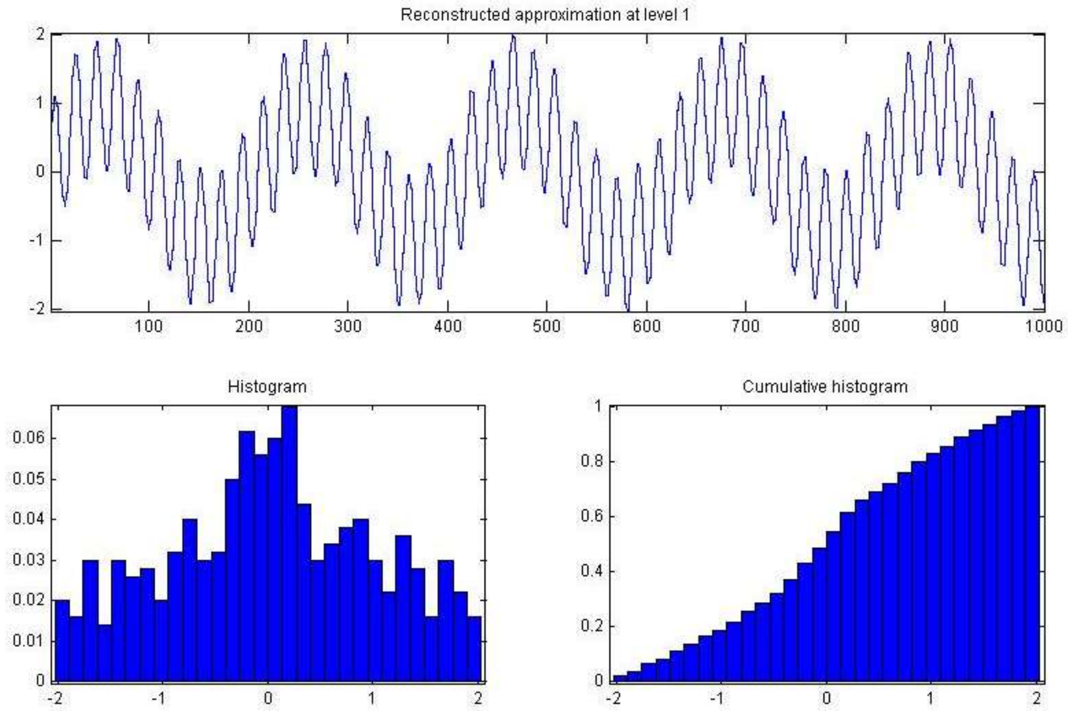

Fig.5 reconstruct signal

7. Conclusion:

The OMI of the first CW and its product have been given in general and applied for solving the nonlinear IDE. The work included numerical examples to prove the superiority of the method that we used, In addition to numerical solutions that prove that we have reached that the error is equal to zero between the exact solution and the approximation solution, we have moved to signal processing to prove that the proposed wave is valid for many applications.

8. References

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بعض تطبيقات طريقة التجميع لحل المعادلات التفاضلية التكاملية ومعالجة الاشارة

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الخلاصة:

في هذا العمل، تم عرض دوال شيبيشيف الموجية والمصفوفات التكاملية الجاهزة، وتستند هذه الطريقة على تقريب شيبيشيف الموجية الاولى.وقد تم تطبيقها من أجل حل معالجة الإشارات مثل إشارة استهلاك الكهرباء بالإضافة إلى التطبيقات الطبية بالإضافة الى حل بعض المعادلات التكاملية التفاضلية. وفي النهاية بعض الأمثلة المعطاة التي تم حلها بالطريقة المعروضة والتي توصلنا من خلالها بانها الطريقة الأكفأ، وتم معالجة الاشارة باستخدام الطريقة المقترحة ورفع الضوضاء منها وتم استخدام برنامج Matlab بعد تجهيزه بالنظرية المقترحة لاستخدامها في حل جميع المسائل اعلاه.