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Solving Linear and Nonlinear Systems of Fuzzy Differential Equations by Using Differential Transform Method

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Abstract:
In this paper the differential transform method (DTM) is used to solve linear and nonlinear system of fuzzy differential equations. The approximate solution of the system is calculated in the form of a rapid convergent series using descriptions and properties of fuzzy set theory, this method captures the exact solution so to illustrate the ability and subtleties of this process, some systems of fuzzy differential equations are solved as numerical examples. A comparison of evaluated numerical results with the exact solutions for each fuzzy level set is displayed in the form of tables.
Keywords: Fuzzy numbers, Fuzzy α-level sets, Differential transform method, System of fuzzy differential equations, Series solution.

Introduction

There are several applications that involve finding many unknown functions at the same time, these unknown functions are related to a set of equations that include the unknown functions and their first derivatives, and in order to infer in uncertain circumstances, and get a realistic system we must take into account the fuzzy set ,it also provides practical solutions to realistic problems, which are solutions at an effective and reasonable cost compared to other solutions that offer other technologies, for example, the medicine [1],golden mean [2], engineering problems [3], quantum optics and gravity [4]and practical systems [5]. The concept of fuzzy sets was originated by Zadeh when he published his pioneering paper [5], it led to the introduction of the concepts of fuzzy number and fuzzy arithmetic [6] and approximate reasoning problems [7]. It was later developed by Mizumoto and Tanaka [8], [9] and Nahmias [10], they all noticed the fuzzy number as a collection of α-levels, $0 \le \alpha \le 1$ [11]. The concept of fuzzy derivative was first introduced by Chang and Zadeh, Dubois and Prade followed in [12], who defined and used the extension principle. In addition Fuzzy systems of differential equations one of the important applications of fuzzy numbers [13] and [14].

In this work, The approximate solution of a system of fuzzy differential equations was obtained with the differential transformation method (DTM), is a numerical as well as analytical method initializes the solution as convergent series with easily computable components introduced first by Zhou[15] to solve linear and nonlinear initial value problems in electric circuit analysis, this method structures an analytical solution for differential equations in the form of a polynomial. It is different from the classic highly-ordered Taylor series method, it is possible to obtain accurate results of differential equations.

This paper contains five sections as follows. In section two, we provide some important definitions to be used in this paper, in section three we general analysis system of fuzzy differential equations. In Section four we discuss Differential Transformation Method for solving systems of linear and nonlinear differential equations for finding the numerical solution, finally, the suggested method is applied by solving two examples in section 5.

2. Preliminaries

In this section, some basic concepts for fuzzy calculus are presented as follows:

Definition 2.1 [16] A fuzzy number y is a fuzzy set: R → [0, 1] which satisfies the following requirements:

(i) y is upper semicontinuous function,

(ii) $y(\mathrm{x})$ = 0 outside some interval [c, d],

(iii) There are real numbers a, b such that c ≤ a ≤ b ≤ d for which

(a) $\underline{y}(\mathrm{x})$ is monotonic increasing on [c, a],

(b) $\overline{y}(\mathrm{x})$ is monotonic decreasing on [b, d],

(c) $\tilde{y}(\mathrm{x})$ = 1 on [a, b].

We will let $\mathrm{R_F}$ denote the set of fuzzy numbers on R. Obviously R $\subset \mathrm{R_F}$, where R is understood as R $\cong R\chi = \{\chi_{\{x\}} : x \in R\} \subset R_F$. The α -level represent of a fuzzy number y, denoted by $[y]^\alpha$, is defined as: $\{\tilde{y}(\mathrm{x};\alpha)\}$ It is clear that the α -Level representation of a fuzzy number y is a compact convex subset of R. Thus, if y is a fuzzy number, then $[y]^\alpha = [\underline{\mathrm{y}}(\alpha), \overline{\mathrm{y}}(\alpha)]$, where $\underline{\mathrm{y}}(\alpha)$= min {s : s $\in [y]^\alpha$ } and $\overline{\mathrm{y}}(\alpha)$= max{s : s $\in [y]^\alpha$ } $\forall\ \alpha \in$ [0, 1]. Sometimes, we will write $\underline{\mathrm{y}}_\alpha$ and $\overline{\mathrm{y}}_\alpha$ replace of $\underline{\mathrm{y}}(\alpha)$ and $\overline{\mathrm{y}}(\alpha)$, respectively, $\forall\ \alpha \in [0,1]$.

Theorem 2.1 [16] suppose that $\underline{y}$: [0, 1] → R and $\overline{y}$: [0, 1] → R satisfies the following conditions:

(i) $\underline{y}(\alpha)$ is a bounded increasing left continuous function on (0, 1],

(ii) $\overline{y}\,(\alpha)$ is a bounded decreasing left continuous function on (0, 1],

(iii) $\underline{y}(\alpha)$ and $\overline{y}\,(\alpha)$ are right continuous functions at $\alpha = 0$,

(iv) $\underline{y}(\alpha) \leq \overline{y}(\alpha)$ on [0, 1], Then y: R → [0, 1] defined by

$$y(s) = \sup\{\alpha: \underline{y}(\alpha) \leq s \leq \overline{y}(\alpha)\}$$

is a fuzzy number given by [$\underline{y}(\alpha)$, $\overline{y}(\alpha)$]. Moreover, if y : R → [0, 1] is a fuzzy number given by [$\underline{y}(\alpha)$,$\overline{y}(\alpha)$], then the functions $\underline{y}(\alpha)$ and $\overline{y}(\alpha)$ satisfy conditions (i-iv).

Definition 2.2 [17] Let $y = R \to E$ be a fuzzy function and let $x_0 \in R$ the derivative $y'(x_0)$ of y at the point x_0 is defined by

$$y'(x_0) = \lim_{h\to 0} \frac{y(x_0+h) - y(x_0)}{h} \qquad (1)$$

The element $y(x_0 + h)$, $y(x_0)$ at the right-hand side of (1)are observed as elements in the Banach space $B = C[0,1] \times C[0,1]$. This ,if $y(x_0 + h) = (\underline{a}\,,\overline{a}\,)$ and $y(x_0) = (\underline{b}\,,\overline{b}\,)$, the difference is simply $y(x_0 + h) - y(x_0) = \left(\underline{a} - \underline{b}\,,\overline{a} - \overline{b}\,\right)$. clearly $y(x_0 + h) - y(x_0)/h$ may not be a fuzzy number for all h .however, if it approaches $y'(x_0)$ (in B) and $y'(x_0)$ is also a fuzzy number, this number is the fuzzy derivative of $y(x)$ at x_0. In this case, if $y = \left(\underline{y}\,,\overline{y}\,\right)$, it can be easily shown that

$$y'(x_0) = \left(\underline{y}'(x_0),\overline{y}'(x_0)\right)$$

Where $\underline{y}'$ and $\overline{y}'$ are the classic derivatives of $\underline{y}$ and $\overline{y}$, respectively.

3. System of Fuzzy Differential Equations

In this section, we discuss a system of fuzzy differential equations in the following form:

$$\sum_{i=0}^{n} A_i\,(x)Y^{(i)}(x) = F(x)$$

with the initial values (2)

$$Y^{(i)}(0) = G_i \quad , \ i = 0, \ldots\ldots, n-1$$

Where x is a scalar and $A_0\,(x)\ , A_1\,(x),\ \ldots\ldots, A_{n-1}\,(x)$ are s × s matrixes and all of the components are a real function of x. Y,G_i , and F are fuzzy s-dimensional vectors. The α component of $Y \in E$ will be denoted by y_α , so we can write

$$Y = [y_1\,(x,\alpha)\ , y_1\,(x,\alpha), \ldots., y_S\,(x,\alpha)]^T$$

$$y_i\,(x,\alpha) = \left((\underline{y})_i(x,\alpha), (\overline{y})_i(x,\alpha) \right) \ , \quad i = 1, \ldots, s,$$

$$F = [\,f_1\,(x,\alpha)\ , f_2\,(x,\alpha), \ldots., f_S\,(x,\alpha)]^T, \tag{3}$$

$$f_i\,(x,\alpha) = \left((\underline{f})_i(x,\alpha), (\overline{f})_i(x,\alpha) \right) \ , \quad i = 1, \ldots, s,$$

$$G_i \ = [g_{i1}\,(\alpha)\ , g_{i2}\,(\alpha), \ldots., g_{is}\,(\alpha)]^T$$

Where

$$g_{ij}\,(\alpha) = \left((\underline{g})_{ij}(\alpha),\ (\overline{g})_{ij}(\alpha) \right) \tag{4}$$

$$i = 1, \ldots, n-1\ , \ j = 1, \ldots, s.$$

T is represent transpose, Then, (2) can be replaced by the following equivalent system:

$$\underline{\sum_{i=0}^{n}\sum_{k=1}^{s} A_i\,(x) Y_k^{(i)}(x,\alpha)} \ = \underline{f_d\,(x,\alpha)} \quad , \qquad d = 1, \ldots, s,$$

$$\overline{\sum_{i=0}^{n}\sum_{k=1}^{s} A_i\,(x) Y_k^{(i)}(x,\alpha)} \ = \overline{f_d\,(x,\alpha)}\,, \qquad d = 1, \ldots, s,$$

(5)

$$\underline{Y_j^{(i)}}(0,\alpha) = \underline{g_{ij}}(\alpha) \quad , i = 0, \ldots, n-1\ , \quad j = 1, \ldots, s.$$

$$\overline{Y_j^{(i)}}(0,\alpha) = \overline{g_{ij}}(\alpha) \quad , i = 0, \ldots, n-1\ , \quad j = 1, \ldots, s.$$

Where $\alpha \in [0,1]$.

4-Basic Ideas of Fuzzy Differential Transformation Method

It is well known that if a function u is infinitely continuously differentiable, then u can be
expressed in Taylor series as

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If fuzzy function $y(x,\alpha)$ is infinitely continuously differentiable, and then $y(x,\alpha)$ can be written in Taylor's series as:

$$y(x,\alpha)=\begin{cases}\underline{y}(x,\alpha)=\sum_{k=0}^{\infty}\frac{1}{k!}\left[\frac{d^k\underline{y}(x,\alpha)}{dx^k}\right]_{x=x_0}(x-x_0)^k\\ \overline{y}(x,\alpha)=\sum_{k=0}^{\infty}\frac{1}{k!}\left[\frac{d^k\overline{y}(x,\alpha)}{dx^k}\right]_{x=x_0}(x-x_0)^k\end{cases}\quad(6)$$

we define the differential transformation of $y(x,\alpha)$, denoted by $Y(k,\alpha)$,by

$$Y(k,\alpha)=\begin{cases}\underline{Y}(k,\alpha)=\frac{1}{k!}\left[\frac{d^k\underline{y}(x,\alpha)}{dx^k}\right]_{x=x_0}\\ \overline{Y}(k,\alpha)=\frac{1}{k!}\left[\frac{d^k\overline{y}(x,\alpha)}{dx^k}\right]_{x=x_0}\end{cases}\quad(7)$$

Now, to solve a given SFDE by differential transform, we make use of the differential transformation of order (k,α) in eq.(7), the inverse differential transformation of $Y(k,\alpha)$ is defined by:

$$y(x,\alpha)=\begin{cases}\underline{y}(x,\alpha)=\sum_{k=0}^{\infty}\underline{Y}(k,\alpha)\,(x-x_0)^k\\ \overline{y}(x,\alpha)=\sum_{k=0}^{\infty}\overline{Y}(k,\alpha)\,(x-x_0)^k\end{cases}\quad(8)$$

From Eq. (6) and Eq. (7) following theorem can be concluded

Theorem 4.1 [18]:
Let $y(x,\alpha)$, $y_1(x,\alpha)$ and $y_2(x,\alpha)$ are fuzzy functions, and let $Y(k,\alpha)$, $Y_1(k,\alpha)$ and $Y_2(k,\alpha)$ are their differential transforms respectively, and then we get the following:

i. If $y(x,\alpha)=y_1(x,\alpha)\pm y_2(x,,\alpha)$, then$Y(k,\alpha)=Y_1(k,\alpha)\pm Y_2(k,\alpha)$.
ii. If $y(x,\alpha)=cy_1(x,\alpha)$ then $Y(k,\alpha)=cY_1(k,\alpha)$, where c is a constant.
iii. If $y(x,\alpha)=\frac{d^n y_1(x,\alpha)}{dx^n}$, then $Y(k,\alpha)=\frac{(k+n)!}{k!}Y_1(k+n,\alpha)$
iv. If $y(x,\alpha)=x^n$, then $Y(k,\alpha)=\delta(k-n,\alpha)$ where $\delta(k-n)=\begin{cases}1, & k=n\\ 0, & k\neq n\end{cases}$
v. If $Y(x,\alpha)=y_1(x,\alpha)y_2(x,\alpha)$, then $Y(k,\alpha)=\sum_{k_1=0}^{k}Y_1(k_1)Y_2(k-k_1,\alpha)$
vi. If $y(x,\alpha)=e^{\lambda x}$, then $Y(k,\alpha)=\frac{\lambda^k}{k!}$, where λ is a constant.

vii. If $y(x,\alpha)=\sin(ax+b,\alpha)$,then $Y(k,\alpha)=\frac{a^k}{k!}\sin(\frac{k\pi}{2}+b,\alpha)$, (where a and b constant.

viii. If $y(x,\alpha)=\cos(ax+b,\alpha)$,then $Y(k,\alpha)=\frac{a^k}{k!}\cos(\frac{k\pi}{2}+b,\alpha)$, (where a and b constant.

ix. If $y(x,\alpha)=y_1(x,\alpha)\,y_2(x,\alpha)\ldots y_n(x,\alpha)$ then

$$Y(k,\alpha)=\sum_{k_{n-1}=0}^{k}\sum_{k_{n-2}=0}^{k_{n-1}}\ldots\sum_{k_1}^{k_2}\mathrm{Y}_1(k_1)\mathrm{Y}_2(k_2-k_1)\ldots\mathrm{Y}_n(k-k_{n-1},\alpha)$$

5-Numerical Examples

Example1. Consider the system of fuzzy differential equations

$$\boldsymbol{y_1}'(\mathbf{x})-\boldsymbol{y_2}(\mathbf{x})=\left((-\mathbf{2x}\alpha+\mathbf{2x}+\mathbf{2}\alpha+\mathbf{3})\boldsymbol{e^{2x}},(\mathbf{2x}\alpha-\mathbf{2x}-\mathbf{2}\alpha+\mathbf{7})\boldsymbol{e^{2x}}\right)$$

$$\boldsymbol{y_1}(\mathbf{x})+\boldsymbol{y_2}'(\mathbf{x})=\left((-\alpha x+x+3\alpha+2)\boldsymbol{e^{2x}},(\alpha x-x-3\alpha+8)\boldsymbol{e^{2x}}\right)$$

Subject to the fuzzy initial conditions (9)

$\boldsymbol{y_1}(\mathbf{0})=(\mathbf{2}+\boldsymbol{\alpha},\mathbf{4}-\boldsymbol{\alpha})$, $\boldsymbol{y_2}(\mathbf{0})=(\boldsymbol{\alpha},\mathbf{2}-\boldsymbol{\alpha})$, $x\in[0,1]$

The exact solution is as follows:

$$\boldsymbol{y_1}(\boldsymbol{x},\boldsymbol{\alpha})=\left((\mathbf{2}+\boldsymbol{\alpha})\boldsymbol{e^{2x}}+(\mathbf{1}-\boldsymbol{\alpha})\boldsymbol{x}\boldsymbol{e^{2x}}\right),\left((\mathbf{4}-\boldsymbol{\alpha})\boldsymbol{e^{2x}}+(\boldsymbol{\alpha}-\mathbf{1})\boldsymbol{x}e^{2x}\right)$$

$$\boldsymbol{y_2}(\boldsymbol{x},\boldsymbol{\alpha})=(\boldsymbol{\alpha}\,\boldsymbol{e^{2x}},(2-\boldsymbol{\alpha})\boldsymbol{e^{2x}})$$

We may replace (9) by the equivalent system with α -levels: $y_{\alpha}(x)=[\underline{y},\overline{y}]$, $\alpha\in(0,1]$, Hence, to create solutions in the(lower case of solution) $\underline{y}$, suppose the problem:

$$\underline{\boldsymbol{y_1}}'(\mathbf{x})-\underline{\boldsymbol{y_2}}(\mathbf{x})=(-\mathbf{2x}\alpha+\mathbf{2x}+\mathbf{2}\alpha+\mathbf{3})\boldsymbol{e^{2x}}$$

$$\underline{\boldsymbol{y_1}}(\mathbf{x})+\underline{\boldsymbol{y_2}}'(\mathbf{x})=(-\alpha x+x+3\alpha+2)\boldsymbol{e^{2x}}$$

With initial conditions (10)

$\underline{\boldsymbol{y_1}}(\mathbf{0})=\mathbf{2}+\boldsymbol{\alpha}$, $\underline{\boldsymbol{y_2}}(\mathbf{0})=\boldsymbol{\alpha}$

By using (iii),(iv) and (vi) of Theorem 4.1 choosing $\boldsymbol{x_0}=\mathbf{0}$, we transform equation (10) as follows:

$$(k+1)\underline{Y_1}(k+1)-\underline{Y_2}(k)=(-\mathbf{2x}\alpha+\mathbf{2x}+\mathbf{2}\alpha+\mathbf{3})\frac{2^k}{k!},$$

$$\underline{Y}_1(\mathrm{k})+(\mathrm{k}+1)\underline{Y}_2(\mathrm{k}+1)=(-\alpha\mathrm{x}+\mathrm{x}+3\alpha+2)\,\frac{2^k}{k!},$$

$\underline{Y}_1(0,\alpha)=2+\alpha,\qquad \underline{Y}_2(0,\alpha)=\boldsymbol{\alpha}$

Consequently, we find

$\underline{Y}_1(1,\alpha)=\mathbf{3}+3\alpha\ ,\qquad \underline{Y}_2(1,\alpha)=2\alpha$

$\underline{Y}_1(2,\alpha)=\ 5+\alpha\ ,\qquad \underline{Y}_2(2,\alpha)=\dfrac{3+\alpha}{2}$

$\underline{Y}_1(3,\alpha)=\dfrac{15+9\alpha}{6}\ ,\qquad \underline{Y}_2(3,\alpha)=\dfrac{-1+5\alpha}{3}$

$\underline{Y}_1(4,\alpha)=\dfrac{11+13\alpha}{12}\ ,\qquad \underline{Y}_2(4,\alpha)=\dfrac{1+15\alpha}{24}$

$\underline{Y}_1(4,\alpha)=\dfrac{49+47\alpha}{120}\ ,\qquad \underline{Y}_2(4,\alpha)=\dfrac{5+11\alpha}{60}$

Therefore, from (7) equation (10) is solved by

$$\underline{Y}_1(x,\alpha)=(2+\alpha)+(\mathbf{3}+3\alpha)x+(5+\alpha)x^2+\left(\frac{15+9\alpha}{6}\right)x^3+\left(\frac{11+13\alpha}{12}\right)x^4+\left(\frac{49+47\alpha}{120}\right)x^5+\cdots$$

$$\underline{Y}_2(x,\alpha)=(\alpha)+(2\alpha)x+\left(\frac{3+\alpha}{2}\right)x^2+\left(\frac{-1+5\alpha}{3}\right)x^3+\left(\frac{1+15\alpha}{24}\right)x^4+\left(\frac{5+11\alpha}{60}\right)x^4+\cdots$$

Similarly, for the upper solution $\overline{\mathrm{y}}$, suppose the problem:

$\overline{y_1}'(\mathbf{x})-\overline{y_2}(\mathbf{x})=(\mathbf{2x}\alpha-\mathbf{2x}-\mathbf{2}\alpha+\mathbf{7})\boldsymbol{e^{2x}}$

$\overline{y_1}(\mathbf{x})+\overline{y_2}'(\mathbf{x})=(\alpha\mathrm{x}-\mathrm{x}-3\alpha+8)\boldsymbol{e^{2x}}$

With initial conditions (11)

$\overline{y_1}(0,\alpha)=4-\alpha\ ,\ \overline{y_2}(0,\alpha)=2-\alpha$

By using (iii),(iv) and (vi) of Theorem 4.1choosing $\boldsymbol{x_0}=\mathbf{0}$, we transform equation (11) as follows:

$$(\mathrm{k}+1)\overline{Y}_1(\mathrm{k}+1)-\overline{Y}_2(\mathrm{k})=\big((\mathbf{2x}\alpha-\mathbf{2x}-\mathbf{2}\alpha+\mathbf{7})\big)\frac{2^k}{k!},$$

$$\overline{Y}_1(\mathrm{k})+(\mathrm{k}+1)\overline{Y}_2(\mathrm{k}+1)=(\alpha\mathrm{x}-\mathrm{x}-3\alpha+8)\frac{2^k}{k!}$$

$\overline{Y}_1(0,\alpha)=4-\alpha,\qquad \overline{Y}_2(0,\alpha)=2-\alpha,$

Consequently, we find

$\overline{Y}_1(1,\alpha) = \mathbf{9} - \mathbf{3}\alpha$, $\overline{Y}_2(1,\alpha) = \mathbf{4} - \mathbf{2}\alpha$

$\overline{Y}_1(2,\alpha) = \mathbf{7} - \alpha$, $\overline{Y}_2(2,\alpha) = \dfrac{5-\alpha}{2}$

$\overline{Y}_1(3,\alpha) = \dfrac{33-9\alpha}{6}$, $\overline{Y}_2(3,\alpha) = \dfrac{9-5\alpha}{3}$

$\overline{Y}_1(4,\alpha) = \dfrac{37-13\alpha}{12}$, $\overline{Y}_2(4,\alpha) = \dfrac{31-15\alpha}{24}$

$\overline{Y}_1(5,\alpha) = \dfrac{143-47\alpha}{120}$, $\overline{Y}_2(5,\alpha) = \dfrac{27-11\alpha}{60}$

Therefore, from (7) equation (11) is solved by

$$\overline{Y}_1(x,\alpha) = (4-\alpha) + (\mathbf{9}-\mathbf{3}\alpha)x + (\mathbf{7}-\alpha)x^2 + \left(\frac{33-9\alpha}{6}\right)x^3 + \left(\frac{37-13\alpha}{12}\right)x^4 + \left(\frac{143-47\alpha}{120}\right)x^5 + \cdots$$

$$\overline{Y}_2(x,\alpha) = (2-\alpha) + (\mathbf{4}-\mathbf{2}\alpha)\,x + \left(\frac{5-\alpha}{2}\right)x^2 + \left(\frac{9-5\alpha}{3}\right)x^3 + \left(\frac{31-15\alpha}{24}\right)x^4 + \left(\frac{27-11\alpha}{60}\right)x^5 + \cdots$$

Combining $\underline{Y}_1$, $\underline{Y}_2$ $\underline{y}$ and $\overline{Y}_1, \overline{Y}_2$ represent the fuzzy solution of the system of fuzzy differential equations (11) as $y_{1\alpha}(x) = [\underline{Y}_1(x), \overline{Y}_1(x)]$ and $y_{2\alpha}(x) = [\underline{Y}_2(x), \overline{Y}_2(x)]$, $\forall\ \alpha \in (0, 1]$, $x \in [0, 1]$. It is clear that for $\alpha = 1$, we find $\underline{Y}_1(x) = \overline{Y}_1(x)$ and $\underline{Y}_2(x) = \overline{Y}_2(x)$ which is the same exact solution of the system of non-fuzzy differential equations. The results of the calculations for all above case are given in table (1):

Table (1)

The lower and upper levels of fuzzy solution for example (1)

x	$\underline{Y}_1$	$\overline{Y}_1$	**exact solution of** Y_1	$\underline{Y}_2$	$\overline{Y}_2$	**exact solution of** Y_2
0	**3**	**3**	**3**	**1**	**1**	**1**
0.1	**3.6642**	**3.6642**	**3.6642**	**1.2214**	**1.2214**	**1.2214**
0.2	**4.3284**	**4.3284**	**4.4755**	**1.4428**	**1.4428**	**1.4918**
0.3	**4.9926**	**4.9926**	**5.4664**	**1.6642**	**1.6642**	**1.8221**
0.4	**5.6568**	**5.6568**	**6.6766**	**1.8856**	**1.8856**	**2.2255**
0.5	**6.3210**	**6.3210**	**8.1548**	**2.1070**	**2.1070**	**2.7183**

0.6	**6.9852**	**6.9852**	**9.9604**	**2.3284**	**2.3284**	**3.3201**
0.7	**7.6494**	**7.6494**	**12.1656**	**2.5498**	**2.5498**	**4.0552**
0.8	**8.3136**	**8.3136**	**14.8591**	**2.7712**	**2.7712**	**4.9530**
0.9	**8.9779**	**8.9779**	**18.1489**	**2.9926**	**2.9926**	**6.0496**
1	**21.6000**	**21.6000**	**22.1672**	**7.1333**	**7.1333**	**7.3891**

Example2. Consider the following nonlinear system of fuzzy differential equations

$y_1'(\mathrm{x}) = 2e^{3x} y_4^2(x)$
$y_2'(\mathrm{x}) = y_1(\mathrm{x}) - y_3(\mathrm{x}) + \cos(\mathrm{x}) - e^{2x}$
$y_3'(\mathrm{x}) = y_2(\mathrm{x}) - y_4(\mathrm{x}) + e^{-x} - \sin(\mathrm{x})$
$y_4'(\mathrm{x}) = -e^{-3x}\, y_1^2(x)$

Subject to the fuzzy initial conditions (12)

$y_1(0) = (0.5 + 0.5\alpha, 2 - \alpha)$, $y_2(0) = (0.3 + 0.7\alpha, 1.1 - 0.1\alpha)$
$y_3(0) = (0.1\alpha - 0.1, 0.1 - 0.1\alpha)$, $y_4(0) = (\alpha,\ 3 - 2\alpha)$
, $x \in [0,1]$

The exact solution is:

$$y(x) = \big(y_1(x), y_2(x), y_3(x), y_4(x)\big) = (e^{2x}, \sin(\mathrm{x}) + \cos(\mathrm{x}),\ \sin(\mathrm{x}), e^{-x})$$

We may replace (12) by the equivalent system with α -levels: $y_\alpha(\mathrm{x}) = [\underline{y}, \overline{y}]$, $\alpha \in (0, 1]$, Hence, to create solutions in the (lower case of solution) $\underline{y}$, suppose the problem:

$\underline{y_1}'(\mathrm{x}) = 2e^{3x} \underline{y_4}^2(x)$
$\underline{y_2}'(\mathrm{x}) = \underline{y_1}(\mathrm{x}) - \underline{y_3}(\mathrm{x}) + \cos(\mathrm{x}) - e^{2x}$
$\underline{y_3}'(\mathrm{x}) = \underline{y_2}(\mathrm{x}) - \underline{y_4}(\mathrm{x}) + e^{-x} - \sin(\mathrm{x})$
$\underline{y_4}'(\mathrm{x}) = -e^{-3x}\, \underline{y_1}^2(x)$

With initial conditions (13)

$\underline{y_1}(0) = 0.5 + 0.5\alpha$, $\underline{y_2}(0) = 0.3 + 0.7\alpha$
$\underline{y_3}(0) = 0.1\alpha - 0.1$, $\underline{y_4}(0) = \alpha$

By using (iii),(vi),(vii),(viii) and (ix) of Theorem 4.1 choosing $x_0 = 0$, we transform equation (13) as follows:

$$(k+1)\underline{Y}_1(k+1) = 2\sum_{k_2=0}^{k}\sum_{k_1=0}^{k_2}\frac{3^k}{k!}\,\underline{Y}_4(k_2-k_1)\,\underline{Y}_4(k-k_2)$$

$$(k+1)\underline{Y}_2(k+1) = \underline{Y}_1(k)-\underline{Y}_3(k)+\frac{1}{k!}\cos\frac{k\pi}{2}-\frac{2^k}{k!}$$

$$(k+1)\underline{Y}_3(k+1) = \underline{Y}_2(k)-\underline{Y}_4(k)+\frac{(-1)^k}{k!}-\frac{1}{k!}\sin\frac{k\pi}{2}$$

$$(k+1)\underline{Y}_4(k+1) = -\sum_{k_2=0}^{k}\sum_{k_1=0}^{k_2}\frac{(-3)^{k_1}}{k!}\,\underline{Y}_1(k_2-k_1)\,\underline{Y}_1(k-k_2)$$

$$\underline{Y}_1(0,\alpha) = 0.5+0.5\alpha\,,\quad \underline{Y}_2(0,\alpha) = 0.3+0.7\alpha$$

$$\underline{Y}_3(0,\alpha) = 0.1\alpha-0.1\,,\qquad \underline{Y}_4(0,\alpha) = \alpha$$

Consequently, we find

$$\underline{Y}_1(1,\alpha) = 2\alpha^2$$

$$\underline{Y}_2(1,\alpha) = 0.6+0.4\alpha$$

$$\underline{Y}_3(1,\alpha) = 1.3-0.3\alpha,$$

$$\underline{Y}_4(1,\alpha) = -(0.25\alpha^2+0.5\alpha+0.25)$$

$$\underline{Y}_1(2,\alpha) = -0.25\alpha^3+2.5\alpha^2-0.25\alpha$$

$$\underline{Y}_2(2,\alpha) = -\frac{1}{2}(-2\alpha^2-0.3\alpha+3.3)$$

$$\underline{Y}_3(2,\alpha) = 0.125\alpha^2+0.45\alpha-0.575$$

$$\underline{Y}_4(2,\alpha) = \frac{1}{2}(-\alpha^3-0.25\alpha^2+1.5\alpha+0.75)$$

$$\underline{Y}_1(3,\alpha) = \frac{2}{3}(-0.5\alpha^4-0.875\alpha^3+3.75\alpha^2-0.375\alpha),$$

$$\underline{Y}_2(3,\alpha) = -\frac{1}{3}(0.25\alpha^3-2.375\alpha^2+0.7\alpha+1.925)$$

$$\underline{Y}_3(3,\alpha) = -\frac{1}{3}(-0.5\alpha^3-1.1255\alpha^2+0.6\alpha+1.525)$$

$$\underline{Y}_4(3,\alpha) = -\frac{1}{3}(-0.125\alpha^4-1.875\alpha^3-0.75\alpha^2+2.125\alpha+1.125)$$

$$\underline{Y}_1(4,\alpha) = \frac{2}{4}(0.0417\alpha^5-0.875\alpha^4-1.25\alpha^3+3.7917\alpha^2-0.375\alpha),$$

$$\underline{Y}_2(4,\alpha) = \frac{1}{4}(-0.3333\alpha^4-0.75\alpha^3+2.125\alpha^2-0.05\alpha-0.825)$$

$$\underline{Y}_3(4,\alpha) = \frac{1}{4}(-0.0417\alpha^4 - 0.7083\alpha^3 + 0.5417\alpha^2 + 0.475\alpha - 0.2667)$$

$$\underline{Y}_4(4,\alpha) = \frac{1}{4}(0.1667\alpha^5 + 0.0834\alpha^4 - 2.0833\alpha^3 - 1.125\alpha^2 + 2\alpha + 1.125)$$

$$\underline{Y}_1(5,\alpha) = \frac{2}{5}(0.0417\alpha^6 + 0.1459\alpha^5 - 0.8958\alpha^4 - 1.2188\alpha^3 + 2.875\alpha^2 - 0.2812\alpha)$$

$$\underline{Y}_2(5,\alpha) = \frac{1}{5}(0.0209\alpha^5 - 0.4375\alpha^4 - 0.625\alpha^3 + 1.8959\alpha^2 - 0.1875\alpha - 0.625)$$

$$\underline{Y}_3(5,\alpha) = \frac{1}{5}(-0.0417\alpha^5 - 0.1042\alpha^4 + 0.3333\alpha^3 + 0.8126\alpha^2 - 0.5125\alpha - 0.4458)$$

$$\underline{Y}_4(5,\alpha) = \frac{-1}{5}(0.0104\alpha^6 + 0.2916\alpha^5 + 0.2812\alpha^4 - 1.6771\alpha^3 - 1.1146\alpha^2 + 1.4062\alpha + 0.8438)$$

Therefore, from (7) equation (13) is solved by

$$\begin{aligned}\underline{Y}_1(x,\alpha) = {} & (0.5 + 0.5\alpha) + (2\alpha^2)x + (-0.25\alpha^3 + 2.5\alpha^2 - 0.25\alpha)x^2 \\ & + \frac{2}{3}(-0.5\alpha^4 - 0.875\alpha^3 + 3.75\alpha^2 - 0.375\alpha)x^3 \\ & + \frac{2}{4}(0.0417\alpha^5 - 0.875\alpha^4 - 1.25\alpha^3 + 3.7917\alpha^2 - 0.375\alpha)x^4 \\ & + \frac{2}{5}(0.0417\alpha^6 + 0.1459\alpha^5 - 0.8958\alpha^4 - 1.2188\alpha^3 + 2.875\alpha^2 - 0.2812\alpha)x^5 + \cdots\end{aligned}$$

$$\begin{aligned}\underline{Y}_2(x,\alpha) = {} & (0.3 + 0.7\alpha) + (0.6 + 0.4\alpha)x - \frac{1}{2}(-2\alpha^2 - 0.3\alpha + 3.3)x^2 \\ & - \frac{1}{3}(0.25\alpha^3 - 2.375\alpha^2 + 0.7\alpha + 1.925)x^3 \\ & + \frac{1}{4}(-0.3333\alpha^4 - 0.75\alpha^3 + 2.125\alpha^2 - 0.05\alpha - 0.825)x^4 \\ & + \frac{1}{5}(0.0209\alpha^5 - 0.4375\alpha^4 - 0.625\alpha^3 + 1.8959\alpha^2 - 0.1875\alpha - 0.625)x^5 + \cdots\end{aligned}$$

$$\underline{Y}_3(x,\alpha) = (0.1\alpha - 0.1) + (1.3 - 0.3\alpha)x + (0.125\alpha^2 + 0.45\alpha - 0.575)x^2 - \frac{1}{3}(-0.5\alpha^3 - 1.1255\alpha^2 + 0.6\alpha + 1.525)x^3 + \frac{1}{4}(-0.0417\alpha^4 - 0.7083\alpha^3 + 0.5417\alpha^2 + 0.475\alpha - 0.2667)x^4 + \frac{1}{5}(-0.0417\alpha^5 - 0.1042\alpha^4 + 0.3333\alpha^3 + 0.8126\alpha^2 - 0.5125\alpha - 0.4458)x^5 + \cdots$$

$$\underline{Y}_4(x,\alpha) = (\alpha) - (0.25\alpha^2 + 0.5\alpha + 0.25)x + \frac{1}{2}(-\alpha^3 - 0.25\alpha^2 + 1.5\alpha + 0.75)x^2 - \frac{1}{3}(-0.125\alpha^4 - 1.875\alpha^3 - 0.75\alpha^2 + 2.125\alpha + 1.125)x^3 + \frac{1}{4}(0.1667\alpha^5 + 0.0834\alpha^4 - 2.0833\alpha^3 - 1.125\alpha^2 + 2\alpha + 1.125)x^4 - \frac{1}{5}(0.0104\alpha^6 + 0.2916\alpha^5 + 0.2812\alpha^4 - 1.6771\alpha^3 - 1.1146\alpha^2 + 1.4062\alpha + 0.8438)x^5 + \cdots$$

Similarly, for the upper solution $\overline{y}$, suppose the problem:

$$\overline{y}_1^{\,\prime}(x) = 2e^{3x}\overline{y}_4^{\,2}(x)$$
$$\overline{y}_2^{\,\prime}(x) = \underline{y}_1(x) - \overline{y}_3(x) + \cos(x) - e^{2x}$$
$$\overline{y}_3^{\,\prime}(x) = \overline{y}_2(x) - \overline{y}_4(x) + e^{-x} - \sin(x)$$
$$\overline{y}_4^{\,\prime}(x) = -e^{-3x}\,\overline{y}_1^{\,2}(x)$$

With initial conditions (14)

$\overline{y}_1(0) = 2 - \alpha$, $\overline{y}_2(0) = 1.1 - 0.1\alpha$

$\overline{y}_3(0) = 0.1 - 0.1\alpha$, $\overline{y}_4(0) = 3 - 2\alpha$

By using (iii), (vi), (vii), (viii) and (ix) of Theorem 4.1 choosing $x_0 = 0$, we transform equation (14) as follows:

$$(k+1)\overline{Y}_1(k+1) = 2\sum_{k_2=0}^{k}\sum_{k_1=0}^{k_2}\frac{3^k}{k!}\overline{Y}_4(k_2 - k_1)\overline{Y}_4(k - k_2)$$

$$(k+1)\overline{Y}_2(k+1) = \overline{Y}_1(k) - \overline{Y}_3(k) + \frac{1}{k!}\cos\frac{k\pi}{2} - \frac{2^k}{k!}$$

$$(\mathrm{k}+1)\overline{Y}_3(\mathrm{k}+1)=\overline{Y}_2(k)-\overline{Y}_4(k)+\frac{(-1)^k}{k!}-\frac{1}{k!}sin\frac{k\pi}{2}$$

$$(\mathrm{k}+1)\overline{Y}_4(\mathrm{k}+1)=-\sum_{k_2=0}^{k}\sum_{k_1=0}^{k_2}\frac{(-3)^{k_1}}{k!}\overline{Y}_1(k_2-k_1)\overline{Y}_1(k-k_2)$$

$$\overline{Y}_1(0,\alpha)=2-\alpha \quad , \overline{Y}_2(0,\alpha)=1.1-0.1\alpha$$

$$\overline{Y}_3(0,\alpha)=0.1-0.1\alpha \ , \quad \overline{Y}_4(0,\alpha)=3-2\alpha$$

Consequently, we find

$$\overline{Y}_1(1,\alpha)=8\alpha^2-24\alpha+18$$
$$\overline{Y}_2(1,\alpha)=1.9-0.9\alpha$$
$$\overline{Y}_3(1,\alpha)=1.9\alpha-0.9$$
$$\overline{Y}_4(1,\alpha)=-(\alpha^2-4\alpha+4)$$
$$\overline{Y}_1(2,\alpha)=2\alpha^3+\alpha^2-16\alpha+15$$
$$\overline{Y}_2(2,\alpha)=-\frac{1}{2}(-8\alpha^2+25.9\alpha-16.9)$$
$$\overline{Y}_3(2,\alpha)=0.5\alpha^2-2.45\alpha+1.95$$
$$\overline{Y}_4(2,\alpha)=\frac{1}{2}(8\alpha^3-37\alpha^2+54\alpha-24)$$
$$\overline{Y}_1(3,\alpha)=\frac{2}{3}(-8\alpha^4+55\alpha^3-124.5\alpha^2+111\alpha-31.5)$$
$$\overline{Y}_2(3,\alpha)=-\frac{1}{3}(-2\alpha^3-0.5\alpha^2+13.55\alpha-10.55)$$
$$\overline{Y}_3(3,\alpha)=-\frac{1}{3}(4\alpha^3-22.5\alpha^2+39.95\alpha-20.95)$$
$$\overline{Y}_4(3,\alpha)=-\frac{1}{3}(-2\alpha^4+27\alpha^3-97.5\alpha^2+133\alpha-60)$$
$$\overline{Y}_1(4,\alpha)=\frac{2}{4}(-1.3333\alpha^5+20\alpha^4+40\alpha^3-11.8333\alpha^2+16\alpha-61.5)$$
$$\overline{Y}_2(4,\alpha)=\frac{1}{4}(-5.3333\alpha^4+38\alpha^3-90.5\alpha^2+87.3167\alpha-29.3166)$$
$$\overline{Y}_3(4,\alpha)=\frac{1}{4}(-0.6667\alpha^4+9.6667\alpha^3-32.3333\alpha^2+39.8166\alpha-16.4833)$$
$$\overline{Y}_4(4,\alpha)=\frac{1}{4}(-5.3333\alpha^5+41.3334\alpha^4-147.3333\alpha^3+154.5\alpha^2-31\alpha-12)$$

$$\overline{Y}_1(5,\alpha) = \frac{2}{5}(2.6667\alpha^6 - 28.6667\alpha^5 + 128.6667\alpha^4 - 234.25\alpha^3 + 161.125\alpha^2 - 14.25\alpha - 14.625)$$

$$\overline{Y}_2(5,\alpha) = \frac{1}{5}(-0.6666\alpha^5 + 10\alpha^4 + 20\alpha^3 - 5.9166\alpha^2 + 8\alpha - 31.375)$$

$$\overline{Y}_3(5,\alpha) = \frac{1}{5}(1.3333\alpha^5 - 11.6666\alpha^4 + 46.3333\alpha^3 - 61.25\alpha^2 + 29.5792\alpha - 4.2875)$$

$$\overline{Y}_4(5,\alpha) = \frac{-1}{5}(0.6666\alpha^6 - 27.3333\alpha^5 + 133\alpha^4 - 373.5833\alpha^3 + 604.5417\alpha^2 - 388.25\alpha + 51)$$

Therefore, from (4) equation (16) is solved by

$$\begin{aligned}\overline{Y}_1(x,\alpha) = {} & (2-\alpha) + (8\alpha^2 - 24\alpha + 18)x + (2\alpha^3 + \alpha^2 - 16\alpha + 15)x^2 \\ & + \frac{2}{3}(-8\alpha^4 + 55\alpha^3 - 124.5\alpha^2 + 111\alpha - 31.5)x^3 \\ & + \frac{2}{4}(-1.3333\alpha^5 + 20\alpha^4 + 40\alpha^3 - 11.8333\alpha^2 + 16\alpha - 61.5)x^4 \\ & + \frac{2}{5}(2.6667\alpha^6 - 28.6667\alpha^5 + 128.6667\alpha^4 - 234.25\alpha^3 + 161.125\alpha^2 - 14.25\alpha - 14.625)x^5 + \cdots\end{aligned}$$

$$\begin{aligned}\overline{Y}_2(x,\alpha) = {} & (1.1 - 0.1\alpha) + (1.9 - 0.9\alpha)x - \frac{1}{2}(-8\alpha^2 + 25.9\alpha - 16.9)x^2 \\ & - \frac{1}{3}(-2\alpha^3 - 0.5\alpha^2 + 13.55\alpha - 10.55)x^3 \\ & + \frac{1}{4}(-5.3333\alpha^4 + 38\alpha^3 - 90.5\alpha^2 + 87.3167\alpha - 29.3166)x^4 + \frac{1}{5}(-0.6666\alpha^5 + 10\alpha^4 + 20\alpha^3 - 5.9166\alpha^2 + 8\alpha - 31.375)x^5 + \cdots\end{aligned}$$

$$\overline{Y}_3(x,\alpha) = (0.1 - 0.1\alpha) + (1.9\alpha - 0.9)x + (0.5\alpha^2 - 2.45\alpha + 1.95)x^2$$

$$-\frac{1}{3}(4\alpha^3 - 22.5\alpha^2 + 39.95\alpha - 20.95)x^3$$
$$+\frac{1}{4}(-0.6667\alpha^4 + 9.6667\alpha^3 - 32.3333\alpha^2 + 39.8166\alpha - 16.4833)x^4$$
$$+\frac{1}{5}(1.3333\alpha^5 - 11.6666\alpha^4 + 46.3333\alpha^3 - 61.25\alpha^2 + 29.5792\alpha$$
$$- 4.2875)x^5 + \cdots$$

$$\overline{Y}_4(x,\alpha) = (3-2\alpha) - (\alpha^2 - 4\alpha + 4))x + \frac{1}{2}(8\alpha^3 - 37\alpha^2 + 54\alpha - 24)x^2$$
$$-\frac{1}{3}(-2\alpha^4 + 27\alpha^3 - 97.5\alpha^2 + 133\alpha - 60)x^3$$
$$+\frac{1}{4}(-5.3333\alpha^5 + 41.3334\alpha^4 - 147.3333\alpha^3 + 154.5\alpha^2$$
$$- 31\alpha - 12)x^4$$
$$+\frac{-1}{5}(0.6666\alpha^6 - 27.3333\alpha^5 + 133\alpha^4 - 373.5833\alpha^3$$
$$+ 604.5417\alpha^2 - 388.25\alpha + 51)x^5 + \cdots$$

Combining $\underline{Y}_1$, $\underline{Y}_2$, $\underline{Y}_3$, $\underline{Y}_4$ and $\overline{Y}_1, \overline{Y}_2, \overline{Y}_3, \overline{Y}_4$represent the fuzzy solution of the system of fuzzy differential equations (14) as $y_{1\alpha}(x) = [\underline{Y}_1(x), \overline{Y}_1(x)]$ and $y_{2\alpha}(x) = [\underline{Y}_2(x), \overline{Y}_2(x)]$, $y_{3\alpha}(x) = [\underline{Y}_3(x), \overline{Y}_3(x)]$, $y_{4\alpha}(x) = [\underline{Y}_4(x), \overline{Y}_4(x)]$, □□□□ (0, 1], x □ [0, 1]. It is clear that for □□□1, we find $\underline{Y}_1(x) = \overline{Y}_1(x)$ and $\underline{Y}_2(x) = \overline{Y}_2(x)$, $\underline{Y}_3(x) = \overline{Y}_3(x)$, $\underline{Y}_4(x) = \overline{Y}_4(x)$ which is the same exact solution of the system of non-fuzzy differential equations. The results of the calculations are shown in table (2):

Table (1)

The lower and upper levels of fuzzy solution for example (1)

x	$\underline{Y}_1$	$\overline{Y}_1$	**exact solution of** Y_1	$\underline{Y}_2$	$\overline{Y}_2$	**exact solution of** Y_2
0	1	1	1	1	1	1
0.1	1.2214	1.2214	1.2214	1.0948	1.0948	1.0948
0.2	1.4918	1.4918	1.4918	1.1787	1.1787	1.1787
0.3	1.8220	1.8220	1.8221	1.2509	1.2509	1.2509
0.4	2.2251	2.2251	2.2255	1.3105	1.3105	1.3105
0.5	2.7167	2.7167	2.7183	1.3570	1.3570	1.3570
0.6	3.3151	3.3151	3.3201	1.3901	1.3901	1.3900
0.7	4.0422	4.0422	4.0552	1.4093	1.4093	1.4091

0.8	4.9231	4.9231	4.9530	1.4145	1.4145	1.4141
0.9	5.9869	5.9869	6.0496	1.4058	1.4058	1.4049
1	7.2667	7.2667	7.3891	1.3834	1.3834	1.3818
x	$\underline{Y}_3$	$\overline{Y}_3$	**exact solution of** Y_3	$\underline{Y}_4$	$\overline{Y}_4$	**exact solution of** Y_4
0	0	0	0	1	1	1
0.1	0.0998	0.0998	0.0998	0.9048	0.9048	0.9048
0.2	0.1987	0.1987	0.1987	0.8187	0.8187	0.8187
0.3	0.2955	0.2955	0.2955	0.7408	0.7408	0.7408
0.4	0.3894	0.3894	0.3894	0.6703	0.6703	0.6703
0.5	0.4794	0.4794	0.4794	0.6065	0.6065	0.6065
0.6	0.5646	0.5646	0.5646	0.5488	0.5488	0.5488
0.7	0.6442	0.6442	0.6442	0.4964	0.4964	0.4966
0.8	0.7174	0.7174	0.7174	0.4490	0.4490	0.4493
0.9	0.7834	0.7834	0.7833	0.4059	0.4059	0.4066
1	0.8417	0.8417	0.8415	0.3667	0.3667	0.3679

6-Conclusions

We conclude from this paper, the following:

1. The upper and lower solutions are equal when $\alpha = 1$this confirms the accuracy of the results.
2. the system of fuzzy Differential Equations can be considered as a generalization to the system of non-fuzzy Differential Equations, in which the solution of a system of non-fuzzy Differential Equations is obtained from the fuzzy solution by setting $\alpha = 1$.
3. The Method has proved its effectiveness, accuracy of results and gives rapidly converging series solutions in solving linear and nonlinear system of Fuzzy Differential Equations.

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حل المعادلات التفاضلية الخطية والغير الخطية باستخدام طريقة التحويل التفاضلي

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مستخلص البحث:
في هذا البحث تم استخدام طريقة التحويل التفاضلية(DTM) لحل النظام الخطي والغير الخطي للمعادلات التفاضلية الضبابية. يتم حساب الحل التقريبي للنظام على شكل سلسلة متقاربة سريعة باستخدام أوصاف وخصائص نظرية المجموعة الضبابية , وهذه الطريقة تلتقط الحل الدقيق لتوضيح القدرة والخواص الدقيقة لهذه العملية , يتم حل بعض أنظمة المعادلات التفاضلية الضبابية كأمثلة عددية.يتم عرض مقارنة النتائج العادية المقيمة مع الحلول الدقيقة لكل مجموعة مستويات ضبابية في شكل جداول .

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