More On MC-Functions

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Abstract

In this work we introduce a new concept of Mc-functions namely M-gc functions, these are the functions f: X⟶Y in which the inverse image of every compact set in Y is g-closed in X. Several theorems about this concept were proved, and the relationships of M-gc functions with other types of functions were discussed.

1. INTRODUCTION

In [7] the concept of Mc-functions introduced a function f: X⟶ Y is said to be a Mc-function if $f^{-1}(F)$ is closed subset of X, whenever F is a compact subset of Y. Also in [3], the concept of Mθ-c functions introduced a function f: X⟶ Y is said to be an Mθ-c function if $f^{-1}(F)$ is θ-closed subset of X, whenever F is a compact subset of Y. In this work we introduce and study the concept of M-gc functions. We proved several theorems concerning this concept.

2. PRELIMINARIES

In this section, we recall the basic definitions and facts that needed in this work. All spaces X and Y are topological spaces and no function is assumed to be continuous unless explicitly stated to be so. For any concepts which we do not define or elaborate upon, the reader is referred to Kelley's book [4]. If A is a subset of a space X, the family of all g-open sets containing x in a space X is denoted by GN(x).

Definition 2.1[6]:

Let (X, T) be a topological space, then a subset A of X is said to be generalized closed set (written g-closed set) if the closure of A is a subset of every open set U contain A. A subset A is generalized open set (written g-open set if its complement X- A is g-closed set.

Definition 2.٢ [5]:

A space X is said to be g-T_2-space if for every two distinct points x and y in X, there exist two disjoint g-open sets U and V containing x and y respectively.

Definition 2.٣ [5]:

A function f from a space X into a space Y is said to be g^*-continuous, if

$f^{-1}(F)$ is open (closed) subset of X, whenever F is g-open (g-closed) subset of Y.

Definition 2.٤ [5]:

A function f: X → Y is said to be g^{**}-continuous (or g-irresolute) if $f^{-1}(H)$ is a g-open (g-closed) subset of X, whenever H is a g-open (g-closed) subset of Y.

Remark 2.1:

Every g*-continuous function is continuous and g**-continuous functions.

Definition 2.٥ [9]:

Let X be a topological space and let A ⊆X, then A is compact set if every open cover of A has finite subcover.

Definition 3.٦ [1]:

A function f: X⟶Y is said to be a compact if $f^{-1}(M)$ is compact subset of X, whenever M is a compact subset of Y.

Definition 2.7 [2]:

A space X is said to be a Kc-space if every compact subset of X is closed.

Definition 2.8 [8]:

A subset K of a space X is said to be generalized compact relative to X (briefly g-compact) if for every g-open cover of K has a finite sub cover. Also a space X is said to be g-compact if for every g-open cover of X has a finite sub cover.

Every g-compact set is compact, but the converse is not true in general as in the following example.

Example (2.1) [2]:

Let R be the real line, N be the subset of R and $\zeta = \{U \subseteq R \mid U = R$ or $U \cap N = \phi\}$ It is clear that (R, ζ) is a topological space. Then R is compact space but it is not g-compact.

Definition 2.9 [2]:

A function f from a space X into a space Y is said to be a g-compact function if $f^{-1}(K)$ is a g-compact subset of X, whenever K is a compact subset of Y.

Definition 2.10[2]:

A function f from a space X into a space Y is said to be a g^{*}-compact function if $f^{-1}(K)$ is a compact subset of X, whenever K is a g-compact subset of Y.

Definition 2.11 [2]:

A function f: X→Yis said to be g^{**}-compact if $f^{-1}(M)$ is a g-compact subset of X, whenever M is a g-compact subset of Y.

Remark 2.2 [2]:

1. Every g-compact function is compact,
2. Every g-compact function is g^{**}- compact, which they g^{*}- compact.

The converse of this remark is not true in general.

Example 2.2 [2]:

Let R be the real line, N be the subset of R and $\zeta = \{U \subseteq R \mid U = R$ or $U \cap N = \phi\}$ It is clear that (R, ζ) is a topological space. Then the identity function I_R: R → R is compact but not g-compact function.

3. ON M-gC FUNCTIONS

In [3] the authors introduced Mθ-c functions and proved some results. Here we introduce the concept of M-gc functions and proved several theorems concerning this concept.

Definition 3.1:

Let f: X⟶Y be a function; we say that f is an M-gc function if $f^{-1}(F)$ is g-closed subset of X, whenever F is a compact subset of Y.

Remark 3.1:

Every Mc-function is M-gc function, but the converse may be not true.

Example 3.1:

Let f be a function from indiscrete space (R,τ_{ind}) into any space, then f is an M-gc function, but not Mc-function.

Theorem 3.1:

Every g-compact set in T_2-space is g-closed.

Proof: Let K be a g-compact subset of Y, so K is a compact subset of Y. But Y is a T_2-space then K is closed subset of Y, and hence K is a g-closed subset of Y.

Proposition 3.1:

Every continuous function f from a space X into a T_2-space Y is an M-gc function.

Proof: Let L be a compact subset of Y, then L is a closed subset of Y (since Y is a T_2-space). But f is a continuous so $f^{-1}(L)$ is a closed subset of X which implies that it is a g-closed. Therefore f is an M-gc function.

Definition 3.٢:

A function f: X→ Y is said to be gM-c function if $f^{-1}(K)$ is a closed subset of X, whenever K is a g-compact subset of Y. So every Mc-function is gM-c function, but the converse may be not true.

Proposition 3.2:

Every continuous function f from a space X into a T_2-spaceY is a gM-c function.

Proof: Let F be a g-compact subset of Y, then F is a compact subset of Y. F is a closed subset of Y (since Y is a T_2-space). But f is a continuous so $f^{-1}(F)$ is a closed subset of X. Therefore f is a gM-c function.

Proposition 3.3:

If f: X⟶Y is g^*-continuous function and Y is a T_2-space then f is a gM-c function.

Proof: Let L be a g-compact subset of Y, which means that L is a compact subset of Y which is a T_2-space, so L is a closed subset of Y and then a g-closed subset of Y. But f is g^*-continuous, then $f^{-1}(L)$ is a closed subset of X. Therefore f is a gM-c function.

Definition 3.٣:

A function f: X⟶Y is said to be gM-gc function if $f^{-1}(K)$ is a g-closed subset of X, whenever K is a g-compact subset of Y.

Remark 3.2:

1. Every M-gc function is gM-gc functions,
2. Every gM-c function is gM-gc functions,

There is no relation between M-gc function and gM-c function.

Proposition 3.4:

Every continuous function f from a space X into a T_2-space Y is a gM-gc function.

Proof: Let M be a g-compact subset of Y, then M is a compact subset of Y. But Y is a T_2- space, so M is a closed subset of Y and since f is a continuous function then, $f^{-1}(M)$ is closed subset of X, so $f^{-1}(M)$ is a g-closed subset of X. Therefore f is a gM-gc function.

Corollary 3.١:

If f: X⟶Y is g^*-continuous function and Y is a T_2-space then f is a gM-gc function.

Theorem 3.2:

Every continuous function from X into a Kc-space Y is a Mc (M-gc) function.

Proof: Let G be a compact subset of Y, which is a Kc-space, then it is a closed subset of Y, but f is continuous then $f^{-1}(G)$ is closed subset of X, that is f is a Mc (M-gc) function.

Definition 3.٤:

A space X is said to be gK-c space if every g-compact subset of X is closed.

Theorem 3.3:

Every continuous function f from X into a gK-c space Y is a gM-c function.

Proof: Let M be a g-compact subset of Y, which is a gK-c space, then it is a closed subset of Y, but f is continuous then, $f^{-1}(M)$ is a closed subset of X that is f is a gM-c function.

Corollary 3.٢:

Every continuous function f from X into a Kc-space Y is a gM-c function.

Definition 3.٥:

A function f from a space X into a space Y is said to be an Lc-function if $f^{-1}(M)$ is a closed subset of X, whenever M is a lindelof subset of Y.

Remark 3.3:

Every Lc-function is a Mc-function, but the converse may be not true.

Theorem 3.4:

Every compact function from a T_2-space X into any space Y is a Mc-function.

Proof: Let M be a compact subset of Y. But f is a compact function, so $f^{-1}(M)$ is a compact subset of X which is a T_2-space, so $f^{-1}(M)$ is a closed subset of X. Therefore f is a M-c function.

Corollary 3.3:

Every compact function f from a T_2-space X into any space Y is a gM-c function.

Remark 3.4:

The converse of the above theorem may be not true, as following example

Example 3.2:

The identity function f from discrete space X into indiscrete space Y is Mc-function, but it is not compact function. Where X and Y are infinite set.

Theorem 3.5:

Every Mc-function from a compact space X into any space Y is a compact function.

Proof: Let K be a compact subset of Y. But f is a Mc-function, then $f^{-1}(K)$ is a closed subset of X. But X is a compact space and any closed set in a compact space is compact, so $f^{-1}(K)$ is a compact subset of X. Therefore f is a compact function.

Corollary 3.4:

If X is a compact and a T_2-space then a function f from X into any space Y is a compact iff f is a Mc-function.

Proposition 3.5:

Every gM-c function from a compact space X into any space Y is a g^*-compact function.

Proof: Let K be a g-compact subset of Y and f is a gM-c function, then $f^{-1}(K)$ is a closed subset of X, which is a compact space, so $f^{-1}(K)$ is a compact subset of X, that is f is a g^*-compact function.

Proposition 3.٥:
Every g^*-compact function from a T_2-space X into any space Y is a gM-c function.

Theorem 3.6:
If f: X⟶Y is a gM-c function and W is a subset of X, then $f|_w: w \longrightarrow Y$ is also a gM-c function.

Proof: Let K be a g-compact subset of Y, so $f^{-1}(K)$ is a closed subset of X, since f is a gM-c function. But $f|_w^{-1}(K) = f^{-1}(K) \cap w$ is a g-closed subset of W, (By definition of relative topology). Therefore $f|_w$ is a gM-c function.

Theorem 3.7:
Let $f: X \to Y$ and $g: Y \to Z$ be a functions then

1. If $f: X \to Y$ be a continous function and $g: Y \to Z$ is a Mc-function. Then g∘f is an M-gc function.
2. If f: X→Y be g^{**}-continuous and g: Y→ Z be M-gc function, then g∘f is an M-gc function.
3. If f be a continuous function and g is a gM- c function. Then gof is also a gM-c function.
4. If f: X→Y be a g**-continuous function and g: Y⟶Z be a gM-gc function. Then gof is a gM-gc function.
5. If f: X→Y be a g*-continuous function and g: Y⟶Z is a gM-gc function, then gof is a gM-c function.
6. If f: X⟶Y and g: Y⟶Z such that f is a Mc-function and g is a compact function. Then gof is a Mc-function.
7. If f: X⟶Y and $g: Y \to Z$ such that f is a gM-c function and g is a g**-compact function. Then gof is a gM-c function.

Proof (1): Let K be a compact subset of Z, then $g^{-1}(K)$ is closed subset of Y (since g is an Mc-function) and since f is continuous function, then $f^{-1}(g^{-1}(K))$ is closed subset of X. But each closed set is g-closed set and $f^{-1}(g^{-1}(K)) = (gof)^{-1}(K)$. Therefore gof is an M-gc function.

Proof (2): Let M be a compact subset of X, then $g^{-1}(M)$ is a g-closed subset of Y. But f is g^{**}-continuous function, then $f^{-1}(g^{-1}(M))=(gof)^{-1}(M)$ is g-closed subset of X, and hence gof is an M-gc function .

Proposition 3.6:
Every compact function f from a Kc-space X into any space Y is a Mc-function.

Proof: Let M be a compact subset of Y, but f is a compact function, so $f^{-1}(M)$ is a compact subset of X which is Kc-space, hence $f^{-1}(M)$ is a closed subset of X. Therefore f is a Mc-function

Theorem 3.8:

Let $f: X \to Y$ and $g: Y \to Z$ be a functions then

1. If f is a continuous function, g is a compact function and Y is a Kc-space. Then gof is a Mc-function
2. If f is a continuous function, g is a compact function and Y is a Kc-space. Then gof is an M-gc function.
3. If f is a continuous function, g is a compact function and Y is a Hausdorff-space. Then gof is a Mc- function.
4. If f is continuous function, g is compact function and Y is a Hausdorff space. Then gof is an M-gc function.
5. Let Y be a Kc-space, f is a continuous function and g is a compact function. Then gof is a gM-c and a gM-gc function.
6. If f: X⟶Y and g: Y⟶Z are Mc-functions. If Y is a compact space then gof is also a Mc-function.
7. If f: X⟶Y and g: Y⟶Z be an M-gc functions and Y is a g-compact space, then gof is also an M-gc function.
8. If f: X⟶Y be an M-gc function, g: Y→Z is a gM-c function and Y is a compact space, then gof is a gM-gc-function.
9. If f is a gM-c function, g is an M-gc function and Y is a g-compact space then gof is a gM-gc function.
10. If f is a gM-c function, g is a Mc-function and Y is a g-compact space then gof is a gM-c function.

REFERENCES

[1] E. Halfar, "Compact mappings", proc. Amer. Math. Soc. 8 (1957), 828-830.

[2] H. J. Ali and A. J. Mohammed, "On g, g^*, g^{**}-Compact Functions", journal of AL-Nahrain University, Vol. 13 (2) (2010), 205-209.

[3] H. J. Mustafa and Ali Al-Tanni, "On Mθ-c Multifunctions", University, of Jordan Journals, Dirasat, Pure Sciences, Vol. 32, N. 2, (2005), 252-257.

[4] J. L. Kelley, "General Topology", D. Van Nostrand Co., Princeton, (1955).

[5] J. M. Esmail, "On G-closed Mapping", M. Sc Thesis, College of Education, University of AL-Mustansiriya (2002).

[6] Levine. N, "Generalized closed sets in Topology", Rend. Circ. Math. Paleremo (2) 19 (1970), 89-96.

[7] M. Farhan, F. N. Nassar, "On-MC-Functions", Tikrit Journal of Pure Science 16 (4) (2011), 287-289.

[8] Sundaram P., Maki. H. and Baiacnandran. K., "On generalized continuous maps in topological spaces", Mem. Fac. Sci. Kochi Uni. Ser A Math.12 (1991), 5-13.
[9] Willard S., "General Topology", Addison-wesley, London, (1970).

الخلاصة

في عملنا هذا قدمنا مفهوم جديد الى الدوال –Mc يدعى الدوال –M-gc تلك الدوال $f:X \to Y$ التي فيها الصورة العكسية لأي مجموعة متراصة في المستقرY تكون مغلقه–gفي المنطلق X. عدة مبرهنات بخصوص هذا المفهوم برهنت وكذلك علاقته مع دوال أخرى قد نوقشت.