ON (α – β) - CONTRACTIVE MAPPING OF PARTIAL b-METRIC SPACES AND FIXED POINT THEOREM

Moayad Mhmoud Khlil
Dept. of mathematics - College of education for pure Sciences
University of Tikrit

Abstract

we prove in this article a unique common fixed point theorem for α – β contractive condition for rational contraction and also we give example that explain the main result.

Keywords: Partial, b-metric space, weakly , compatible mapping, rational contraction. metric space.

١.Preliminaries

In the year ١٩٩٢,a partial metric space is a generalization of the notion of the metric space defined in ١٩٠٦,by (Maurice.frechet) such that the distance of a point from itself isn't necessarily ٠.well known banach fixed point theorem also known as banach contaction principle,was.a foundation for a development of metric fixed point theory and found applications in different areas. There was much popularization of that result in tha last seventy years.At ١٩٨٩ [١] submitted the notion of Quasi-Metric Space as a special general concept of Metric Spaces. At١٩٩٣,[٢][٣] put many theorems due to the b–metric spaces.In (١٩٩٤)[٤] assumptive the notion of partial metric space in which the Self-distance of every point of space not equal ٠.At(١٩٩٦)o'neill generalized the concept of (P.M.SP) by introduced negative distances.At(٢٠١٣) [٥] popularized both the notion of (b-M.SP) and (P.M.SP) by presenting the partial b-metric spaces (P.b-M.SP)as example alot of researchers at present day have studying this presupposition and its generalization in different types of (M.SP).

some authors close to our interest were studying some fixed point theorems in the so called b-metric space. After then, some authors started to prove ((α – ψ)) versions of certain fixed point theorems in different type metric spaces [٦,٧,٨]. Mustafa in [٩], gave a generalization of Banach's contraction principles in a complete ordered partial b-metric space by introducing a generalized (α-ψ)weakly contractive mapping.

Definition١[١٠] Let Q be a set and $q \geq ١$ be a real no. A mapping $d: Q \times Q \rightarrow [٠, \infty)$ then(Q, d) is said to be a(b-M.SP) and q is called the coefficient of (Q, d) if $\forall e, f, g \in Q$then following properties satisfied:

$(d١١)$ $e = f \; iff \; d(e, f) = ٠$

$(d٢٢)$ $d(e, f) = d(f, e)$

$(d٣٣)$ $d(e, f) \leq q[d(e, g) + d(g, f)]$

Definition٢[٤] Let $Q \neq \emptyset$ A map $p: Q \times Q \rightarrow [٠, \infty)$ is a(P.M.SP) then (Q, p) is called (P.M.SP) if $\forall e, f, g \in Q$ if the following terms are satisfying:

$(p١١)$ $p(e, e) = p(e, f) = p(f, f) \; iff \; e = f$

$(p٢٢)$ $p(e, e) \leq p(e, f)$

$(p٣٣)$ $p(e, f) = p(f, e)$

$(p٤٤)$ $p(e, f) \leq p(e, g) + p(g, f) - p(g, g)$

Remark٣ it's obvious that (P.M.SP) mayn't be a (M.SP), since in a (b-M.SP) if $e = f$ then $d(e, e) = d(e, f) = d(f, f) = zero$. but in (P.M.SP) if $e = f$ then $p(e, e) = p(e, f) = p(f, f)$ doesn't need equalize zero therefore (P.M.SP) doesn't need to be (b-M.SP).

At other hand,[٥] admit the concept of (P.b-M.SP) as:

Definition٤[٥] Let $Q \neq \emptyset$ and $q \geq ١. P_b: Q \times Q \rightarrow [٠, \infty)$is called a (P.b-M.SP) if $\forall e, f, g \in Q$ then

$$(P_b١١)\; P_b(e, e) = P_b(e, f) = P_b(f, f) \; iff \; e = f$$

$$(P_b٢٢)\; P_b(e, e) \leq P_b(e, f)$$

$$(P_b٣٣)\; P_b(e, f) = P_b(f, e)$$

$$(P_b٤٤)\; P_b(e, f) \leq q[P_b(e, g) + P_b(g, f)] - P_b(g, g)$$

then $(Q, P_b)$is a (P.b-M.SP).$q \geq ١$ the coefficient of (Q, P_b).

Remark ٥ the order of (P.b-M.SP)(Q, P_b) is certainly greater than the class of (P.M.SP) , Since a (P.M.SP) is a particular kind of a (P.b-M.SP) $(Q, P_b)$when $q = ١$ also the class of (P.b-M.SP)(Q, P_b) is greater than the order of (b-M.SP) since (b-M.SP) is a particular kind of a (P.b-M.SP)(Q, P_b)while the self-distance $p(e, e) = zero$.

The following example explain that a (P.b-M.SP) on Q need not be a (P.M.SP), nor a (b-M.SP) on Q.

Example ٦[٥] Let $Q = [٠,١)$ define amap $P_b: Q \times Q \rightarrow [٠, \infty)$where

$$P_b(e, f) = [max\{e, f\}]^٢ + |e - f|^٢, \forall e, f \in Q$$

then (Q, P_b) is a(P.b-MS) on Q with coefficient $q = ٢ > ١$ but $P_b$isn't a(b-MS) nor a(P.M.S) on Q.

Definition٧[٩] every partial b-metric P_b defines a b - metric d_{P_b} , where $d_{P_b}(e, f) = ٢P_b(e, f) - P_b(e, e) - P_b(f, f), \forall e, f \in Q$.

Definition ٨[٩] A sequence $\{e_n\}$in a(P.b-M.SP) (Q, P_b) is called:
a) P_b -convergent to a point $e \in Q$ if $\lim_{n\to\infty} P_b\,(e, e_n) = P_b\,(e,e)$
b) A P_b-Cauchy seq.(C.Seq.) if $\lim_{n,m\to\infty} P_b\,(e_n, e_m)$ defined and limited;
c) A (P.b-M.SP) (M, P_b) is called P_b-complete if any P_b - (C.Seq.) $\{e_n\}$ in Q is P_b Approaches to A Point $e \in Q$ where $\lim_{n,m\to\infty} P_b\,(e_n, e_m) = \lim_{n\to\infty} P_b\,(e_n, e) = P_b\,(e,e)$

Lemma ٩[٩] a seq. $\{e_n\}$ is a P_b - (C. Seq.) in a (P.b-M.SP) (Q, P_b) iff b_(C. Seq.) in the (b-M.SP) (Q, d_{P_b}).

Lemma ١٠[٩] a (P.b-M.SP) (Q,P_b) is P_b - Complete iff (b - M.SP) (Q, d_{P_b}) is b - complete. moreover, $\lim_{n,m\to\infty} d_{P_b}(e_n, e_m) =$ ٠ iff $\lim_{n,m\to\infty} P_b\,(e_m, e) = \lim_{n\to\infty} P_b(e_n, e) = P_b(e,e)$.

Definition١١[١١] The twin of a self-maps A and S of a(M.SP.) (Q,d) are said to be weakened compatible if they subrogate at fortuity points that is if $Ae = Se \Longrightarrow ASe = SAe$ for $e \in Q$.

٢. Main Results

Theorem ١ suppose (Q, P_b) be a a (P_b-M.SP) with coefficient $q \geq 1$. Let $T, A : Q \to Q$ be a mapping satisfies the following

$$\alpha\left(q^2 \cdot P_b\,(Ae, Af)\right) \leq \alpha\left(Q_{P_b}\,(e,f)\right) - \beta\left(Q_{P_b}\,(e,f)\right) \quad (٢,١)$$

$\forall\ e,f \in Q$, $\alpha : [0,\infty) \to [0,\infty)$ be continuous and monotonically increasing function with $\alpha(t) = 0$ iff t = ٠,l $\beta : [0,\infty) \to [0,\infty)$ be lower semi-continuous with $\beta(t) = 0$ iff t = ٠ also where

$$Q_{P_b}\,(e,f) = \max\left\{\frac{P_b\,(Te, Ae) \cdot P_b\,(Te, Af)}{1 + P_b\,(Te, Tf)}, P_b\,(Te, Tf)\right\}$$

$A(Q) \leq T(Q)$ and $T(Q)$ is complete subspace of Q (٢,٢)

(T, A) weakly compatible pair. (٢,٣)

Then A and T have unique common fixed point in Q.

Proof: Choose $e_0 \in Q$. From (٢.٢), there is seq. $\langle e_n \rangle$ in Q. Where $Ae_n = Te_{n+1}$,$\forall\ n =$ ٠,١, ...

Case ((a)) Let $Te_{n+1} \neq Te_n$, $\forall\ n$, From (٢,١) we obtain

$$\alpha\left(q^2 P_b\,(Ae_1, Ae_2)\right) \leq \alpha\left(Q_{P_b}\,(e_1,e_2)\right) - \beta\left(Q_{P_b}\,(e_1,e_2)\right)$$

Where

$$Q_{P_b}\,(e_1,e_2) = \max\left\{\frac{P_b\,(Te_1, Ae_1) \cdot P_b\,(Te_2, Ae_2)}{1 + P_b\,(Te_1, Te_2)}, P_b\,(Te_1, Te_2)\right\}$$

$$= \max\left\{\frac{P_b\left(Te_1,Te_1\right)\cdot P_b\left(Te_2,Ae_3\right)}{1+P_b\left(Te_1,Te_2\right)}, P_b\left(Te_1,Te_2\right)\right\}$$

If $\frac{P_b\left(Te_1,Te_2\right)\cdot P_b\left(Te_2,Te_3\right)}{1+P_b\left(Te_2,Tf_2\right)}$ is maximum, then

$$\alpha\left(q^2\cdot P_b\left(Te_2,Te_3\right)\right)\le\alpha\left(\frac{P_b\left(Te_1,Te_2\right)\cdot P_b\left(Te_2,Te_3\right)}{1+P_b\left(Te_1,Te_2\right)}\right)-\beta\left(\frac{P_b\left(Te_1,Te_2\right)\cdot P_b\left(Te_2,Te_3\right)}{1+P_b\left(Te_1,Te_2\right)}\right)$$

$$<\alpha\left(\frac{P_b\left(Te_1,Te_2\right)\cdot P_b\left(Te_2,Te_3\right)}{1+P_b\left(Te_1,Te_2\right)}\right)$$

Since α is monotonically increasing, we have that

$$q^2\cdot P_b\left(Te_2,Te_3\right)\le\frac{P_b\left(Te_1,Te_2\right)\cdot P_b\left(Te_2,Te_3\right)}{1+P_b\left(Te_1,Te_2\right)}$$

It follows that

$$q^2\left[1+P_b\left(Te_1,Te_2\right)\right]\le P_b\left(Te_1,Te_2\right),$$

which is contradiction.

Hence $P_{\mathbf{b}}\left(Te_1,Te_2\right)$ is maximum.

$$\alpha\left[q^2\cdot P_b\left(Te_2,Te_3\right)\right]\le\left[P_b\left(Te_1,Te_2\right)\right]-\beta\left[P_b\left(Te_1,Te_2\right)\right]$$

$$\le\alpha\left(P_b\left(Te_1,Te_2\right)\right)$$

Since α is monotonically increasing, we have that

$$q^2\cdot P_b\left(Te_2,Te_3\right)\le P_b\left(Te_1,Te_3\right)\le\frac{1}{q^2}P_b\left(Te_0,Te_1\right)$$

Similarly we can also prove that

$$q^2\cdot P_b\left(Te_3,Te_4\right)\le P_b\left(Te_2,Te_3\right)$$

$$\le\frac{1}{q^4}P_b\left(Te_0,Te_1\right)$$

Continuing this way we can conclude that

$$\alpha\left(q^2\cdot P_b\left(Te_n,Te_{n+1}\right)\right)\le\alpha\left(P_b\left(Te_{n-1},Te_n\right)\right)-\beta\left(P_b\left(Te_{n-1},Te_n\right)\right)\quad \text{…..(١)}$$

Also it is clear that $\left\{P_b\left(Te_n,Te_{n+1}\right)\right\}$ is decreasing sequence of positive real nos. which approaches to $t\ge 0$.

let $t>0$

assuming n approaches to infinity in (١),we have that

$$\alpha(t)\le\alpha\left(q^2\cdot t\right)\le\alpha(t)-\beta(t)<\alpha(t),$$

which is contradiction.

therefore t = ٠.

Thus $\lim_{n\to\infty}P_{\mathbf{b}}\left(Te_n,Te_{n+1}\right)=0$ …..(٢)

From $(Pb2)$, $\lim_{n\to\infty} P_b(Te_n, Te_n) = 0$ … (٣)

From (٢) and (٣) and by the definition of d_{P_b}, we have that

$$\lim_{n\to\infty} d_{P_b}(Te_n, Te_{n+1}) = 0 \text{ …….. (٤)}$$

We will prove now that $\{Te_n\}$ is (C. seq.) in (Q, P_b). Alternatively, we can prove that

$\{Te_n\}$ is (C. seq.) in (Q, d_{P_b}). On contrary suppose that $\{Te_n\}$ is not (C. seq.) in (Q, P_b). This implies that $d_{P_b}(Te_n, Te_m) \not\to 0$ as $n, m \to \infty$.

Then there exist $\in > 0$ and monotonically increasing sequence of natural numbers $\{m_k\}$ and

$\{n_k\}$ such that $n_k > m_k > k$

$$d_{P_b}\left(Te_{n_k}, Te_{m_k}\right) \geq \in \text{ ….(٥)}$$

and $$d_{P_b}\left(Te_{n_k-1}, Te_{m_k}\right) < \in \text{ …(٦)}$$

From (٥) and (٦), we have that

$$\begin{aligned}\in &\leq d_{P_b}\left(Te_{n_k}, Te_{m_k}\right)\\ &\leq q\left[d_{P_b}\left(Te_{m_k}, Te_{n_k-1}\right) + d_{P_b}\left(Te_{n_k-1}, Te_{n_k}\right)\right]\\ &< q\cdot \in + q\cdot d_{P_b}\left(Te_{n_k-1}, Te_{n_k}\right)\end{aligned}$$

Taking the upper limit as $k \to \infty$ and using (٤), we have that

$$\in \leq \lim_{k\to\infty}\sup d_{P_b}\left(Te_{n_k}, Te_{m_k}\right) \leq q \in \text{ ….. (٧)}$$

Also

$$\begin{aligned}\in &\leq d_{P_b}\left(Te_{m_k}, Te_{n_k}\right)\\ &\leq q\cdot d_{P_b}\left(Te_{m_k}, Te_{n_k+1}\right) + q\cdot d_{P_b}\left(Te_{n_k+1}, Te_{n_k}\right)\end{aligned}$$

Taking upper unit as $k \to \infty$ and using (٤) , we have that

$$\frac{\in}{q} \leq \lim_{k\to\infty} sup\ d_{P_b}\left(Te_{m_k}, Te_{n_k+1}\right) \text{ …..(٨)}$$

On other hand

$$d_{P_b}\left(Te_{m_k}, Te_{n_k+1}\right) \leq q\cdot d_{P_b}\left(Te_{m_k}, Te_{n_k}\right) + q.d_{P_b}\left(Te_{n_k}, Te_{n_k+1}\right)$$

Taking upper limit as $k \to \infty$, using (٤) we have that

$$\lim_{k\to\infty}\sup d_{P_b}\left(Te_{m_k}, Te_{n_k+1}\right) < \in \cdot q^2 \text{ ….. (٩)}$$

Also from (٥), we have that

$$\begin{aligned}\in &\leq d_{P_b}\left(Te_{m_k}, Te_{n_k}\right)\\ &\leq q\left[d_{P_b}\left(Te_{m_k}, Te_{m_k+1}\right) + d_{P_b}\left(Te_{m_k+1}, Te_{n_k}\right)\right]\end{aligned}$$

$$\leq q\cdot d_{P_b}\left(Te_{m_k},Te_{m_k+1}\right)+q^2\cdot d_{P_b}\left(Te_{m_k+1},Te_{n_k+2}\right)+q^2\cdot d_{P_b}\left(Te_{n_k+2},Te_{n_k}\right)$$

$$\leq q\cdot d_{P_b}\left(Te_{m_k},Te_{m_k+1}\right)+q^2\cdot d_{P_b}\left(Te_{m_k+1},Te_{n_k+2}\right)+q^3\cdot d_{P_b}\left(Te_{n_k+2},Te_{n_k}\right)+q^2\cdot d_{P_b}\left(Te_{n_k+2},Te_{n_k}\right)$$

Taking upper limit as $k\to\infty$ and using (٤), we have that

$$\frac{\in}{q^2}\leq\limsup_{k\to\infty} d_{P_b}\left(Te_{m_k+1},Te_{n_k+2}\right)\quad \ldots.\ (١٠)$$

On other hand

$$d_{P_b}\left(Te_{m_k+1},Te_{n_k+2}\right)\leq q\left[d_{P_b}\left(Te_{m_k+1},Te_{m_k}\right)+d_{P_b}\left(Te_{m_k},Te_{n_k+2}\right)\right]$$

$$\leq q\ d_{P_b}\left(Te_{m_k+1},Te_{m_k}\right)+q^2\left[d_{P_b}\left(Te_{m_k},Te_{n_k+1}\right)\right]$$

$$+q\ d_{P_b}\left(Te_{n_k+1},Te_{n_k+2}\right)$$

Taking upper limit as $k\to\infty$ and using (٤), we have

$$\limsup_{k\to\infty} d_{P_b}\left(Te_{m_k+1},Te_{n_k+2}\right)\leq q^4\cdot\in\qquad \ldots.\ (١١)$$

Now

$$\alpha\left(q^2\cdot P_b\left(Te_{m_k+1},Te_{n_k+2}\right)\right)=\alpha\left(q^2\cdot P_b\left(Ae_{m_k},Ae_{n_k+1}\right)\right)$$

$$\leq\alpha\left(Q_{P_b}\left(e_{m_k},e_{n_k+1}\right)\right)-\beta\left(Q_{P_b}\left(e_{m_k},e_{n_k+1}\right)\right)\quad \ldots(١٢)$$

where

$$Q_{P_b}\left(e_{m_k},e_{n_k+1}\right)=\max\left\{\frac{P_b\left(Te_{m_k},Ae_{m_k}\right)\cdot P_b\left(Te_{n_k+1},Ae_{n_k+1}\right)}{1+P_b\left(Te_{m_k},Te_{n_k+1}\right)},P_b\left(Te_{m_k},Ae_{n_k+1}\right)\right\}$$

$$=\max\left\{\frac{P_b\left(Te_{m_k},Te_{m_k+1}\right)\cdot P_b\left(Te_{n_k+1},Te_{n_k+2}\right)}{1+P_b\left(Te_{m_k},Te_{n_k+1}\right)},P_b\left(Te_{m_k},Te_{n_k+1}\right)\right\}$$

Taking $k\to\infty$ (٩) and (٢), we have that

$$\limsup_{k\to\infty} Q_{P_b}\left(e_{m_k},e_{n_k+1}\right)\leq\max\left\{0,q^2\cdot\in\right\}=q^2\cdot\in$$

Taking upper limit as $k\to\infty$ in (١٢) and from (١١), we have that

$$\alpha\left(q^6\cdot\in\right)\leq\alpha\left(q^2\cdot\in\right)-\beta\left(\limsup_{k\to\infty} Q_{P_b}\left(e_{m_k},e_{n_k+1}\right)\right)\quad \ldots(١٣)$$

Subcase (i) if $q=$ ١,from (١٣),we obtain

$$\alpha(\in)\leq\alpha(\in)-\beta\left(\limsup_{k\to\infty} Q_{P_b}\left(e_{m_k},e_{n_k+1}\right)\right)$$

$$<\alpha(\in)$$

which is a contradiction

Subcase (ii) if $q>$ ١, from (١٣), we see

$$\alpha\left(q^6\in\right)\leq\alpha\left(q^2\in\right)-\beta\left(\limsup_{k\to\infty} Q_{P_b}\left(e_{m_k},e_{n_k+1}\right)\right)$$

$$<\alpha\left(q^6\in\right)$$

Since α is monotonically increasing, we have that

$$q^6 \in \leq q^2 \in$$

It follows that $q^4 \leq 1$

which is a contradiction.

Hence we conclude that $\{Te_n\}$ is (C. seq.) in $\left(Q, \mathrm{d}_{P_b}\right)$.

let $T(Q)$ is complete subspace of Q then $\{Te_n\}$ converges to e in

$$\left(T(e), \mathrm{d}_{P_b}\right)$$

Thus $\mathrm{d}_{P_b}(Te_n, u) = 0$ for some u = TV. From Lemma ١٠ we obtain

$$\lim_{n,m\to\infty} P_b(Te_n, Te_m) = P_b(Te_n, u) = P_b(Te_m, u) = p_b(u,u) = 0 \quad .. (١٤)$$

Now we claim that AV =u

From (٢،١), we have that

$$\alpha\left(q^2 \cdot P_b(Av, Ae_n)\right) \leq \alpha\left(Q_{P_b}(v, e_n)\right) \ldots (١٥)$$

Where

$$\mathrm{Q}_{P_b}(v, e_n) = \max\left\{\frac{P_b(Tv, Av)\cdot P_b(Te_n, Ae_n)}{1+P_b(Tv, Te_n)}, P_b(Tv, Te_n)\right\}$$

$$= \max\left\{\frac{P_b(u, Av)\cdot P_b(Te_n, Te_{n+1})}{1+P_b(u, Te_n)}, P_b(u, Te_n)\right\}$$

$$\to ٠ \text{ as } n\to\infty$$

Letting $n\to\infty$ and

From (١٥), we have that

$$\alpha\left(q^2 \cdot P_b(Av, u)\right) \leq 0$$

It follows that $Av = u = Tv$

Since (A, T) is weakly compatible pair, we have that Au = Tu

Suppose $Au \neq u$

Now

$$\alpha\left(q^2 \cdot P_b(Au, Ae_n)\right) \leq \left(\mathrm{Q}_{P_b}(u, e_n)\right) - \beta\left(Q_{P_b}(u, e_n)\right) \ \ldots (١٦)$$

Where

$$\mathrm{Q}_{P_b}(u, e_n) = \max\left\{\frac{P_b(Tu, Au)\cdot P_b(Te_n, Ae_n)}{1+P_b(Tu, Te_n)}, P_b(Tu, Te_n)\right\}$$

$$= \max\left\{\frac{P_b(Au, Au)\cdot P_b(Te_n, Te_{n+1})}{1+P_b(Au, Te_n)}, P_b(Au, Te_n)\right\}$$

$$\to P_b(Au, u) \text{ as } n\to\infty.$$

Hence letting $n\to\infty$ in (١٦), we have that

$$\alpha\left(q^2 \cdot P_b(Au, u)\right) \leq \alpha\left(P_b(Au, u)\right) - \beta\left(P_b(Au, u)\right) \ldots..(١٧)$$

Subcase (iii) if $q = ١$, from (١٧) we see

$$\alpha\left(P_b\left(Au,u\right)\right) \le \alpha\left(P_b\left(Au,u\right)\right) - \beta\left(P_b\left(Au,u\right)\right)$$
$$< \alpha\left(P_b\left(Au,u\right)\right)$$

Which is a contradiction

Subcase(iv) if $\boldsymbol{q > 1}$, from (١٧), we have

$$\alpha\left(q^2 \cdot P_b\left(Au,u\right)\right) \le \alpha\left(P_b\left(Au,u\right)\right) - \beta\left(P_b\left(Au,u\right)\right)$$
$$< \alpha\left(P_b\left(Au,u\right)\right)$$

Since α is monotonically increasing function, we have that
$$q^2 \cdot P_b\left(Au,u\right) \le P_b\left(Au,u\right)$$

It follows that $q^2 \le 1$

Which is a contradiction.

Hence $Au = u = Tu$

hence u is com. fixed pt. of A and T.

For uniqueness let g is another com. fixed pt. of A and T provided $u \ne g$ then

$$\alpha\left(q^2 P_b\left(u,g\right)\right) = \alpha\left(q^2 P_b\left(Au,Ag\right)\right)$$
$$\le \alpha\left(Q_{P_b}\left(u,g\right)\right) - \beta\left(Q_{P_b}\left(u,g\right)\right) \ldots\ldots..(١٨)$$

Where

$$Q_{P_b}\left(u,g\right) = \max\left\{\frac{P_b\left(Tu,Au\right)\cdot P_b\left(Tg,Ag\right)}{1+P_b\left(Tu,Tg\right)}, P_b\left(Tu,Tg\right)\right\}$$
$$= \max\left\{\frac{P_b\left(u,u\right)\cdot P_b\left(g,g\right)}{1+P_b\left(u,g\right)}, P_b\left(u,g\right)\right\}$$
$$= \max\left\{0, P_b\left(u,g\right)\right\}$$
$$= P_b\left(u,g\right)$$

Therefore, from (١٨), we have that

$$\alpha\left(q^2 \cdot P_b\left(u,g\right)\right) \le \alpha\left(P_b\left(u,g\right)\right) - \beta\left(P_b\left(u,g\right)\right)$$
$$< \alpha\left(P_b\left(u,g\right)\right)$$

Which is contradiction

since $u = g$

hence, u a unique fixed pt. of A and T.

Example ٢. if $Q = [٠,١)$and $P_b : QQQ \to [0,\infty)$defined as $P_b\left(e,f\right) = \left(\max\left\{e,f\right\}\right)^2 \ \forall e,f \in Q$. Then it's obviously that $\left(Q,P_b\right)$ is a complete (P.b-MS) with $q = ٢$.

define $A,T:Q\to Q$ by $T(Q)=\frac{e}{2}$, $A(Q)=\frac{e^2}{e+1}$ and $\alpha,\beta:[0,\infty)\to[0,\infty)$ by $\alpha(t)=t$ and

$\beta(t)=\frac{t}{2}$. Then all terms of theorem (٢،١) are holding and zero is unique fixed pt. of A and T.

Result ٣ in theorem (٢،١), if we take $\alpha(t)=\beta(t)=t$ and (P.M.SP) (Q,P_b) is exchanged by (M.SP) (Q,d). Then we will see main result of [٥].

Result ٤ if we put $p(e,f)=$ ٠ in (٢،١) and let $p:Q\times Q\to[$٠$,\infty)$ be a mapping, provided $p(e,f)=$ ٠ iff $e=f=$ ٠ then theorem (٢،١) reduces to theorem (٢،١) in [١٢].

References:

[١] I. A. Bakhtin, The contraction principle in quasi metric spaces, It. Funct. Anal., ٣٠ (١٩٨٩), ٢٦-٣٧.

[٢] S. Czerwik, Nonlinear set-valued contraction mappings in b-metric spaces, Atti Sem. Mat. Fis. Univ. Modena. ٤٦ (١٩٩٨), ٢٦٣ - ٢٧٦.

[٣] S. Czerwik, Contraction mappings in b-metric spaces, Acta Math. Inform. Univ. Osrav., ١ (١٩٩٣), ٥-١١.

[٤] S. G. Matthews, Partal metric topology Proc. ٨th Summer Conference on General Topology and Applications, Ann. N.Y. Acad. Sci., ٧٢٨ (١٩٩٤), ١٨٣-١٩٧.

[٥] S. Shukla, Partial b-metric spaces and fixed point theorems, Mediterranean Journal of Mathematics, doi:١٠١٠٠٧/s٠٠٠٠٩-٠١٣-٠٣٢٧-٤, (٢٠١٣).

[٦] T. Abdeljawad, Meir-Keeler α -contractive fixed and common fixed point theorems, Fixed Point Theory and Applications, ٢٠١٣, ٢٠١٣:١٩, ١٠ pp.

[٧] E. Karapinar, R. P. Agarwal, A note on Coupled fixed point theorems for α - ψ - contractive-type mappings in partially ordered metric spaces, Fixed Point Theory and Applications, ٢٠١٣, ٢٠١٣:٢١٦, ١٦ pp.

[٨] J. Asl, S. Rezapour, N. Shahzad, On fixed points of α - ψ - contractive multi functions, Fixed Point Theory and Applications, ٢٠١٢, ٢٠١٢:٢١٢, ٦ pp.

[٩] Z. Mustafa, J. R. Roshan, V. Parvaneh, Z. Kadelburg, Some common fixed point result in ordered partial b-metric spaces, Journal of Inequalities and Applications, (٢٠١٣), ٢٠١٣:٥٦٢.

[١٠] H. Aydi, M. Bota, E. Karapinar, S. Mitrovic, A fixed point theorem for set valued quasi-contractions in b-metric spaces, Fixed Point Theory and its Applications, ٢٠١٢, ٢٠١٢:٨٨, ٨ pp.

[١١] Jha, K., R.P. Pant and S.L. Singh, On the existence of common fixed points for compatible mappings, Punjab Univ. J. Math., ٣٧ (٢٠٠٥), ٣٩ – ٤٨.

[١٢] C. Chen and C. Zhu, Fixed point theorems for weakly C-contractive mappings in partial metric spaces, Fixed Point Theory and Applications, vol. ٢٠١٣, article ١٠٧, ٢٠١٣.

حول التطبيق الانكماشي $(\alpha - \beta)$ للفضاءات المترية الجزئية من النوع b ومبرهنة النقطة الثابتة

م. مؤيد محمود خليل

قسم الرياضيات / كلية التربية للعلوم الصرفة / جامعة تكريت

مستخلص البحث

تم في هذا البحث برهان مبرهنة النقطة الثابتة الوحيدة للتطبيق الانكماشي $(\alpha - \beta)$ واعطينا مثالاً يدعم ويوضح النتيجة الرئيسية للبحث كما استنتجنا بعض النتائج المفيدة.