Partial Differential Equations

Ahlam Kalef Jabbar

Abstract

Differential equations are used in the study of higher-rank partials with variable coefficients in different areas of the Cartesian coordinate plane, while some scientists and researchers have: N. Rajabov, A. S. Star & F, A Nasim Adeeb Haneen, and others. Accordingly, the partial differential equation, which will be taken from its rank (fourth rank), was studied in this research while its coefficients differ from those of partial differential equations. In the context of that equation, conditions are set for their coefficients to produce a single solution for that partial differential equation in several different cases related to these coefficients. These conditions were summarized in five theories.

Key Word: **Partial**, **Differential**

Introduction:

Partial differential equations of higher rank represent one of the important branches of the partial differential equations theory. There are many applied cases ending with the study of partial differential equations, especially the subject of fluid mechanics and elasticity theory. These equations include possible results that may be the basis of the theory of partial differential equations of the increasing and decreasing anomalies obtained in the work of a number of scientists and researchers.

While the study of partial differential equations results in access to integral Volterra equations of type II, the solution of these integral equations is based on four optional successives of a single transformer.

In our division of the research, we have included the basic methods for reaching the solutions of partial differential equations and their transformation into a differential equation in terms of the performance of four partial differential effects of the first rank, which makes it easier for us to solve the partial differential equation studied. In addition, via which solving the integral equations resulting from solving the partial differential equation using successive approximation method [6] and obtaining integral formulas, and then clarifying the conditions and placing them in integral formulas and their inclusion in the theories of existence and singularity solving of partial differential equation.

Chapter One
Basic methods of partial differential equations

In the beginning we must give a definition of differential equations. The name of differential equations may be called on equations containing derivatives and differentials of some mathematical functions. The equations are designed to find these mathematical functions whose derivatives achieve these equations. Differential equations Significant in chemical, physical and even mathematical applications related to biological, social and economic processes.

The basic methods of partial differential equations include the divisible rows according to the symbols that we will substitute the continuous successive of rows. In area D identified by the relationship..

$$D=\{0<x<\delta_1, 0<y<\delta_2\}$$

Where as:

$$\Delta_1=\{0<x<\delta_1\ , y=0\} \quad , \quad \Delta_2=\{0<y<\delta_2\ , x=0\}$$

We would study the following partial integral equation:

$$L_{a_1,b_1}^{c_1}\left(L_{a_2,b_2}^{c_2}V\right)=\frac{f(x,y)}{r^{\alpha+\beta}} \qquad (1)$$

where as:

$$L_{a_j,b_j}^{c_j}\equiv\frac{\partial^2}{\partial x\partial y}+y\frac{a_j(x,y)}{r^{\alpha}}\frac{\partial}{\partial x}+x\frac{b_j(x,y)}{r^{\beta}}\frac{\partial}{\partial y}+\frac{c_j(x,y)}{r^{\alpha+\beta}},\ j=1,2$$

$$r=\sqrt{x^2+y^2}$$

.true facts α,β

Then we use the following symbols $C(\overline{D}), C_x^1(\overline{D}), C_{xy}^2(\overline{D}), C_{xxy}^3(\overline{D})$

Where as (D) is the continuous successive within area D

Is row of partial derivatives continuous successive of the second $C_x^1(\overline{D})$ rank for the y and x transformers In area D

Is row of partial derivatives continuous successive of the third rank $C_x^1(\overline{D})$ for the y transformers In area D

Assume:

$$L_{a_2,b_2}^{c_2}V=U \qquad (2)$$

$$L_{a_1,b_1}^{c_1}U=\frac{f(x,y)}{r^{\alpha+\beta}} \qquad (3)$$

Also assume that:

$$a_1(x,y)\in C_x^1(\overline{D}), b_1(x,y), c_1(x,y), f(x,y)\in C(\overline{D})$$

Then equation (3) takes the following form:

$$\left(\frac{\partial}{\partial x}+x\frac{b_1(x,y)}{r^{\beta}}\right)\left(\frac{\partial}{\partial y}+y\frac{a_1(x,y)}{r^{\alpha}}\right)U=\frac{f(x,y)+[xya_1(x,y)b_1(x,y)-c_1(x,y)]U}{r^{\alpha+\beta}}+$$

$$U\frac{\partial}{\partial x}\left(y\frac{a_1(x,y)}{r^{\alpha}}\right) \qquad (4)$$

We assume that:

$$c_1^0(x,y)=x.ya_1(x,y)b_1(x,y)-c_1(x,y)+r^{\alpha+\beta}\frac{\partial}{\partial x}\left(y\frac{a_1(x,y)}{r^{\alpha}}\right)$$

Then the equation be as follows:

$$\left(\frac{\partial}{\partial x}+x\frac{b_1(x,y)}{r^{\beta}}\right)\left(\frac{\partial}{\partial y}+y\frac{a_1(x,y)}{r^{\alpha}}\right)U=\frac{f(x,y)+c_1{}^0(x,y)U}{r^{\alpha+\beta}}\equiv f_1(x,y) \qquad (5)$$

On the other side if we assume that:

$$a_2(x,y)\in C^3_{xxy}\left(\overline{D}\right),\{b_2(x,y),c_2(x,y)\}\in C\left(\overline{D}\right)$$

We find writing(2) equation in this way:

$$\left(\frac{\partial}{\partial x}+x\frac{b_2(x,y)}{r^{\beta}}\right)\left(\frac{\partial}{\partial y}+y\frac{a_2(x,y)}{r^{\alpha}}\right)V=c^{(2)}(x,y)V+U \qquad (*)$$

where

$$c^{(2)}(x,y)=\frac{c_2^0(x,y)}{r^{\alpha+\beta}}$$

$$c_2{}^0(x,y)=x.y.a_2(x,y)b_2(x,y)-c_2(x,y)+r^{\alpha+\beta}\frac{\partial}{\partial x}\left(y\frac{a_2(x,y)}{r^{\alpha}}\right)$$

The equation(*) takes the following formula

$$\left(\frac{\partial}{\partial x}+x\frac{b_2(x,y)}{r^{\beta}}\right)\left(\frac{\partial}{\partial y}+y\frac{a_2(x,y)}{r^{\alpha}}\right)V=\frac{c_2{}^0(x,y)}{r^{\alpha+\beta}}V+U \qquad (6)$$

To find a solution for(5) equation we assume that:

$$\frac{\partial U}{\partial y}+y\frac{a_1(x,y)}{r^{\alpha}}U=U_1 \qquad (7)$$

We substitute in (5) and write the resulting equation after substitution in the following way:

$$\frac{\partial U_1}{\partial x}+x\frac{b_1(x,y)-b_1(0,0)+b_1(0,0)}{r^{\beta}}U_1=f_1(x,y) \qquad (8)$$

The result of this first rank linear integral equation , its general solution from the formula,.

$$U_1 = e^{-\Omega_{b_1}^{\beta}(x,y)+W_{b_1}^{\alpha}(x,y)}$$

$$\left[\psi_1(y)+\int_0^x e^{\Omega_{b_1}^{\beta}(t,y)-W_{b_1}^{\beta}(t,y)}\left(t^2+y^2\right)^{-\left(\frac{\alpha+\beta}{2}\right)}\left(f(t,y)+c_1^0(t,y)U(t,y)dt\right)\right] \qquad (9)$$

In the same way we find solution of the equation (7)

$$U(x,y)=e^{-\Omega_{a_1}^{\alpha}(x,y)+W_{a_1}^{\alpha}(x,y)}\left[\varphi_1(x)+\int_0^y e^{\Omega_{a_1}^{\alpha}(x,s)-W_{a_1}^{\alpha}(x,s)}U_1(x,s)ds\right] \qquad (10)$$

Where as

$$,\Omega_{b_1}^{\beta}(x,y)=\int_0^x \frac{b_1(t,y)-b_1(0,0)}{\left(t^2+y^2\right)^{\frac{\beta}{2}}}tdt \quad , \quad \Omega_{a_1}^{\alpha}(x,y)=\int_0^y \frac{a_1(t,y)-a_1(0,0)}{\left(x^2+s^2\right)^{\frac{\alpha}{2}}}sds$$

$$W_{a_1}^{\alpha}(x,y)=a_1(0,0)(\alpha-2)^{-1}r^{2-\alpha} \quad , \quad W_{b_1}^{\beta}(x,y)=b_1(0,0)(\beta-2)^{-1}r^{2-\beta}$$

We substitute(10) for (9) we find

$$U(x,y)-\int_0^y ds_1\int_0^x e^{\Omega_{a_1}^{\alpha}(x,s)-W_{a_1}^{\alpha}(x,s_1)-\Omega_{b_1}^{\beta}(x,s_1)+W_{b_1}^{\beta}(x,s_1)+\Omega_{b_1}^{\beta}(t_1,s_1)-W_{b_1}^{\beta}(t_1,s_1)}$$

$$\left(t_1^2+s_1^2\right)^{-\left(\frac{\alpha+\beta}{2}\right)}c_1^0(t_1,s_1)U(t_1,s_1)dt_1 =$$

$$e^{W_{a_1}^{\alpha}(x,y)-\Omega_{a_1}^{\alpha}(x,y)}\left[\varphi_1(x)+\int_0^y e^{\Omega_{a_1}^{\alpha}(x,s_1)-W_{a_1}^{\alpha}(x,s_1)-\Omega_{b_1}^{\beta}(x,s_1)+W_{b_1}^{\beta}(x,s\ )}\psi_1(s_1)\right.$$

$$\left.+\int_0^x e^{\Omega_{b_1}^{\beta}(t_1,s_1)-W_{b_1}^{\beta}(t_1,s_1)}.\left(t_1^2+s_1^2\right)^{-\frac{(\alpha+\beta)}{2}}f(t_1,s_1)dt_1)ds_1\right] \qquad (11)$$

In the same way we find solution of the equation (6)

$$V(x,y)-\int_0^y ds_2\int e^{\Omega_{a_2}^{\alpha}(x,s_2)-W_{a_2}^{\alpha}(x,s_2)-\Omega_{b_2}^{\beta}(x,s_2)+W_{b_2}^{\beta}(x,s_2)+\Omega_{b_2}^{\beta}(t_2,s_2)-W_{b_2}^{\beta}}$$

$$\left(t_2^2+s_2^2\right)^{-\frac{(\alpha+\beta)}{2}}c_2^0(t_2,s_2)V(t_2,s_2)dt_2 =$$

$$e^{W_a^{\alpha}(x,y)-\Omega_{a_2}^{\alpha}(x,y)}\left[\varphi_2(x)+\int_0^y e^{\Omega_{a_2}^{\alpha}(x,s_2)-W_{a_2}^{\alpha}(x,s_2)-\Omega_{b_2}^{\beta}(x,s_2)+W_{b_2}^{\beta}(x,s_2)}\right.$$

$$\left(\psi_2(s_2)+\int_0^x e^{\Omega_{b_2}^{\beta}(t_2,s_2)-W_{b_2}^{\beta}(t_2,s_2)}\left(t_2^2+s_2^2\right)^{-\frac{(\alpha+\beta)}{2}}U(t_2,s_2)dt_2)ds_2\right] \qquad (12)$$

Chapter Two
Resulting cases of differential equations

In this chapter we will study the first and fourth cases, the first case is considered the easiest case, the fourth is the most difficult cases, and the second and third cases can be deduced from the fourth case
From (11) and (12) equations we could study four cases:

First case $c_1^0(x,y)=0 \ , \ c_2^0(x,y)=0$

Second case $c_1^0(x,y)\neq 0 \ , \ c_2^0(x,y)=0$

Third case $c_1^0(x,y)=0 \ , \ c_2^0(x,y)\neq 0$

Forth case $c_1^0(x,y)\neq 0 \ , \ c_2^0(x,y)\neq 0$

The first case, then from(11) and(12) we $c_1^0(x,y)=0, c_2^0(x,y)=0$ obtain the following equations

$$U(x,y)=e^{-\Omega_{a_1}^{\alpha}(x,y)+W_{a_1}^{\alpha}(x,y)}\left[\varphi_1(x)+\int_0^y e^{\Omega_{a_1}^{\alpha}(x,s_1)-W_{a_1}^{\alpha}(x,s_1)-\Omega_{b_1}^{\beta}(x,s_1)+W_{b_1}^{\beta}(x,s_1)}\right.$$
$$\left.\left(\psi_1(s_1)+\int_0^x e^{\Omega_{b_1}^{\beta}(t_1,s_1)-W_{b_1}^{\beta}(t_1,s_1)}.\left(t_1^2+s_1^2\right)^{-\frac{\alpha+\beta}{2}}.f(t_1,s_1)dt_1\right)ds_1\right] \tag{13}$$

$$V(x,y)=e^{-\Omega_{a_2}^{\alpha}(x,y)+W_{a_2}^{\varepsilon}(x,y)}\left[\varphi_2(x)+\int_0^y e^{\Omega_{a_2}^{\alpha}(x,s_2)-W_{a_2}^{\alpha}(x,s_2)-\Omega_{b_2}^{\beta}(x,s_2)+W_{b_2}^{\beta}(x,s_2)}\right.$$
$$\left.\left(\psi_2(s_2)+\int_0^x e^{\Omega_{b_2}^{\beta}(t_2,s_2)-W_{b_2}^{\beta}(t_2,s_2)}.\left(t_2^2+s_2^2\right)^{-\frac{(\alpha+\beta)}{2}}U(t_2,s_2)\,dt_2\right)ds_2\right] \tag{14}$$

We substitutes(14) for(13)

$$V(x,y)=e^{-\Omega_{a_2}^{\alpha}(x,y)+W_{a_2}^{\alpha}(x,y)}\left\{\varphi_2(x)+\int_0^y e^{\Omega_{a_2}^{\alpha}(x,s_2)-W_{a_2}^{\alpha}(x,s_2)-\Omega_{b_2}^{\beta}(x,s_2)+W_{a_2}^{\beta}(x,s_2)}\right.$$
$$\left[\psi_2(s_2)+\int_0^x e^{\Omega_{b_2}^{\beta}(t_2,s_2)-W_{b_2}^{\beta}(t_2,s_2)-\Omega_{a_2}^{\alpha}(t_2,s_2)+W_{a_1}^{\alpha}(t_2,s_2)}\left(t_2^2+s_2^2\right)^{\frac{(\alpha+\beta)}{2}}\left(\varphi_1(t_2)\right.\right.$$
$$+\int_0^{s_2} e^{\Omega_{a_1}^{\alpha}(t_2,s_1)-W_{a_1}^{\alpha}(t_2,s_1)-\Omega_{b_1}^{\beta}(t_2,s_1)+W_{b_1}^{\beta}(t_2,s_1)}$$
$$\left.\left.\left.\left\langle\psi_1(s_1)+\int_0^{t_2} e^{\Omega_{b_1}^{\beta}(t_1,s_1)-W_{b_1}^{\beta}(t_1,s_1)}\left(t_1^2+s_1^2\right)^{\frac{(\alpha+\beta)}{2}}f(t_1,s_1)dt_1\right\rangle ds_1\right)dt_2\right]ds_2\right\} \tag{15}$$

Whereas:

$$\Omega_{a_j}^{\alpha}(x,y)=\int_0^y \frac{a_j(x,s_j)-a_j(0,0)}{\left(x^2+s_j^2\right)^{\frac{\alpha}{2}}} s_j ds_j \ ,\Omega_{b_j}^{\beta}(x,y)=\int_0^x \frac{b_j(t_j,y)-b_j(0,0)}{\left(t_j^2+y^2\right)^{\frac{\beta}{2}}} t_j dt_j$$

$$W_{a_j}^{\alpha}(x,y)=a_j(0,0)(\alpha-2)^{-1}r^{2-\alpha}\ ,\ W_{b_1}^{\beta}(x,y)=b_j(0,0)(\beta-2)^{-1}r^{2-\beta},\ j=1,2$$

we conclude from what has mentioned that the following equation was proved

Theories: theory(1): assume that the differential equation coefficients(1) also its right side achieving the following conditions:

1) $a_2(x,y)\in C_{xxy}^3(\overline{D}),\ \{b_2(x,y),c_2(x,y)\}\in C(\overline{D})$

2) $a_1(x,y)\in C_x^1(\overline{D}),\ \{b_2(x,y),c_2(x,y)\}\in C(\overline{D})$

3) $\left|a_j(x,y)-a_j(0,0)\right|\le H_{a_j} r^{-\gamma_{j1}}\ ;\ 0<\gamma_{j1}<2-\alpha\ ,\ j=1,2$

4) $\left|b_j(x,y)-b_j(0,0)\right|\le H_{b_j} r^{-\gamma_{j2}}\ ;\ 0<\gamma_{j2}<2-\beta\ ,\ j=1,2$

5) $r^{-(\alpha+\beta)}.f(x,y)=0(r^{-\delta})\ ;\ 0<\delta<1\ ,\ for: r\to 0$

6) $c_j^0(x,y)=x.y\ a_j(x,y)b_j(x,y)-c_j(x,y)+r^{\alpha+\beta}\dfrac{\partial}{\partial x}\left(y\dfrac{a_j(x,y)}{r^{\alpha}}\right)$

$$j=1,2\quad,\quad \alpha<2\ ,\ \beta<2$$

there is one single solution for the differential equation(1) given by relation(15) where as:

optional successive achieve $\varphi_1(x),\varphi_2(x),\psi_1(y),\psi_2(y)$ the following

We will process the most complicated case which is:

Forth case $c_1^0(x,y)\neq 0\ ,\ c_2^0(x,y)\neq 0$

To take relation(11) as follows:

$$U(x,y)-\int_0^y ds_1\int_0^x e^{\Omega_{a_1}^{\alpha}(x,s_1)-\Omega_{b_1}^{\beta}(x,s_1)-W_{a_1}^{\alpha}(x,s_1)+W_{b_1}^{\beta}(x,s_1)+\Omega_{b_1}^{\beta}(t_1,s_1)-W_{b_1}^{\beta}(t_1,s_1)}.\left(t_1^2+s_1^2\right)^{-\frac{(\alpha+\beta)}{2}}.$$

$$c_1^0(t_1,s_1).U(t_1,s_1)dt_1=F_1[\varphi_1(x),\psi_1(y),f(x,y)] \qquad (16)$$

We assumed that the right side of the relation(11) equals to:

$$F_1[\varphi_1(x),\psi_1(y),\ f(x,y)]$$

We take relation (12) as follows:

$$V(x,y)-\int_0^y ds_2\int_0^x e^{\Omega_{a_2}^{\alpha}(x,s_2)-\Omega_{b_2}^{\beta}(x,s_2)-W_{a_2}^{\alpha}(x,s_2)+W_{b_2}^{\beta}(x,s_2)+\Omega_{b_2}^{\beta}(t_2,s_2)-W_{b_2}^{\beta}(t_2,s_2)}$$

$$\left(t_2^2+s_2^2\right)^{-\frac{(\alpha+\beta)}{2}}\ c_2^0(t_2,s_2)V(t_2,s_2)dt_2=F_2[\varphi_2(x),\psi_2(y)] \qquad (17)$$

We assume that the right side of (12) relation matching

$$F_2[\varphi_2(x),\psi_2(y)]$$

The integrated equation(16) is integrated volterra equation of second type ,to solve we use the successive approximation. First we assue it approximate to zero

$$e^{\Omega_{a_2}^{\alpha}(x,s_2)-\Omega_{b_2}^{\beta}(x,s_2)-W_{a_2}^{\alpha}(x,s_2)+W_{b_2}^{\beta}(x,s_2)+\Omega_{b_2}^{\beta}(t_2,s_2)-W_{b_2}^{\beta}(t_2,s_2)}c_1^0(x,y)=0\left(r^{\gamma_1}\right)$$

(3) Then the core of integral equation(16) achieves the following:-

$$I=\int_0^y ds_1\int_0^x \frac{\left(\sqrt{t_1^2+s_1^2}\right)^{\gamma_1}}{\left(\sqrt{t_1^2+s_1^2}\right)^{\alpha+\beta_1}}dt_1 \Rightarrow$$

$$I_1=\int_0^x \frac{\left(\sqrt{t_1^2+s_1^2}\right)^{\gamma_1}}{\left(\sqrt{t_1^2+s_1^2}\right)^{\alpha+\beta}}dt_1 \le \int_0^x \frac{(2t_1)^{\gamma_1}}{\left(\sqrt{t_1^2+s_1^2}\right)^{\alpha+\beta}}dt_1 \le 2^{\gamma_1}\int_0^x \frac{(t_1)^{\gamma_1-1}.t_1}{\left(\sqrt{t_1^2+s_1^2}\right)^{\alpha+\beta}}dt_1$$

$$I_1 \le \frac{2^{\gamma_1}}{\gamma_1-\alpha-\beta+1}\left(x^2+s_1^2\right)^{\frac{\gamma_1-\alpha-\beta+1}{2}} \Rightarrow$$

$$I \le \frac{2^{\gamma_1}}{\gamma_1-\alpha-\beta+1}\int_0^y \left(x^2+s_1^2\right)^{\frac{\gamma_1-\alpha-\beta+1}{2}}ds_1 \le \frac{2^{\gamma_1}}{\gamma_1-\alpha-\beta+1}\int_0^y (2s_1)^{\gamma_1-\alpha-\beta+1}ds_1$$

$$I \le \frac{2^{\gamma_1-\alpha-\beta+1}}{(\gamma_1-\alpha-\beta+1)(\gamma_1-\alpha-\beta+2)}\left(x^2+y^2\right)^{\frac{2\gamma_1-\alpha-\beta+2}{2}}$$

From the other side that:

$$|AU| \leq \frac{2^{\gamma_1-\alpha-\beta+1}\|U\|}{(\gamma_1-\alpha-\beta+1)(\gamma_1-\alpha-\beta+2)}\left(x^2+y^2\right)^{\frac{2\gamma_1-\alpha-\beta+2}{2}}$$

$$|A2U| = \left|\int_0^y ds_1 \int_0^x \frac{c_1^0(t_1,s_1)}{\left(\sqrt{t_1^2+s_1^2}\right)^{\alpha+\beta}} AU(t_1,s_1)dt_1\right| =$$

$$\frac{2^{\gamma_1-\alpha-\beta+1}}{(\gamma_1-\alpha-\beta+1)(\gamma_1-\alpha-\beta+2)}\int_0^y ds_1 \int_0^x \frac{\left(t_1^2+s_1^2\right)^{\frac{2\gamma_1-\alpha-\beta+2}{2}}}{\left(t_1^2+s_1^2\right)^{\frac{\alpha+\beta}{2}}} dt_1 \Rightarrow$$

$$|A2U| \leq \frac{2^{6\gamma_1-4\alpha-4\beta+6}\left(x^2+y^2\right)^{\frac{2\gamma_1-2\alpha-2\beta+4}{2}}\|U\|}{(\gamma_1-\alpha-\beta+1)(\gamma_1-\alpha-\beta+2)(2\gamma_1-2\alpha-2\beta+3)(2\gamma_1-2\alpha-2\beta+4)}$$

Thus we could find that:

$$|AnU| \leq \frac{2^{(4n-2)\gamma_1-(3n-2)(\alpha+\beta)+5n-4}\|U\|\left(x^2+y^2\right)^{\frac{n(\gamma_1-\alpha-\beta+2)}{2}}}{(\gamma_1-\alpha-\beta+1)(\gamma_1-\alpha-\beta+2)\ldots\ldots n(\gamma_1-\alpha-\beta+1)}$$

We now study the continuity of the effect A, [6],

$$|AU_1 - AU_2| = \left|\int_0^y ds_1 \int_0^x \frac{\left(\sqrt{t_1^2+s_1^2}\right)^{\gamma_1}}{\left(\sqrt{t_1^2+s_1^2}\right)^{\alpha+\beta}}\left[U_2(t_1,s_1)-U_1(t_1,s_1)\right]dt_1\right|$$

$$\leq \frac{2^{2\gamma_1-\alpha-\beta}\left(x^2+y^2\right)^{\frac{2\gamma_1-\alpha-\beta+2}{2}}}{(\gamma_1-\alpha-\beta+1)(\gamma_1-\alpha-\beta+2)}\rho\left[U_2(x,y),U_1(x,y)\right]$$

We choose $\varepsilon > 0$ also $\delta = \dfrac{\varepsilon(\gamma_1-\alpha-\beta+1)(\gamma_1-\alpha-\beta+2)}{2^{\gamma_1-\alpha-\beta+1}\left(x^2+y^2\right)^{\frac{2\gamma_1-\alpha-\beta+2}{2}}}$

Then from the conditions $\rho\left[U_2(x,y),U_1(x,y)\right] < \delta$

We find $\rho\left[AU_2(x,y),AU_1(x,y)\right] < \varepsilon$

Means that A effect is continuous and from the other side we could conclude the following:-

$$\left|A^nU_2(x,y) - A^nU_1(x,y)\right| \leq$$

$$\frac{2^{(4n-2)\gamma_1-(3n-2)(\alpha+\beta)+5n+4}\rho\left(U_2,U_1\right)\left(x^2+y^2\right)^{\frac{n(\gamma_1-\alpha-\beta+2)}{2}}}{(\gamma_1-\alpha-\beta+1)(\gamma_1-\alpha-\beta+2)\ldots.n\left[\gamma_1-(\alpha+\beta)+1\right]}$$

We take big n sufficiently to be:

$$\frac{2^{(4n-2)\gamma_1-(3n-2)(\alpha+\beta)+5n+4}\left(x^2+y^2\right)^{\frac{n(\gamma_1-\alpha-\beta+2)}{2}}}{(\gamma_1-\alpha-\beta+1)(\gamma_1-\alpha-\beta+2)\ldots.n\left[\gamma_1-(\alpha+\beta)+1\right]} < 1$$

Then we can say that A is an pressing effect for big n values accordingly due to the principle of the pressing effects, there is single fixed point which is in its self the single fixed point of A effect represented solution for the integrated equation(16), the successive approximation give the following relation:

$$U_{n+1}(x,y)=F_1[\varphi_1(x),\psi_1(y),\ f(x,y)]+\int_0^y ds_1\int_0^x K(t_1,s_1)U_n(t_1,s_1)dt_1 \quad (**)$$

for

$$n=0 \Rightarrow U_1(x,y)=F_1[\varphi_1(x),\psi_1(y),\ f(x,y)]$$

For n=1 we substitute in $(**)$ we find:

$$U_2(x,y)=F_1+\int_0^y ds_1\int_0^x K(t_1,s_1)U_1(t_1,s_1)dt_1$$

$$U_2(x,y)=F_1+\int_0^y ds_1\int_0^x K(t_1,s_1)F_1(t_1,s_1)dt_1$$

Thus we reach the following relation:

$$U_{n+1}(x,y)=F_1+\int_0^y ds_1\int_0^x K_1(t_1,s_1)F_1(t_1,s_1)dt_1+\int_0^y ds_1\int_0^x K_1(t_1,s_1)F_1(t_1,s_1)dt_1$$

$$+\ldots\ldots+\int_0^y ds_1\int_0^x K_n(t_1,s_1)F_1(t_1,s_1)dt_1$$

where as:

$$K_1(t_1,s_1)=\frac{\left(\sqrt{t_1^2+s_1^2}\right)^{\gamma_1}}{\left(\sqrt{t_1^2+s_1^2}\right)^{\alpha+\beta}}$$

$$K_2(t_1,s_1)=\int_0^y ds_1\int_0^x K_1(t_1,s_1)\left[\int_0^{s_1} ds\int_0^{t_1}\frac{\left(\sqrt{t_1^2+s^2}\right)^{\gamma_1}}{\left(\sqrt{t_1^2+s^2}\right)^{\alpha+\beta}}dt\right]dt_1$$

$$K_n(t_1,s_1)=\int_0^y ds_1\int_0^x K_{n-1}(t_1,s_1)\left[\int_0^{s_1} ds\int_0^{t_1}\frac{\left(\sqrt{t_1^2+s^2}\right)^{\gamma_1}}{\left(\sqrt{t_1^2+s^2}\right)^{\alpha+\beta}}dt\right]dt_1$$

:[3] we study now the approximation of the following series

$$\Gamma_1(t_1,s_1)=K_1(t_1,s_1)+K_2(t_1,s_1)+\ldots\ldots\ldots\ldots+K_n(t_1,s_1) \quad (***)$$

we find that:

$$|K_1(t_1,s_1)|=\left|\int_0^y ds_1\int_0^x\frac{\left(\sqrt{t_1^2+s_1^2}\right)^{\gamma_1}}{\left(\sqrt{t_1^2+s_1^2}\right)^{\alpha+\beta_1}}dt_1\right|\le\frac{2^{\gamma_1-\alpha-\beta+1}\left(x^2+y^2\right)^{\frac{2\gamma_1-\alpha-\beta+2}{2}}}{(\gamma_1-\alpha-\beta+1)(\gamma_1-\alpha-\beta+2)}$$

$$|K_2(t_1,s_1)|\le\frac{2^{6\gamma_1-4\alpha-4\beta+6}\left(x^2+y^2\right)^{\frac{2\gamma_1-2\alpha-2\beta+4}{2}}}{(\gamma_1-\alpha-\beta+1)(\gamma_1-\alpha-\beta+2)(2\gamma_1-2\alpha-2\beta+3)(2\gamma_1-2\alpha-2\beta+4)}$$

………………………………………………

……………………………….

$$|K_n(t_1,s_1)|\le\frac{2^{(4n-2)\gamma_1-(3n-2)(\alpha+\beta)+5n-4}\left(x^2+y^2\right)^{\frac{n(\gamma_1-\alpha-\beta+2)}{2}}}{(\gamma_1-\alpha-\beta+1)(\gamma_1-\alpha-\beta+2)\ldots\ldots n(\gamma_1-\alpha-\beta+1)}\le$$

$$\frac{2^{(4n-2)\gamma_1-(3n-2)(\alpha+\beta)+5n-4}\left(\delta_1^2+\delta_2^2\right)^{\frac{n(\gamma_1-\alpha-\beta+2)}{2}}}{(\gamma_1-\alpha-\beta+1)(\gamma_1-\alpha-\beta+2)\ldots\ldots n(\gamma_1-\alpha-\beta+1)}=b_n$$

The last relation represented the b_n Fornumercal series approximation for: general limit

$$\lim_{n\to\infty}\frac{b_{n+1}}{b_n}=0<1$$

Subsequently the(***) series according to Firestrasse test would be regularly approximated, we multiply its sides successively:

$$F_1=F_1[\varphi_1(x),\psi_1(y),\ f(x,y)]$$

Then integrated the transformers y , x

$$\int_0^y ds_1\int_0^x \Gamma_1(t_1,s_1)F_1[\varphi_1(t_1),\psi_1(s_1),\ f(t_1,s_1)]dt_1 =$$

$$\int_0^Y ds_1\int_0^x K_1(t_1,s_1)F_1[\varphi_1(t_1),\psi_1(s_1),\ f(t_1,s_1)]dt_1 + \ldots.. +$$

$$\int_0^Y ds_1\int_0^X K_n(t_1,s_1)F_1[\varphi_1(t_1),\psi_1(s_1),\ f(t_1,s_1)]dt_1$$

ACCORDINGLY

$$U_{n+1}(x,y)=F_1[\varphi(x),\psi_1(y),\ f(x,y)]+\int_0^y ds_1\int_0^x \Gamma_1(t_1,s_1)F_1(x,y;t_1,s_1)dt_1$$

THAT IS:

$$U(x,y)=F_1[\varphi_1(x),\psi_1(y),\ f(x,y)]+\int_0^y ds_1\int_0^x \Gamma_1(x,y;t_1,s_1)F_1[\varphi_1(t_1),\psi_1(s_1),\ f(t_1,s_1)]dt_1 \qquad (18)$$

By substitution(18) in the right side by(17) we again obtain integral Volterra equations with function of solution coefficient, $\Gamma_1(x,y;t_1,s_1)$ [2] , for the integrated equation(16):

$$V(x,y)-\int_0^y ds_2\int_0^x e^{\Omega_{a_2}^{\alpha}(x,s_2)-\Omega_{b_2}^{\beta}(x,s_2)-W_{a_2}^{\alpha}(x,s_2)+W_{b_2}^{\beta}(x,s_2)+\Omega_{b_2}^{\beta}(t_2,s_2)-W_{b_2}^{\beta}(t_2,s_2)}.\left(t_2^2+s_2^2\right)^{-\frac{(\alpha+\beta)}{2}}$$

$$c_2^0(t_2,s_2)V(t_2,s_2)dt_2=e^{W_{a_2}^{\alpha}(x,y)-\Omega_{a_2}^{\alpha}(x,y)}\left\{\varphi_2(x)+\int_0^y e^{\Omega_{a_2}^{\alpha}(x,s_2)-W_{a_2}^{\alpha}(x,s_2)-\Omega_{b_2}^{\beta}(x,s_2)+W_{b_2}^{\beta}(x,s_2)}\right.$$

$$\left[\psi_2(s_2)+\int_0^x e^{\Omega_{b_2}^{\beta}(t_2,s_2)-W_{b_2}^{\beta}(t_2,s_2)}\left(t_2^2+s_2^2\right)^{-\frac{(\alpha+\beta)}{2}}F_1(\varphi_1(t_2),\psi_1(s_2),\ f(t_2,s_2))+\right.$$

$$\left.\left.\int_0^{s_2} ds_1\int_0^{t_2}\Gamma_1(x,y;t_1,s_1).F_1[\varphi_1(t_1),\psi_1(s_1),\ f(t_1,s_1)]dt_1)dt_2\right]ds_2\right\}$$

we assume the first side of equation equals:

$$F_3\left[\varphi_1(x),\varphi_2(x),\psi_1(y),\psi_2(y),\Gamma_1(x,y)\right]$$

Then written as follows:

$$V(x,y)-\int_0^y ds_2\int_0^x e^{\Omega_{a_2}^{\alpha}(x,s_2)-\Omega_{b_2}^{\beta}(x,s_2)-W_{a_2}^{\alpha}(x,s_2)+W_{b_2}^{\beta}(x,s_2)+\Omega_{b_2}^{\beta}(t_2,s_2)-W_{b_2}^{\beta}(t_2,s_2)}\left(t_2^2+s_2^2\right)^{-\frac{(\alpha+\beta)}{2}}$$

$$c_2^0(t_2,s_2)V(t_2,s_2)dt_2=F_3\left[\varphi_1(x),\varphi_2(x),\psi_1(y),\psi_2(y),\Gamma_1(x,y)\right] \qquad (19)$$

To solve this integral equation we assume initially that approximate to zero achieve the following:

$$e^{\Omega_{a_2}^{\alpha}(x,s_2)-\Omega_{b_2}^{\beta}(x,s_2)-W_{a_2}^{\alpha}(x,s_2)+W_{b_2}^{\beta}(x,s_2)+\Omega_{b_2}^{\beta}(t_2,s_2)-W_{b_2}^{\beta}(t_2,s_2)}c_2^0(x,y)=0\left(r^{\gamma_2}\right)$$

In the same way we have found there is solution for the integral equation(16) we obtain solution for the integrated equation(19) with the following relation:

$$V(x,y)=F_3\left[\varphi_1(x),\varphi_2(x),\psi_1(y),\psi_2(y),f(x,y),\Gamma_1(x,y)\right]+\int_0^y ds_2\int_0^x \Gamma_2(x,y;t_2,s_2).$$

$$F_3\left[\varphi_1(t_2),\psi_1(s_2),\varphi_2(t_2),\psi_2(s_2),f(t_2,s_2),\Gamma_1(x,y;t_2,s_2)\right]dt_2 \qquad (20)$$

subsequently due to what has mentioned we proved that the following theory:

Theory(2) assume that coefficients of differential equation(1) also its right side achieve the following conditions:

1) $a_2(x,y)\in C_{xxy}^3\left(\overline{D}\right),\ \{b_2(x,y),c_2(x,y)\}\in C\left(\overline{D}\right)$

2) $a_1(x,y)\in C_x^1\left(\overline{D}\right),\ \{b_2(x,y),c_2(x,y)\}\in C\left(\overline{D}\right)$

3) $\left|a_j(x,y)-a_j(0,0)\right|\le H_{a_j}r^{-\gamma_{j1}}\ ;\ 0<\gamma_{j1}<2-\alpha\ ,\ j=1,2$

4) $\left|b_j(x,y)-b_j(0,0)\right|\le H_{b_j}r^{-\gamma_{j2}}\ ;\ 0<\gamma_{j2}<2-\beta\ ,\ j=1,2$

5) $r^{-(\alpha+\beta)}.f(x,y)=0\left(r^{-\delta}\right)\ ;\ 0<\delta<1\ ,\ for: r\to 0$

6) $r^{-(\alpha+\beta)}c_j^0(x,y)=0\left(r^{-\gamma_j}\right)\ ;\ 0<\gamma_j<1\ ,\ for: r\to 0$

$$H_{a_j},H_{b_j}=const$$

Where as:

$$c_j^0(x,y)=x.ya_j(x,y)b_j(x,y)-c_j(x,y)+r^{\alpha+\beta}\frac{\partial}{\partial x}\left(y\frac{a_j(x,y)}{r^{\alpha}}\right)$$

$$j=1,2\ ,\ \ \alpha<2,\beta<2$$

Then we find that there is single solution for With relation(20) where as (1) equation $\Gamma_1(x,y),\Gamma_2(x,y)$ coefficient to solve the two integrated equations(19) and(16) respectively, besides achiving the following:

$$\varphi_1(x)\in C(\Delta_1),\varphi_2(x)\in C^2(\Delta_1),\psi_1(y)\in C(\Delta_2),\psi_2(y)\in C^1(\Delta_2)$$

The former results including the two theories(1) and (2) are for the two conditions:

As for the other values for these two approximations , we obtain the same solutions but in another conditions differ from the conditions mentioned in the first and the second theories, subsequently for $.\alpha<2,\beta<2$ We obtained the following theory

Theory(3) assume that coefficients of differential equation(1) and its right side achieve the following conditions:

1) $a_2(x,y)\in C^3_{xxy}(\overline{D}),\{b_2(x,y),c_2(x,y)\}\in C^2_{xy}(\overline{D})$

2) $a_1(x,y)\in C^1_x(\overline{D}),\{b_1(x,y),c_1(x,y),\ f(x,y)\}\in C(\overline{D})$

3) $|a_j(x,y)-a_j(0,0)|\le H_{a_j}r^{-\gamma_{j1}}\ \ ;0<\gamma_{j1}<2-\alpha,\ j=1,2$

4) $|b_j(x,y)-b_j(0,0)|\le H_{b_j}r^{-\gamma_{j2}}\ \ ;0<\gamma_{j2}<\beta-2,\ j=1,2$

$$H_{aj},H_{bj}=const$$

5) $r^{-(\alpha+\beta)}.f(x,y)=0(r^{-\delta});0<\delta<1\ ,\ for:r\to 0$

6) $e^{-W^{\beta}_{b_j}(x,y)}.r^{-(\alpha+\beta)}.c^0_j(x,y)=0(r^{-\delta_j});0<\delta_j<1,\ for:r\to 0,\ \ j=1,2$

Then there is single solution for the differential equation(1) with relation(20) whereas :

$$\varphi_2(x)\in C^2(\Delta_1),\psi_2(y)\in C^1(\Delta_2),\varphi_1(x)\in C(\Delta_1),\psi_1(y)\in C(\Delta_2)$$

For $\alpha>2,\beta<2$ we obtained the following theory:

Theory(4)assume that coefficients of differential equitation(1) and its right side achieving the following conditions:

1) $a_2(x,y)\in C^3_{xxy}(\overline{D}),\{b_2(x,y),c_2(x,y)\}\in C^2_{xy}(\overline{D})$

2) $a_1(x,y)\in C^1_x(\overline{D}),\{b_1(x,y),c_1(x,y),\ f(x,y)\}\in C(\overline{D})$

3) $|a_j(x,y)-a_j(0,0)|\le H_{a_j}r^{-\gamma_{j1}}\ \ ;0<\gamma_{j1}<\alpha-2,\ j=1,2$

4) $|b_j(x,y)-b_j(0,0)|\le H_{b_j}r^{-\gamma_{j2}}\ \ ;0<\gamma_{j2}<2-\beta,\ j=1,2$

$$H_{aj},H_{bj}=const$$

5) $r^{-(\alpha+\beta)}.f(x,y)=0(r^{-\delta});0<\delta<1\ ,\ for:r\to 0$

6) $e^{-W^{\beta}_{b_j}(x,y)}.r^{-(\alpha+\beta)}.c^0_j(x,y)=0(r^{-\delta_j});0<\delta_j<1,\ for:r\to 0,\ \ j=1,2$

Then there is single solution for the differential equation(1) gives with the relation(20) whereas:

$$\varphi_2(x)\in C^2(\Delta_1),\psi_2(y)\in C^1(\Delta_2),\varphi_1(x)\in C(\Delta_1),\psi_1(y)\in C(\Delta_2)$$

for w $\alpha>2$, $\beta>2$ We obtained the following theory:

Theory(5) assume that coefficients of differential equation(1) also its right side achieve the first ,second and fifth conditions, besides we assume achieving the following condition:

1) $a_2(x,y)\in C^3_{xxy}(\overline{D}),\{b_2(x,y),c_2(x,y)\}\in C^2_{xy}(\overline{D})$

2) $a_1(x,y)\in C^1_x(\overline{D}),\{b_1(x,y),c_1(x,y),f(x,y)\}\in C(\overline{D})$

3) $\left|a_j(x,y)-a_j(0,0)\right|\le H_{a_j}r^{-\gamma_{j1}}$ $;0<\gamma_{j1}<\alpha-2,\ j=1,2$

4) $\left|b_j(x,y)-b_j(0,0)\right|\le H_{b_j}r^{-\gamma_{j2}}$ $;0<\gamma_{j2}<\beta-2,\ j=1,2$

$H_{aj},H_{bj}=const$

5) $r^{-(\alpha+\beta)}.f(x,y)=0(r^{-\delta});0<\delta<1,\ for:r\to 0$

6) $e^{-W^{\beta}_{b_j}(x,y)}.r^{-(\alpha+\beta)}.c^0_j(x,y)=0(r^{-\delta_{j1}});0<\delta_{j1}<1,\ for:r\to 0,\ j=1,2$

7) $e^{-W^{\beta}_{a_j}(x,y)}.r^{-(\alpha+\beta)}.c^0_j(x,y)=0(r^{-\delta_{j2}});0<\delta_{j2}<1,\ for:r\to 0,\ j=1,2$

There is single solution for equation(1) with relation(20) where as:

$$\varphi_2(x)\in C^2(\Delta_1),\psi_2(y)\in C^1(\Delta_2),\varphi_1(x)\in C(\Delta_1),\psi_1(y)\in C(\Delta_2)$$

References

Cohclusions Recommendations

1 - Mikhlin SG, 1959 - Lectures on linear integral equations "Moscow" 232 p.

2 - Nusrat Radzhabov, 1985 - Integral representations and boundary value problems for certain differential equations with a sinus line. Part 4.

3 -Nusrat Radzhabov., 1992 - Introduction to the theory of differential equations "Manual for special courses", "Tajikistan" Dushanbe 239 p.

4 - Petrovsky IG, 1970 - Lectures on the theory of ordinary differential equations "Moscow" 279 p.

5 - Vladimirov VS, 1981 - Equations of mathematical physics "Moscow" 512 p.

6 -EDWARDS C.H, 2000 -Differential equations and boundary value problems, Prentice-Hall, XII ,787, pages .

7 -Khalid Ahmed Al-Samarrai, Methods of Solving Equations, College of Science, University of Baghdad Khalid Ahmed Al-Samarrai, Methods of Solving Differential Equations, Faculty of Science, University of Baghdad ,2004.

8 -Salim Ismail Al-Ghabrabi and Sabah Hadi Jassim, Differential Equations, Statistical Number, Faculty of Management and Economics, University of Baghdad, 1988 -1409 e

المستخلص

تستخدم المعادلات التفاضلية في دراسة جزيئات ذات مرتبات عالية مع معاملات المتغيرات في مجالات مختلفة من المستوى الاحداثي الكارتيزي في حين ان بعض العلماء والباحثين قد درسوا ذاك امثال : ان ريكابوف وآي اس ستار ونسيم اديب حنين ، وغيرهم . وبناء على ذلك ، تمت دراسة المعادلة الجزئية التي سيتم اخذها من رتبتها (المرتبة الرابعة) في هذا البحث في حين تختلف معاملاتها عن المعادلات التفاضلية الجزئية وفي سياق هذه المعادلة تحدد الشروط لمعاملاتها لانتاج حل وحيد لتلك المعادلة التفاضلية الجزئية في عدة حالات مختلفة تتعلق بهذه المعاملات وقد تم تلخيص هذه الظروف في خمس نظريات.