Some properties of intuitionistic fuzzy cone metric space
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Abstract

In this paper, we will study a concepts of sectional intuitionistic fuzzy cone continuous and prove the schauder fixed point theorem in intuitionistic fuzzy cone metric space as a generalization of fuzzy cone metric space and prove another version of schauder fixed point theorem in intuitionistic fuzzy cone metric space as a generalization to the other types of fixed *Point theorems in intuitionistic fuzzy cone metric space considered by other researchers,* as well as, to the usual intuitionistic fuzzy cone metric space.

Keywords: Fuzzy − Cone Metric space,
Intuitionistic in Fuzzy − Cone Metric space,
Schauder Fixed Point theorem.

Introduction

The concept to know intuitionistic fuzzy sets (IFSs) to fuzzy set popularization, Insert through Atanassov [1], after that, many authors developed the theory of intuitionistic fuzzy- metric spaces by park in [2], as well in intuitionistic fuzzy topology (both in metric and normed) spaces Saadati and park in[3],Kramosil & Michalek[4] introduced the fuzzy metric space through popularizing that notion from probabilistic- metric space through popularizing the notion from probabilistic metric spaces for fuzzy condition. George & Veeramani[5] modified the notion from fuzzy metric spaces. Huangad Zhang [6] inserts the concept from cone spaces through displacing real numbers for an arranging banach space & show several theorems in fixed point to contractive -mappings through that spaces. In (2008) some notes on paper cone metric spaces by S. Rezapour and R. Hamlbarani [7],After that, may authors prove fixed point theorems using different mapping in such spaces[8],[9],[10],[11],[12], the aim of this paper the concept of this is introducing to intuitionistic fuzzy cone- metric space, Schauder theorem in fixed point on intuitionistic fuzzy cone-metric space are stated in & show ,when another prove version from theorem fixed point of many authors from years.

2-Preliminaries:

Some basic concepts of fuzzy cone metric spaces and intuitionistic fuzzy cone metric spaces are given in this section.

Definition (2.1), [11]: *A subset q of E is called a cone if*

1. q *is closed, non* $-$ *empty* $\&\{ q \neq 0\}$,
2. *If* $x,y \in R$ & $x,y \geq 0$ *for* $a,b \in q$ *then* $(ax+by) \in q$,
3. *If both* $a \in q$ *and* $-a \in q$ *then* $a = 0$.

For a given cone, a partial ordering $\leq$ *on* E *via* P *is defined by* $a \leq b$ *when* $(b-a) \in q\}$. $(a < b)$ *will stand for*$(a \leq b)$, $(a \neq b)$, *while* $\{(a \ll b)$ *will stand for* $(b-a) \in int(q)\}$

Where E, is the set *of real numbers* throughout that sheet, we let that all cones have a nonempty interior.

Definition (2.2), [11]:

A cone metric space is an ordered (X,d), *where* X *is any set and* $d: X \times X \to E$ *is a mapping satisfying*:

1. $[0 \leq d(a,b)$ *for all* $a,b \in X]$,
2. $[d(a,b) = 0]$ *if* a *equal* b,
3. $[d(a,b)$ *equal* $d(b,a)$ *for all* a & $b \in X]$,
4. $[d(a,z) \leq d(a,b) + d(b,z)]$ *for all* x,y & $z \in X$.

Definition (2.3),
[11]

: *Let* (X,d) *be a cone metric space. Then, for each* $c1 \gg 0$ *and* $c2 \gg 0, c_1, c_2 \in E$, *there exists* $c \gg 0, c \in E$ *such that* $c \ll c_1$ *and* $c \ll c_2$.

Definition (2.4), [13]

A binary operation $*: [0,1] \times [0,1] \to [0,1]$ *is a continuous t_norm then* $*$ *satisfy the following condition*:

a) $*$ *is commutative & associative*

b) $*$ *is continuous,*

c) $a * 1 = a$ *for all* $a \in [0,1]$,

d)) $a * b \leq c * d$,$a \leq c$ *and* $b \leq d$ *to each* $a,b,c,d \in [0,1]$.

Definition (2.5), [13]:

A binary operation $\lozenge: [0,1] \times [0,1] \to [0,1]$ *is a continuous* $t-$ *conorm if* $*$ *satisfies the following conditions*:

1) $\lozenge$ *is commutative and associative,*

2) $\lozenge$ *is continuous,*

3) $a \lozenge 0 = a$ *for all* $a \in [0,1]$,
4) $a \lozenge b \le c \lozenge d$ *whenever* $a \le c$ *and* $b \le d$ *for all* $a, b, c, d \in [0,1]$.
Note: If we*take* $E = R, P = [0, \infty)$ *and* $a * b = min\{a, b\}, a \lozenge b = max\{a, b\}$, for all intuitionistic fuzzy - metric space become intuitionistic in fuzzy cone - metric spaces.
Definition(2.6),[9]: *A* $3-tuple\ (X, M, *)$ is known for through fuzzy cone-metric space*if* q, *is at cone of* E, X *is an arbitrary set*,$*$ *is a continuous* $T-norm$ & M *is a fuzzy set at* $x^2 \times int\ (q)$,
satisfying the following next case to whole
a, b & $c \in X$, n & $m \in int\ (q)$ (*that is* $n \gg \theta, m \gg \theta$) :
1)$[M(a, b, t) > 0]$.
2)$[M(a, b, t) = 1]$ *if & only if* $a = b$.
3) $[M(a, b, t) = M(b, a, t)]$.
4) $[M(a, b, n) * M(b, c, m) \le M(a, c, n + m)]$.
5) $[M(a, b, .): int\ (q) \to [0, 1]$ *is continuous*$]$.
<u>Definition (2.7):</u> A $4-tuple\ (X, M, *)$ is known fuzzy in a metric space if & only if X is as arbitrary(non-empty)set,M *is a fuzzy set of* $X * X * (0, \infty)$& $*$ *is continuous* $T-norm$, satisfying that following next case to every a, b, c & $e \in X, n$ & $m > 0$:
1) $[M\ (a, b, c, n) > 0]$.
2) $[M(a, b, c, n) = 1]$ *if & only if* a *equal* b *equal* c.
3) $[M(a, b, c, n)$ *equal* $M\ (p\{a, b, c\}, n)]$, When p is at permutation function of a, b and c.
4) $[M\ (a, b, e, t) * M(e, c, c, m) \le M\ (a, b, c, n + m)]$.
5)$\{M\ (a, b, c, *): (0, \infty) \to [0, 1]\}$ Is a continuous.
<u>Definition (2.8)</u>
A $4-tuple\ (X, M, *)$ Is known fuzzy cone- metric space*if* P *is a cone of* E, X is an arbitrary (nonempty) set, $*$ *is continuous* $T-norm$ & M *is a fuzzy* set of $X * X * int\ (q)$ satisfying the following conditions to eacha, b, c & $e \in X$, n & $m \in int\ (q)$(*that is* $n > 0, m > 0$).
1)$[(a, b, c, n) > 0]$.
2) $[M(a, b, c, n) = 1]$ *if & only if* a *equal* b *equal* c.
3) $[M(a, b, c, n)$ *equal* $M\ (p\{a, b, c\}, n)]$, When p is a permutation function of a, b and c.
4)$[M\ (a, b, e, n) * M\ (e, c, c, m)$ *less then* $M\ (a, b, c, n + m)]$.
5) $[M\ (a, b, c, *): int(q) \to [0, 1]$ *is a continuous*$]$.

<u>Definition (2.9),[11]:</u>

A 5 –tuple (X, M, N, *,◊) is said to be an intuitionistic fuzzy cone metric space (i. f. m. s) if q is cone of$E$,

$*$ $is\ a\ continuous\ t-norm, X\ is\ an\ arbitrary\ set,$

$\lozenge$ $is\ a\ continuous\ t-\ conorm,$

$M\ \&\ N\ are\ fuzzy\ sets\ on\ X2 \times int(q)\ sach\ that\ cases: \{to\ every\ a,b\&\ c\ \in X\}, \{\ n\ \&\ m\ \in int(q)\ that\ is\ n\ \gg 0, m \gg 0\}$

1) $[M\ (a,b,n)\ +\ N(a,b,n)\ \ \leq\ 1]$,
2) $[M(a,b,0)\ >\ 0]$,
3) $[M\ (a,b,n)\ =\ 1]\ if\ \&\ only\ if\ a\ =\ b$;
4) $[M\ (a,b,n)\ =\ M(b,a,n)]$,
5) $[M(a,b,n) * M(b,c,m)\ \leq\ M(a,c,n\ +\ m)]$;
6) $[M(a,b,.): int\ (q) \to\ [0,1] is\ continuous]$,
7) $[N(a,b,n)\ > 0]$,
8) $[N(a,b,n)\ =\ 0]\ if\ \&\ only\ if\ a\ equal\ b.$
9) $[N(a,b,n)\ =\ N(b,a,n)]$,
10) $[N(a,b,n) \lozenge\ N(b,c,m) \geq\ N(a,c,n\ +\ m)]$,
11) $[N(a,b,.)\ : int\ (q)\ \to\ [0,1]\ is\ continuous]$.

$Then\ (M,N)\ is\ called\ an\ intuitionistic\ fuzzy\ cone\ metric\ on\ X.$ The function M(x, y, t) &

$(x,y,t)\ denote\ the\ degree\ of\ nearness\ and\ the\ degree\ of\ non-nearness\ between\ x\ \&\ y\ with\ respect\ to\ t, respectively.$

An alternative definition of convergent and Cauchy sequence in intuitionistic in fuzzy - metric space is given next.

Definition (2.10), [11]:

$Let\ (X,\ M,\ N,*,\lozenge)\ be\ at\ intuitionsitic\ fuzzy\ cone\ metric\ space, x \in X. Then$

1)$A\ sequence\ \{xn\}\ in\ X$ is known for becoming (Cauchy sequence) if at all $t\ > \theta\ \&\ 0 < \epsilon < 1\ There\ \exists$ a natural number n_0 show that [

$M\ (x_n,x_m,\ t) > 1-\epsilon, N\ (x_n,x_m,t) < \epsilon$] to each n, $m\ \geq\ n_0$.

2)$A\ sequence \{x_n\}\ in\ X\ is\ called\ for\ become\ convergent\ to\ a\ point\ x\ on\ X\ if,$
There exists a natural n_0 to each

$[M\ (x_n, x, t) > 1 - r] \& [N\ (x_n, x, t) < r]\ if\ for\ any\ n \geq n_0, r \in (0,1)\ \&\ t \gg 0$

We denote this by $\lim_{n\to\infty} x_n$ = x or $x_n \rightarrow$ x as n $\rightarrow \infty$.

3) $A\ sequence\ x_{n\,in}\ X$ (Cauchy sequence is convergent) $, is\ called\ for\ become$ Complete.

4)$If\ each\ sequence\ in\ X\ includes\ a\ convergent\ subsequence,\ is\ called\ to$ become compact.

Note1: $If\ E = R, P = [0, \infty)\ and\ a * b = a\, b$, then each fuzzy in metric space became a fuzzy -cone in metric spaces.

Example1. [9]: $a * b = a\, b$

Example2. [9]: $a * b$ = min $\{a, b\}$

Example3. [9]; when $E = R^2, then\ q = \{(K1, K2): K1, K2 \geq 0\} \subset E$ known is a normal- cone together normal stable K=1. Where $X = R,\ [a * b = ab]\ and\ M: X^2 \times int\ (q) \rightarrow [0,1]$ known through $[M(x, y, t) = \frac{1}{e^{\frac{|x-y|}{\|t\|}}}]\ to\ each\ x, y \in X$ & t $\gg \theta$.

Note2: $If\ E = R, P = [0, \infty)\ \&\ a * b = ab, a \lozenge b = max\ \{a, b\},$ then for all intuitionistic in fuzzy metric- spaces be an intuitionistic in fuzzy - cone metric spaces.

Proposition1. [9]:$when\ p\ became\ a\ cone\ of\ E$. Then,

1) $[\ int\ (p) + int\ (p) \subset int\ (p)],$

2)$[\ \lambda\ int\ (p) \subset int(p) to\ all\ \lambda \in R+]$

3) $[to\ all\ \theta \leq c1\ \&\ \theta \leq c2, there\ is\ an\ element\ \theta \leq c\ show\ at\ c \leq c1, c \leq c2]$

If that $\exists$ a stable $K > 0$ that cone P is known normal $show\ there\ to\ each\ t, s \in E, \theta \leq t \leq s\ \ it\ \|t\| \leq K\ \|s\|$& that least positive number K such that property is known normal stable of P [6]. Rezapour, Hamlbarani [7] we are no cones together normal stable $K < 1$ & it cones of normal stable 1, & cones for normal stable$[M > K\ to\ that\ K > 1]$.

Example4:

$let E = R^2, [P = \{(x, y) \in E: x, y \geq 0\}], X = R\ \&[\ d = X \times X \rightarrow E\ ] known\ through$

{d(x, y) = ($|x - y|$,$\alpha|x - y|$)}

Where $\alpha \geq 0$ is a stable? That (x, d) is a cone metric space.

Definition (2.11),

[11]: $Let\ (X, M, N, *, \lozenge)\ be\ at\ intuitionistic\ fuzzy\ cone\ metric\ space$. A subset A of X is called for become (FC-bounded) if that $\exists\ t > \theta\ \&\ r \in (0,1)$ show their$\{M(x, y, t) > 1 - r,$
$N(x, y, t) < r\}\ to\ each\ x, y \in A.$

Definition (2.12): $Let\ (X, M, N, *, \lozenge)\ be\ at\ intuitionistic\ fuzzy\ cone\ metric\ space$ A subset A of X and B⊑X is known for become (FC- closed) if that ∃ r ∈(0, 1), t >0 if & only if for any sequence $\{x_n\}$ in B converges to x ∈ X such that $\lim_{n\to\infty} M(x_n, x, t) \geq 1-r$ and $\lim_{n\to\infty} N(x_n, x, t) \leq r, x \in B$.

Definition (2.13):
$Let\ (X,\ M,\ N, *, \lozenge)\ be\ at\ intuitionistic\ fuzzy\ cone\ metric\ space$. Let subset A for X is called for through FC-compact for each sequence in X contains a convergent subsequence.

Definition (2.14): $Let\ (X, M, N, *, \lozenge)\ be\ at\ intuitionistic\ fuzzy\ cone\ metric\ space$. A continuous mapping $W: X2 \times [0,1] \to X$ is said to be FC- convex structure on X if for all $x, y \in X$ and $r \in [0,1], t > 0$,
$[\emptyset(u, w(x, y, r), t)] \leq [\lambda M(u, x, t) + (1 - \lambda) N(y, y, t)]$
Holds for all u ∈ X. $(X, M, N, *, \lozenge)\ be\ at\ intuitionsitic\ fuzzy\ cone\ metric\ space$ together with a convex structure is called an
$intuitionsitic\ fuzzy\ cone\ convex\ metric\ space.$

Example5: $Let\ (X, M, N, *, \lozenge)\ be\ at\ intuitionsitic\ fuzzy\ cone\ metric\ space.\ Define\ t - norm\ ab = min\ \{a, b\}\ and\ a \lozenge b = max\ \{a, b\}\ for\ all\ a, b \in X\ \&\ t > 0,$ Let us define

$$\left[M(a, b, n) = \frac{n}{(n + M(a, b))}\right] \quad and \quad \left[N(a, b, n) = \frac{N(a, b)}{(n + N(a, b))}\right]$$

Example6: $Let\ (X, M, N, *, \lozenge)\ through\ at\ intuitionistic\ fuzzy\ cone\ metric\ space$. Let q be any cone, $X = N, a * b = ab, M, N: X2 \times int\ (q) \to [0,1]$ known through

$$[M(x, y, t) = \begin{cases} \frac{x}{y} & if\ x \leq y \\ \frac{y}{x} & if\ y \leq x \end{cases} \quad \text{for all x, y} \in \text{X \& t} > 0].$$

$$[N(x, y, t) = \begin{cases} \frac{y-x}{y} & if\ x \leq y \\ \frac{x-y}{x} & if\ y \leq x \end{cases} \quad \text{for all x, y} \in \text{X \& t} > 0].$$

Example7:
$Let\ (X, M, N, *, \lozenge)\ be\ at\ intuitionsitic\ fuzzy\ cone\ metric\ space. E = R2, P = \{(a, b) \in$ E: $a, b \geq 0\}, X = R$, where R is real number, and M, N: $X^2 \times int\ (q) \to E$ defined by

$$\begin{cases} M(a, b, n) = (|a - b|, \alpha|a - b|) \\ N(a, b, n) = (1 - |a - b|, 1 - \alpha|a - b|) \end{cases} where\ \alpha \geq 0\ is\ aconstant\ for\ all\ t > 0$$

In what follows A = (X, N_1, M_1, *,◊) and B = (Y, N_2, M_2, *',◊ ') will denote two $intuitionistic\ fuzzy\ cone\ metric\ space$, where X & Y are cone metric spaces.

Definition11: $Let\ (X, M, N,*,\Diamond)\ be\ at\ intuitionsitic\ fuzzy\ cone\ metric\ space$. A subset A, B of X. A mapping T: A→B is said to be sectional intuitionistic fuzzy cone continuous as $x_0 = (x_{01}, x_{02}, \dots, x_{0n}) \in x^n, if\ \exists\ r \in (0,1)$ such that for each$\in > 0$, there exist $\delta > 0$ such that, $[N_1(x, x_0, \delta) \le r]$ & $[M_1(x, x_0, \delta) \ge 1 - r]$ & $[N_2(T(x), T(x_0), \epsilon) \le r]$ & $[M_2(T(x), T(\ x_0), \epsilon) \ge 1 - r]$ for every X= $(x_1,\ x_2, \dots, x_n) \in X^n$.

Note3: let A be intuitionistic fuzzy cone metric space. As same the condition that (proposition) for all t > 0, N (x, y, t) < r define [d(x, y)$_r$ = inf $\{t > 0 = [N\ (x,y,t) \le r] and\ [M\ (x,y,t) \ge 1 - r]\}$], r∈(0,1).

Then $\{d(x,y)_r : r \in (0,1)\}$ is at ascending family for cone metric space on X. we call that cone metric space as r- cone metric space on X corresponding to intuitionistic fuzzy cone metric space A.

Remark1: if (X, M) is a fuzzy cone metric space and k ⊂ X, then $\bar{K}$ is closed in$(X, d_r\ (x,y))$ and$\bar{k}^r \subset \bar{K}\ for\ all\ r\ \in (0,1)$, where $\bar{k}^r$denotes the closure of K in (X, $d_r\ (x,y)$).

Shchauder fixed point theorem

Theorem1: let A bean $intuitionistic\ fuzzy\ cone\ metric\ space.$ then X is $an\ intuitionsitic\ fuzzy\ cone\ bounded$ if and only if X is cone bounded with respect to $d_r(x,y) for\ all\ r\ \in (0,1)\ where\ d_r(x,y)\ denotes\ the\ r - cone\ metric\ of\ X.$

Proof: first suppose that x is $an\ intuitionistic\ fuzzy\ cone\ bounded\ then$for each $r \in (0,1)\ \exists\ t(r) > 0$ such that

$N(x,y,t(r))\ \le r\ \&\ M(x,y,t(r)) \ge 1 - r\ for\ all\ x,y \in X \dots_{\dots\dots} *$

Now from the definition we have

$[d(x,y)_r] = [\inf\{t > 0: [N(x,y,t) \le r]\ \&[\ M(x,y,t) \ge (1 - r)], r \in (0,1)\}] \dots \dots **$

From (*) we have, $[d(x,y)r\ \le t(r)] to\ each\ x,y\ \in X$ and then X is cone bounded with respect $to d(x,y)r$.

Conversely, suppose that, X is cone bounded together $to\ [d(x,y)r]$,$0 < r\ < 1$. then to each r $\in (0,1)\ \exists\ t(r)\ such\ that\ [d(x,y)_r \le t(r)] for\ all\ x,y \in X$.that is $[d(x,y)r\ \le t(r) < t(r) + 1]$ $for\ all\ (x,y) \in X$. By $(*)[N(x,y,t(r) +\ 1) \le r\ ], [M\ (x,y,t(r) + 1) \ge 1 - r]\ for\ all\ x,y \in X\ then$ X is intuitionistic fuzzy cone bounded.

Theorem2: let A be intuitionistic fuzzy cone metric space and B ⊂ X. Then B is intuitionistic fuzzy cone closed if and only if B is cone closed with respect to [d(x, y)$_r$] to every r∈ $(0,1)$.

Proof: $first\ suppose\ that\ B$ intuitionistic in fuzzy cone closed. Take $r_0\ \in (0,1)$. Let $\{x_n\}$ become a sequence at B show their [$\lim_{n\to\infty} d(x,y)_r$=0]. Then for a

given $\in > 0, \exists$ appositive integer N (ϵ) such that [d(x, y)$_{r0}$ $< \epsilon$], for all n $\geq$ N($\in$).
$[N(X, Y, \epsilon) \leq r_0 \; for\, all\, n \geq N(\epsilon)]. [lim_{n\to\infty} N(X, Y, \epsilon) \leq r_0 \; for\, all\, \epsilon > 0].$
Sine ϵ is arbitrary and by condition (1) of definition (2.9)
It follows that $\lim_{n\to\infty} M(x, y, \epsilon) \geq 1 - \alpha_0 \; then\; x \in B.$ B is cone closed together for$d(x, y)_{r_0}$.
Since 0< r_0 <1 is arbitrary, it follows that B is cone closed together for$[d(x, y)r, 0 < r < 1]$.
To show for that converse follows by conditions (1), (7) and (11) of definition (2.9)
Theorem 3: let A be intuitionistic fuzzy cone metric space satisfy (proposition1) that $\subset$ B for X is cone (intuitionistic Fuzzy compact) when B is cone compact together for d(x, y)$_r$ to every $r \in (0,1)$.
Proof: let that first B is cone-intuitionistic fuzzy compact if $r_0 \in$ (0, 1). Suppose {x_n} through a sequence in B. that $\exists$ a sub sequence $\{x_{n_k}\}$ and x in B (both depending on r_0) such that $[\lim_{k\to\infty} N(x_{nk}, x, t) \geq r_0]$ & $[\lim_{k\to\infty} M(x_{n_k}, x, t) \geq 1 - r_0 \; \forall\, t > 0]$.
$For\; a\; given\; \epsilon > 0\; with\; r_0 - \epsilon > 0\; and\; for\; a\; t > 0\; \exists\; appositive\; integer\; k\,(\epsilon, t)$
Such that
$[N\,(x_{n_k}, x, t) \leq r_0 - \in]$&
$[M\,(x_{n_k}, x, t) \geq 1 - r_0 + \epsilon\; \forall k \geq k(\epsilon, t)]\,, [lim_{k\to\infty}\, d_{r_{n-\epsilon}}(x_{n_k}, x) = 0]$
B is compact with respect to$d_{r_{0-\epsilon}}(x_{n_k}$, x)
Since
$[r0 \in (0,1)\; \&\; \in >$
$0]\; arbitrary\; it\; follows\; their\; B\; is\; compact\; with\; respect\; to\; dr\; (x_{n_k}, x$
) for each r $\in$(0,1). Converse follows from theorem (1&2).
Theorem4: let A and B be an intuitionistic fuzzy cone metric space then a mapping $T\colon A \to B$ is a fuzzy cone – sectional in intuitionistic fuzzy continuous if & only if $[T\colon (X, d_r^1(x, y)) \to (Y, d_r^2(x, y))]$is continuous to r $\in (0,1)$.
Proof: let that first [T: A$\to B$] is fuzzy cone – sectional intuitionistic continuous. Thus $for\; any$ y$\in$ X, $\exists\, r_0 \in (0,1)$such that to every $\epsilon > 0, \exists\, \delta > 0$ $(N_1(x, y, \delta) \leq r_0)$, (M_1(x, y,$\delta) \geq 1 - r_0)$ & $(N_2(T(x), T(y), \epsilon) \leq r_0)$, $(M_2(T(x), T(y), \epsilon) \geq 1 - r_0)$ ⍰ $x \in X$.
Choose η_0 such that
$\delta_1 = \delta - \eta_0 > 0, let\; d_{r_n}^1(x, y) \leq \delta - \eta_0 = \delta_1\; then\; d_{r_n}^1(x, y) \leq \delta - \eta_0 < \delta.$
$(N_1(x, y, \delta) \leq r_0), (M_1(x, y, \delta) \geq 1 - r_0), [N_2(T(x), T(y), \epsilon) \leq r_0], [M_2(T(x), T(y), \epsilon) \geq 1 - r_0].$

Then $[d^2_{r_n}(T(x),T(y)) \leq \epsilon]$.
Thus T is continuous with respect to $d^1_{r_n}(x,y)$ $and d^2_{r_n}(x,y)$.
Next let T is continuous together for $[d^1_{r_n}(x,y)$ $and d^2_{r_n}(x,y)]$thus □y∈ X and $\epsilon > 0$ $\exists\delta > 0$ $such$ $that$
$[d^1_{r_0}(x,y) \leq \delta]$, $[d^2_{r_0}(T(x),T(y)) \leq \frac{\epsilon}{2}]$.
[
$N_1(x,y,\delta) \leq r_0$] ,$[M_1(x,y,\delta) \geq 1 - r_0$]$then[$ $d^1_{r_0}(x,y) \leq \delta \rightarrow d^2_{r_0}(T(x),T(y)) \leq \frac{\epsilon}{2} < \epsilon$].
$[N_2(T(x),T(y),\epsilon) \leq r_0]$,$[M_2(T(x),T(y),\epsilon) \geq 1 - r_0]$.
Theorm5: [theorem of Schauder for (fixed point)]
suppose K through a non-empty convex, ($intuitionistic$ $fuzzy$ $compact$) subset from an ($intuitionistic$ $fuzzy$ $cone$ $metric - space$) satisfying (proposition1) & $T: K \rightarrow K$ be fuzzy cone sectional intuitionistic fuzzy -continuous. That it has a (fixed point).
Proof: if A such (proposition (1)), that $(X, d_r(x,y))$ is a $cone$ $metric - space$. let K is an intuitionistic in fuzzy cone - compact subset of X, K is a cone - compact subset of (X, $d_r(x,y)$) by theorem (3) for each r ∈(0, 1).
Again since $T: K \rightarrow K$is a cone sectional intuitionistic fuzzy continuous $\exists r_0 \in (0,1)$ it is prove $T: K \rightarrow K$ is continuous with respect to (X, $d_{r_0}(x,y)$) by theorem (4)
Hence it follows that K is a non-empty, convex and cone compact subset of the cone metric space (X, $d_{r_0}(x,y)$) and $T: K \rightarrow K$ is a continuous - mapping. So throgh [theorem of schauder for (fixed point)] [15]
[let A be a $closed$ $-$
$convex$ $subset$ of $banach$ $space$ & let $there$ $exist$ a $continuous - mapT$
T $sending$ A to a $countable - compact$ $subset$ $T(A)$ of A $then$ T $has($ $fixed$ $points$)], then such that T has a (fixed point).
Theorem6: suppose K through a non- empty, intuitionistic in fuzzy closed - convex subset for at intuitionistic in fuzzy cone metric- space. A satisfying (proposition (1)) & suppose $[T: K \rightarrow K]$ be cone - sectional intuitionistic in fuzzy continuous togther $\bar{T}$ (K) through intuitionistic in fuzzy cone - compact. That T has a (fixed point) in K.
Proof: then satisfies (proposition (1)), then from theorem (3) that(X, $d_r(x,y)$) is a cone metric space
A gain since K is an intuitionistic in fuzzy cone - closed; K is closed with respect to $d_r(x,y)$ for each r $\in (0,1)$. Now By section $T: K \rightarrow K$ is cone – sectional in intuitionistic fuzzy - continuous.

Thus $\exists\, r_0 \in (0,1)$ it is prove $T: K \to K$ is continuous with respect to $d_{r_0}(x,y)$. since also $\bar{T}\,(K)$is an intuitionistic in fuzzy cone - compact by theorem (3) $\bar{T}\,(K)$ is cone - compact with respect to $d_r(x,y)$ for each r$\in (0,1)$.

In part $\bar{T}\,(K)$ is cone- compact on (X, $d_{r_0}(x,y)$)

From remark (1) also $\bar{T}\,(K)$is cone-closed in (X, $d_{r_n}(x,y)) and \bar{T}\,(K)^{r_0} \subset \bar{T}\,(K) where\, \bar{T}\,(K)^{r_0}$ is that closure for T(K) in (X,$d_{r_n}(x,y)). so \bar{T}\,(K)^{r_0}$ is cone compact in (X,$d_{r_0}(x,y)$) thus K is a non- empty closed convex subset for a fuzzy cone metric space (X,$d_{r_0}(x,y)$) $and\ T: K \to K$ is continues together $\bar{T}\,(K)^{r_0}$cone - compact.

Then through [theorem schauder for fixed point] [15] follows that it T is a (fixed point).

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بعض خصائص الفضاء المخروطي الضبابي الحدسي

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المستخلص:
في هذه الورقة ، سوف ندرس مفاهيم مخروطية غامضة حدسية مقطعية مستمرة ونثبت نظرية نقطة شاودر الثابتة في "الفضاءات المترية المخروطية الضبابية الحدسية كتعميم للفضاء المتري المخروطي الضبابي وإثبات نسخة أخرى من نظرية نقطة شاودر الثابتة في الفضاء المتري المخروطي الضبابي الحدسي كتعميم "للأنواع الأخرى من نظريات النقطة الثابتة في الفضاء المتري المخروطي الضبابي الحدسي الذي يدرسه الباحثون الآخرون ، وكذلك إلى الفضاء المتري المخروطي الضبابي الحدسي المعتاد.

الكلمات المفتاحية: الفضاء المتري المخروطي الضبابي ،الفضاء المتري المخروطي في الضبابي الحدسي،نظريات النقطة الصامدة شاودر