Fully Small Stable Quasi-prime Module

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Abstract:

In this paper we study the concept of small fully stable quasi-prime module as a generalization of (f.q.p) module and give some relationships between small fully stable quasi- prime and (f.q.p)

Keywords: fully stable quasi-prime module,fully small stable quasi-prime module,prime module,s.prime module,s.monoform.

1-Introduction:

Let R be commutative ring with unity and let M be unitary R-module ,M is called stable quasi-prime module if there exist homomorphism function f: N→M where N be submodule M;
$Ann_R f(N) = ann_R M$ [2].An R-module M is called fully stable quasi-prime module (briefly f.s.q.p)module iff for each N submodule (s.q.p)module [2]. Of M is

The main purpose of this work to investigate the new definition is called fully small stable quasi- prime module (briefly f.s.s.q.p) Module ,where an R-module M is called (s.s.q.p)module if there exist Homomorphism function f: N→M ,where N be small submodule of M it denoted by N≪M then $ann_R f(N) = ann_R M$, and M is called (f.s.s.q.p)module if for each N≪ M there exist homomorphism function F:N→M ; $ann_R f(N)=ann_R M$

Recall that an R-module M is called s-prime if $ann_R M= ann_R N$ for each N≪M see[4].the concept of s- monoform is introduced in [3] where an R-module M is called s- monoform if for each N≪ M,N≠0 and f∈ Hom (N , M) implies ker f=0 .we show that there is equivalent relation between s-monoform and (f.s.s.q.p)module and we give the basic properties of (f.s.s.q.p) module

2- fully small stable quasi- prime module: in this study,we introduce the concept of fully small stable quasi-prime module and give several result about it

2-1 Definition:

Let N be small submodule of an R-module M ,then N is called small stable quasi-prime module (briefly s.s.q.p)module if there exist homomorphism function $f:N\rightarrow M$; $ann_R f(N)=ann_R M$

2-2 Definition:

An R- module M is called fully small stable quasi-prime module (briefly f.s.s.q.p)module iff for each N small submodule is s.s.q.p module

2-3 Remarks and Examples:

1- Z_8 as Z-module is f.s.s.q.p module since the small submodule
N={[0],[4]} see[1],[4]
Let the homomorphism function $f:N\rightarrow M$; f(N)=N+[5]
$Ann_R f(N)=8Z= ann_R Z_8$

2- clearly every f.s.q.p module is f.s.s.q.p module but the converse is not true for example in (2-3,1) ,Z_8 is f.s.s.q.p module but not f.s.q.p module see[2]

3- it is clear that every simple R-module is f.s.s.q.p module

4-let $N=[0]+Z_2$ be the small submodule of Z_2+Z_2 ;$f:N\rightarrow M$;
$F(N)=Z_2$ $ann_Z f(N)=ann_Z 2Z$
So N is f.s.s.q.p module

Recall that an R-module M is called small prime briefly s-prime $Ann_R M=ann_R N$ for each $N\ll M$ see [4] if

2-4 Theorem:

Let M_1 and M_2 be two s-prime f.s.s.q.p module then
M_2 is f.s.s.q.p module $M=M_1\oplus$

Proof:

Let $N\ll M$ and let $f\in Hom(N,M)$; $f\neq 0$ and $N = N_1\oplus N_2$ where $N_1=N\cap M_1$, $N_2=N\cap M_2$ as $N\ll M$,we get $N_1\ll M_1$ and $N_2\ll M_2$
so there exist two homomorphism functions $f_1:N_1\rightarrow M_1$ and
$F_2:N_2\rightarrow M_2$;since M_1 is f.s.s.q.p module so $ann_R f_1(N_1)=ann_R M_1$
And since M_2 is f.s.s.q.p module implies $ann_R f_2(N_2)=ann_R M_2$
M ; $ann_R F(N)=ann_R M$ To prove there exist $f:N\rightarrow M$;$f:N_1\oplus N_2\rightarrow$
Let $N=N_1\oplus N_2$ so since f is homomorphism so $f(N)=f(N_1)+f(N_2)$
$ann_R f(N)=ann_R [f(N_1)+f(N_2)]$,see [1]
$=ann_R f(N_1)\cap ann_R f(N_2)$
Since M_1 and M_2 are f.s.s.q.p module implies
$Ann_R M_2$ $Ann_R f(N)=ann_R M_1\cap$
$Ann_R f(N)=ann_R M\cap ann_R M_2=ann_R M$ (since M_1 and M_2 are s-prime)
So M is f.s.s.q.p module

2-5 Corollary: If M is s-prime R-module,for each $x \neq 0$ with (x) small submodule of M ,then M is f.s.s.q.p module.

Proof:

For each homomorphism function $f:(x) \rightarrow M$, $f \neq 0$

We must prove $ann_R\, f(x) = ann_R\, M$

But $f(x) \subseteq M$ so $ann_R M \subseteq ann_R f(x)$

To prove $ann_R\, f(x) \subseteq ann_R M$

Let $r \in ann_R\, f(x)$,this implies $rf(x)=0$ so $f(rx)=0=f(0)$,since f is homomorphism,this $rx=0$ and $r \in ann_R(x)$ but $(x) \ll M$ and M is s-prime so $r \in ann_R M$.

s-monoform if Recall that an R-module M is called for each $N \ll M, N \neq 0$ and $f \in Hom(N,M)$ implies ker f=0 see[3]

2-6 Theorem:

Let M be s-prime module then M is s-monoform iff M is f.s.s.q.p module

Proof:

Let $f: N \rightarrow M$ homomorphism and ker f=0

To prove $ann_R M = ann_R f(N)$ where N be small submodule of M

Since $f(N) \subseteq M$ so $ann_R M \subseteq ann_R f(N)$

To prove $ann_R f(N) \subseteq ann_R M$

Let $x \in ann_R f(N)$ then $xf(N)=0$ but f is homomorphism so $f(xN)=0$

Implies $xN \in kerf$ but M is s-monoform so kerf=0 implies $x N=0$

Which mean $x \in ann_R N$ but M is s-prime so $x \in ann_R M$ so $Ann_R f(N) = ann_R M$ which mean M is f.s.s.q.p module.

To prove the converse suppose M is f.s.s.q.p module (i.e to prove M is s-monoform),to prove kerf=0

Suppose $kerf \neq 0$ and $kerf \leq M$ and $N \ll M$ since since $kerf \leq N \ll M$ so $kerf \ll M$, $f:N \rightarrow M$,so there exist $x \in N; f(x) \neq 0$,let $r \in R; r \neq 0$,such that $rx \in \ker f$ so $f(rx)=0$,but f is homomorphism so $rf(x)=0$ which mean $r \in ann_R f(x) = ann_R M$(since M is f.s.s.q.p)Module) ,so $r \in ann_R M$,but M s-prime ,by [4] for each $x \in N \subseteq M$ implice $ann_R M = ann_R(x)$,so $r \in ann_R(x)$,so rx=0 which is controduction ,so ker f=0.

Recall that an R-module M is called multiplication if for every submodule N of M, there exist an ideal I of R such that N=IM equivalently M is multiplication R- module if for every submodule N of M such that $N=(N:_R M)M$ where $(N, M) = \{ r \in R; rM \subseteq N\}$ see [1] , [5]

2-7 Theorem:
Every s-prime multiplication R-module and $I\nsubseteq ann_R M$ then M Is f.s.s,q.p module.

Proof:
To prove $ann_R f(N) = ann_R M$ for each homomorphism function $F:N\rightarrow M$; $N\ll M$, since $f(N)\subseteq M$ so $ann_R M\subseteq ann_R f(N)$. To prove $Ann_R f(N)\subseteq ann_R M$.
Let $x\in ann_R f(N)$ so by definition of annihilator , $xf(N)=0$ but f is homomorphism implies $f(xN)=0$ since f is homomorphism
Implies $xN=0$ but M is multiplication so $xIM=0$ b M is s-prime so either $x\in ann_R M$ or $I\subseteq ann_R M$ but $I\nsubseteq ann_R M$ implies $x\in ann_R M$ so M isf.s.s.q.p module
Recall that an R-module M is called hollow if $M\neq 0$ and Every proper submodule N of M is small in M , see[6]
Recall that an R-module M is Noetherian,if every submodule of M Is finitely generated,see[1]

2-8 Corollary:
If M is s-prime and hollow Noetherian R-module,then M is f.s.s.q.p module.

Proof:
Since M is hollow ,let $N\leq M$ and $N\ll M$, since M is Noetherian so N is finitely generated submodule of M . Hence $N = (x)$,for some $x\in M$, $x\neq 0$.Thus the result obtain by corollary (2-5).

3- Relation between stable function and f.s.s.q.p module

3-1 Definition:
Let N be submodule of an R-module M,we call the function $F:N\rightarrow M$; $ann_R f(N)=ann_R M$ stable function.

3-2 Definition:
Let M be an R-module and N be submodule of an R-module w be submodule of M,let the stable homomorphism function $F:w\rightarrow M$ and epimorphism function $g:N\rightarrow W$ then $ann_R f\circ g(N)= ann_R f(w)$

3-3 Definition:
Let N be small submodule of an R-module M ,the homomorphism function $f:N\rightarrow M$ is called small stable function If $ann_R f(N)=ann_R M$ for each $N\ll M$.

3-4 Theorem:

Let N be small submodule of W where W be be small submodule of an R-module M If $f:W\rightarrow M$ is small stable function and epimorphism function $g:N\rightarrow W$ then $f\circ g$ is small stable function

Proof:

By definition (3-2) we must prove $ann_R M=ann_R f\circ g$

$ann_R f\circ g(N)= ann_R f(g(N))=\{r\in R;rf(g(N))=0\}$

$= \{r\in R;rf(w)=0\}$

$=ann_R f(w)$

$=ann_R M$ (since f is small stable function)

So $f\circ g$ is small stable function

3-5 Theorem:

Let N and W be two submodule of an R-module M,

If $f: N\rightarrow M$ be stable function and $g:W\rightarrow M$ be stable function then $f+g(N+W)\rightarrow M$ be stable function

proof:

since f,g are stable function so $ann_R f(N)=ann_R M$ and $ann_R g(W)=ann_R M$

To prove $ann_R f(N)+ann_R g(W)=ann_R M+ann_R M=ann_R M$

Implies $ann_R(f(N)\cap g(W)=ann_R M$

Let $x\in ann_R(f(N)\cap g(W))$ so $x(f(N)\cap g(W))=0$, implies $xf(N)=0$ and $xg(W)=0$ so $x\in ann_R f(N)$ and $x\in ann_R g(W)$,implies $x\in ann_R M$.

3-6 Corollarly:

Let N and W be two submodule of an R-module M

If $f:N\rightarrow M$ be small stable function and $g:W\rightarrow M$ be small stable function

Then $f+g:(N+W)\rightarrow M$ be small stable function.

Proof: It is clear.

Main Result:

From this research we conclude that every f.s.q.p module is f.s.s.q.p module,but the converse is not true.if M_1 and M_2 be two s-prime modul then $M=M_1\oplus M_2$ is f.s.s.q.p module and if M is s-prime R-module,then for each $x\neq 0$ with (x) small submodule of M , then M is f.s.s.q.p module.

Every s-prime multiplication R-module and $I\not\subseteq ann_R M$,then M is f.s.s.q.p module.if M is s-prime and hollow Notherian R-module then M is f.s.s.q.p module.finally we conclude if $f:N\rightarrow M$be stable function and $g:W\rightarrow M$ stable function then $f+g:N+W\rightarrow M$ be stable function.

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الموديولات الشبه اوليا الاصغر المستقرة كليا

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في هذا العمل قدمت تعريف جديد وهو الموديولات الشبه اولية الاصغر المستقرة كليا وقد برهنا بعض الخواص لهذا النوع من الموديولات

الكلمات المفتاحية:

الموديولات الشبه اولية المستقرة كليا ، ،الموديولات الشبه اولية الاصغر المستقرة كليا، الموديول الاولي ، الموديول الاولي الا صغر ،الشكل الاحادي الاصغر.