Topological Projective Covers for Topological Groups

Mohammed Jabbar Hussein
Department of Mathematics- College of Basic Education
Al-Mustansiriya University

Abstract

The primary objective of this paper is to evaluate the tensor product of topological projective cover for topological groups. After we explain that not every topological group has topological group cover. We depend on tensor product properties of topological groups for algebra and we inputed topological properties that suitable of algebra construction as topology from the definition of topological groups and projective on topological projective groups , such that; a topological group q is called topological projective group if for all topological group epimorphism

g : A $\longrightarrow$ B and for all topological group morphism f : q $\longrightarrow$ B, there exists a topological group morphism f ' :q $\longrightarrow$ A, for which the following diagram commutes":

<u>Difinition:-</u> Amorphism of topologicl group$f: G \rightarrow H$ is a continuous homomorphism between topological groups.

<u>Difinition:-</u> epimorphism in the category of all topological groups are easily seen to be surjective if G is a topological group and any subgroup there exist agroup G' which endow with the indiscrete topology and two homomorphism from G into G' which agree only on it.

Furthermore, new theorems are given at the end of the paper.

Key word: Topological group, Topological subgroup, Projective group, Topological projective group.

Introduction

The aim of this paper is to study the properties of topological algebra groups, We start with simple cases of algebra and topology, but they are very important, so in our study we concerned with topological projective groups and topological projective covers.

In the ending of the twentieth century they begin to concern with study of topological groups and at the ending .

In this paper we study topological groups specially topological projective groups. We didn’t determine topology and countable space, metric spaces and topological linear spaces. The important point in this paper is evaluate the tensor product of topological projective covers of the topological groups. I think the tensor product is a manufacturer of fancy floor lamps. In mathematics, the tensor product of groups is a construction that allows arguments about bilinear maps to be carried out in terms of

linear maps (group homomorphisms). The group construction is analogous to the construction of the tensor product of vector spaces, but can be carried out of a pair of groups, Let G, G' be two topological groups, thus the tensor product (G ⊗ G') is an abelian group together with abelian map ⊗: G×G' ⟶ G⊗G', which is universal of tensor product in the following sense "For abelian group W and every linear map f : G×G' ⟶ W, there is a unique group homomorphism f': G ⊗G' ⟶ W, such that :
$f' \circ \otimes = f$.

$$\begin{tikzcd} G \times G' \arrow[r, "\otimes"] \arrow[dr, "f"'] & G \otimes G' \arrow[d, dotted, "f'"] \\ & W \end{tikzcd}$$

1- Topological groups

In this section, we give the importment propositions and definitions and fundamental concepts of this work:

Definition (1.1):

A topological group is a set G together with two structures:
1. G is a group.
2. Topology T on G.

The two structures are compatible, i.e., the group (binary operation) μ: G×G ⟶ G, and the inversion law ν: G ⟶ G are both continuous maps.

Example (1.2):

Every group is a topological group with discrete topology G = R on the topological group.

Definition (1.3):

Let G,G' be a topological group, a homomorphism $f : G \to G'$ is called a homomorphism topological group if:

1- f is homomorphism group.
2- f is continuous.

Definition (1.4):

A function $f : G \to G'$ is called a homeomorphism from a topological group G into G' if :
1- f is group isomorphism.
2- f is homeomorphism.

Example (1.5):

Let (R,+) be ausuall topological group and (R^+-{0},.) is a relative topological group then there exists group homeomorphism between(R,+) and (R^+-{0},.) .

Definition (1.6):

Let G be a topological group and H be a subset of G ,then H is called topological subgroup if :

1- H is subgroup from G.
2- H is a subspace from topology space G.

Definition (1.7):

The genral Linear group GL(n,R) of all invertible n-by-n matrices with real entries can be viewed GL(n,R) a subspace of Euclidean space $R^{n\times n}$.

Example (1.7):

A special orthognial SO(2,R)= $\left\{\begin{Bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{Bmatrix} : \theta \in R\right\}$ is a topological subgroup from topological group GL(2,R) since SO(2,R) is subgroup from group GL(2,R), therefore SO(2,R) is subspace from topology space GL(2,R).

Note (1.8):

Every subgroup from topological group is topological group but not every subspace from topological group is topological group.

2- Topological Projective group

Definition (2.1):

A topological group q is called topological projective group if for all topological group epimorphism g: A $\longrightarrow$ B and topological group morphism
f :q $\longrightarrow$ B, there exists a topological group morphism
f': q $\longrightarrow$A, which the following diagram commutes:

$$\begin{tikzcd} & q \arrow[dl, dotted, "f'"'] \arrow[d, dotted, "f"] \\ A & B \arrow[l, dotted] \end{tikzcd}$$

Notation (2.2):

Ker f is topological subgroup of q , where f is topological group homomorphism from q into q′.

Proposition (2.3): [7]

Let q be a topological projective and f be discrete topological group, then q is topological projective group.

Theorem (2.4): [1]

Let $\{q_\alpha\}_{\alpha\in L}$ be a family of topological groups, B be a topological group ,the topological group homomorphism g_α: $q_\alpha \longrightarrow$ B for all $\alpha \in$ L, there exists a unique topological group homomorphism g: $\underset{\alpha\in L}{\oplus} q_\alpha \longrightarrow$ B, for which the following diagram commutes:

$$\begin{tikzcd}
& q_\alpha \arrow[dl, "I_\alpha"'] \arrow[d, "g_\alpha"] \\
\oplus_{\alpha\in l} q_\alpha \arrow[r, "g"'] & B
\end{tikzcd}$$

Theorem (2.5): [1]

Let $\{q_\alpha\}_{\alpha\in L}$ be afinite family of topological subgroups of a topological group G, then the topological direct sum q = $\underset{\alpha\in L}{\oplus} q_\alpha$ is a topological projective group iff q_α is a topological group.

3- Topological Projective Covers

In this section, we study the concept of topological projective covers and some results.

Definition (3.1):[1]

The topological subgroup G of a topological group A is called small if for all topological subgroup M of A, then G + M = A, M = A.

Definition (3.2):[1]

The topological group morphism f : q $\longrightarrow$ B is called small if Ker f is small topological subgroup of q.

Definition (3.3):[3]

The topological group epimorphism f ': q $\longrightarrow$ G is called topological group cover of G of q is a topological projective group and f ' is topological group epimorphism and small.

Notation (3.4):

Never all topological group has topological projective cover.

Example (3.5):[1]

Z/2Z has no topological projective cover as a topological group Z.

4-Universal Property of Topological Projective Cover for Topological Groups of Tensor Product

Let q and E be topological projective groups, let $f : q \longrightarrow G$ and $g : E \longrightarrow G'$ be topological morphism, where G and G' be topological group, then there exists a unique topological group morphism from $q\otimes E$ to $q\otimes E$ denoted by $f\otimes g$, such that:
$(f\otimes g)(q\otimes E) = (f(q))\otimes(g(E))$, for all $q \in q$, $e \in E$.

$$\begin{tikzcd}
f \times g \arrow[r] \arrow[d] & f \otimes g \arrow[dl] \\
f \otimes g &
\end{tikzcd}$$

Theorem (4.1):

If $f = \underset{1\le i\le n}{\oplus} f_i : \underset{1\le i\le n}{\oplus} q_i \longrightarrow \underset{1\le i\le n}{\oplus} G_i$ be topological group cover of $\underset{1\le i\le n}{\oplus} G_i$ and $g = \underset{1\le i\le n}{\otimes} g_i : \underset{1\le i\le n}{\otimes} E_i \longrightarrow \underset{1\le i\le n}{\otimes} G'_i$ be topological group cover of $\underset{1\le i\le n}{\otimes} G'_i$, then:
$J = f \oplus g$ be topological group cover of $\underset{1\le i\le n}{\oplus} G_i \oplus \underset{1\le i\le n}{\otimes} G'_i$.

Proof:

Let $J = f \otimes g$ be topological group to $\underset{1\le i\le n}{\oplus} G_i \otimes \underset{1\le i\le n}{\oplus} G'_i$. We prove that $J = f \otimes g$ is a topological group cover to $\underset{1\le i\le n}{\oplus} G_i \otimes \underset{1\le i\le n}{\oplus} G'_i$.

Let $J = f \otimes g$ is small and surjective topological group homomorphism. f is toplogical group cover of $\underset{1\le i\le n}{\oplus} G_i$ and g is toplogical group cover of $\underset{1\le i\le n}{\oplus} G'_i$., thus Ker f is small topological subgroup of $\underset{1\le i\le n}{\oplus} q_i$ and Ker g is small topological subgroup of $\underset{1\le i\le n}{\otimes} E_i$, then:

$\text{Ker } f_i + M_i = q_i \Rightarrow M_i = q_i$
$\text{Ker } g_i + N_i = E_i \Rightarrow N_i = E_i$
$\text{Ker } J = \text{Ker}(f_i \otimes g_i)$
$\text{Ker}(f_i \otimes g_i) + (M_i \otimes N_i) = q_i \otimes E_i$
Thus :
$M_i \otimes N_i = q_i \otimes E_i,\ 1 \le i \le n.$
That is $J = f \otimes g$ is small and surjective topological group homomorphism of $\underset{1\le i\le n}{\oplus} G_i \otimes \underset{1\le i\le n}{\oplus} G'_i$, $1 \le i \le n$.
Thus

$(\underset{1\le i\le n}{\oplus} M_i \oplus \underset{1\le i\le n}{\otimes} N_i) = \underset{1\le i\le n}{\oplus} q_i \oplus \underset{1\le i\le n}{\otimes} E_i$

U = P

Where $M = (\underset{1\le i\le n}{\oplus} M_i \oplus \underset{1\le i\le n}{\otimes} N_i)$.

J be topological homomorphism onto and small.

Theorem (4.2):

If $f = \underset{1\le i\le n}{\oplus} f_i : \underset{1\le i\le n}{\oplus} q_i \longrightarrow \underset{1\le i\le n}{\oplus} G_i$ be topological group cover of $\underset{1\le i\le n}{\oplus} G_i$ and $g = \underset{1\le i\le n}{\otimes} g_i : \underset{1\le i\le n}{\otimes} E_i \longrightarrow \underset{1\le i\le n}{\otimes} G'_i$ be topological group cover of $\underset{1\le i\le n}{\otimes} G'_i$, then:

$J = f \otimes g$ be topological group cover of $\underset{1\le i\le n}{\oplus} G_i \otimes \underset{1\le i\le n}{\otimes} G'_i$.

Proof:

Let $J = f \otimes g$ be topological group to $\underset{1\le i\le n}{\oplus} G_i \otimes \underset{1\le i\le n}{\otimes} G'_i$.

We prove that $J = f \otimes g$ is a topological group cover to $\underset{1\le i\le n}{\oplus} G_i \otimes \underset{1\le i\le n}{\otimes} G'_i$.

Let $J = f \otimes g$ is small and surjective topological group homomorphism. By theorem(4.1) f is toplogical group cover of $\underset{1\le i\le n}{\oplus} G_i$ and g is toplogical group cover of $\underset{1\le i\le n}{\otimes} G'_i$, thus Ker f is small topological subgroup of $\underset{1\le i\le n}{\oplus} q_i$ and Ker g is small topological subgroup of $\underset{1\le i\le n}{\otimes} E_i$, then:

Ker $f_i + M_i = q_i \Rightarrow M_i = q_i$

Ker $g_i + N_i = E_i \Rightarrow N_i = E_i$

Ker J = Ker$(f_i \otimes g_i)$

Ker$(f_i \otimes g_i) + (M_i \otimes N_i) = q_i \otimes E_i$

Thus :

$M_i \otimes N_i = q_i \otimes E_i,\ 1 \le i \le n$.

That is $J = f \otimes g$ is small and surjective topological group homomorphism of $\underset{1\le i\le n}{\oplus} G_i \otimes \underset{1\le i\le n}{\otimes} G'_i,\ 1 \le i \le n$.

Thus

$(\underset{1\le i\le n}{\oplus} M_i \otimes \underset{1\le i\le n}{\otimes} N_i) = \underset{1\le i\le n}{\oplus} q_i \otimes \underset{1\le i\le n}{\otimes} E_i$

U = P

Where $M = (\underset{1\le i\le n}{\oplus} M_i \otimes \underset{1\le i\le n}{\otimes} N_i)$.

J be topological homomorphism onto and small.

Theorem (4.3):

If $f = \underset{1\le i\le n}{\otimes} f_i : \underset{1\le i\le n}{\otimes} q_i \longrightarrow \underset{1\le i\le n}{\otimes} G_i$ is topological group cover to $\underset{1\le i\le n}{\otimes} G_i$ and $\underset{1\le i\le n}{\otimes} q_i$ be topological projective group.

$g = \underset{1\le i\le n}{\otimes} g_i : \underset{1\le i\le n}{\otimes} E_i \longrightarrow \underset{1\le i\le n}{\otimes} G'_i$ is represent topological group cover to $\underset{1\le i\le n}{\otimes} G'_i$ and $\underset{1\le i\le n}{\otimes} E_i$ be topological projective group, then $f \oplus g$ be topological group cover of topological projective group.

Proof:

We prove J be topological projective cover of $G = (\underset{1\le i\le n}{\otimes} G_i \oplus \underset{1\le i\le n}{\otimes} G'_i)$.

Thus to show that:

$q = \underset{1\le i\le n}{\otimes} q_i \oplus \underset{1\le i\le n}{\otimes} E_i$ be topological projective group. Since $\underset{1\le i\le n}{\otimes} q_i$ be topological projective group and $\underset{1\le i\le n}{\otimes} E_i$ be topological projective group.

The tensor product of two topological projective groups be topological projective group.

And J be topological homomorphism onto and small,

$\underset{1\le i\le n}{\otimes} f_i$ be topological homomorphism onto and small

$\underset{1\le i\le n}{\otimes} g_i$ be topological homomorphism onto and small

Thus

$\underset{1\le i\le n}{\otimes} \mathrm{Ker}\, f_i + \underset{1\le i\le n}{\otimes} M_i = \underset{1\le i\le n}{\otimes} q_i \Rightarrow \underset{1\le i\le n}{\otimes} M_i = \underset{1\le i\le n}{\otimes} q_i$

$\underset{1\le i\le n}{\otimes} \mathrm{Ker}\, f_i + \underset{1\le i\le n}{\otimes} N_i = \underset{1\le i\le n}{\otimes} E_i \Rightarrow \underset{1\le i\le n}{\otimes} N_i = \underset{1\le i\le n}{\otimes} E_i$

Thus :

$(\underset{1\le i\le n}{\otimes} \mathrm{Ker}\, f_i \oplus \underset{1\le i\le n}{\otimes} \mathrm{Ker}\, g_i) + (\underset{1\le i\le n}{\otimes} M_i \oplus \underset{1\le i\le n}{\otimes} N_i) = \underset{1\le i\le n}{\otimes} q_i \oplus \underset{1\le i\le n}{\otimes} E_i$.

Theorem (4.4):

If $f = \underset{1\le i\le n}{\oplus} f_i : \underset{1\le i\le n}{\oplus} q_i \longrightarrow \underset{1\le i\le n}{\oplus} G_i$ be a topological group cover of $\underset{1\le i\le n}{\oplus} G_i$ and $g = \underset{1\le i\le n}{\otimes} g_i : \underset{1\le i\le n}{\otimes} E_i \longrightarrow \underset{1\le i\le n}{\otimes} G'_i$ is topological group cover of $\underset{1\le i\le n}{\otimes} G'_i$, then:

$J = f \otimes g$ be a topological group cover of $\underset{1\le i\le n}{\oplus} G_i \otimes \underset{1\le i\le n}{\otimes} G'_i$.

Proof:

Let $J = f \otimes g$ is a topological group of $\underset{1\le i\le n}{\oplus} G_i \otimes \underset{1\le i\le n}{\otimes} G'_i$.

We prove that $J = f \otimes g$ is a topological group cover of $\underset{1\le i\le n}{\oplus} G_i \otimes \underset{1\le i\le n}{\otimes} G'_i$.

Let $J = f \otimes g$ be small and surjective topological group homomorphism. By theorem(4.1) f is toplogical group cover of $\underset{1\le i\le n}{\oplus} G_i$ and g is toplogical group cover of $\underset{1\le i\le n}{\otimes} G'_i$, thus Ker f is small topological subgroup of $\underset{1\le i\le n}{\oplus} q_i$ and Ker g is small topological subgroup of $\underset{1\le i\le n}{\otimes} E_i$, then:

$\mathrm{Ker}\, f_i + M_i = q_i \Rightarrow M_i = q_i$

Ker $g_i + N_i = E_i \Rightarrow N_i = E_i$

Ker J = Ker$(f_i \otimes g_i)$

Ker$(f_i \otimes g_i) + (M_i \otimes N_i) = q_i \otimes E_i$

Thus :

$M_i \otimes N_i = q_i \otimes E_i$, $1 \leq i \leq n$.

That is $J = f \otimes g$ is small and surjective topological group homomorphism of $\underset{1\leq i\leq n}{\oplus} G_i \otimes \underset{1\leq i\leq n}{\otimes} G'_i$, $1 \leq i \leq n$.

Thus

$(\underset{1\leq i\leq n}{\oplus} M_i \otimes \underset{1\leq i\leq n}{\otimes} N_i) = \underset{1\leq i\leq n}{\oplus} q_i \otimes \underset{1\leq i\leq n}{\otimes} E_i$

U = P

Where M = $(\underset{1\leq i\leq n}{\oplus} M_i \otimes \underset{1\leq i\leq n}{\otimes} N_i)$.

J be topological homomorphism onto and small.

Theorem (4.5):

Let q be a topological projective group, G be a topological subgroup and

$f_i : q_i \longrightarrow G_i$, $\forall$ I = 1,2,…, n; be a topological projective covers of topological groups G_i, then $f : \underset{1\leq i\leq n}{\otimes} q_i \longrightarrow \underset{1\leq i\leq n}{\otimes} G_i$, be a topological projective covers of

$\underset{1\leq i\leq n}{\otimes} G_i$.

Proof:

We prove f is a topological projective cover of $\underset{1\leq i\leq n}{\otimes} M_i$; thus to show

$\underset{1\leq i\leq n}{\otimes} q_i$ be a topological projective groups.

Let q = $\underset{\alpha\in L}{\otimes} q_\alpha$ be a topological projective group

for all A and B be a topological groups of R and f, $f\,'$ be a topological groups homomorphism and g be a topological group homomorphism and surjective. Thus q is a topological projective group.

($\Leftarrow$) since q a topological projective group, thus there exist a topological group homomorphism $f\,' : q \longrightarrow A$, such that:

$f_\alpha \circ I_\alpha = g \circ f\,'$

For each g element of G there is an application for the known Acer bus as it comes, the Ig application is a continuous application and has a continuous inverse Ig and therefore the left-hand carrier application is equal to the topology

We define $J_\alpha: q_\alpha \longrightarrow A$, such that:

$$J_\alpha(q_\alpha) = f\,'\circ I_\alpha(q_\alpha) = (g\circ f\,')\circ(I_\alpha(q_\alpha))$$
$$= (f_\alpha \circ u_\alpha)\circ I_\alpha(q_\alpha)$$
$$= f_\alpha \circ (u_\alpha \circ I_\alpha)(q_\alpha) = f_\alpha(q_\alpha)$$

Thus q_α be a topological projective group.
And to show f be topological group homomorphism small and surjective
Since $f_i : q_i \longrightarrow G_i$ be topological projective cover, thus evey q_i be topological projective and every f_i be represented topological group homomorphism small and surjective, Ker f_i be topological subgroup small of q_i. Thus every topological subgroup $M_i \leq q_i$

$$\underset{1\leq i\leq n}{\otimes} \text{Ker } f_i + \underset{1\leq i\leq n}{\otimes} M_i = \underset{1\leq i\leq n}{\otimes} q_i$$

$$\Rightarrow \quad \underset{1\leq i\leq n}{\otimes} M_i = \underset{1\leq i\leq n}{\otimes} q_i$$

We mean:

Ker $f_1 + M_1 = q_1 \Rightarrow M_1 = q_1$
Ker $f_2 + M_2 = q_2 \Rightarrow M_2 = q_2$
Ker $f_3 + M_3 = q_3 \Rightarrow M_3 = q_3$
:
Ker $f_n + M_n = q_n \Rightarrow M_n = q_n$

Thus :

$$\underset{1\leq i\leq n}{\otimes} \text{Ker } f_i + \underset{1\leq i\leq n}{\otimes} M_i = \underset{1\leq i\leq n}{\otimes} q_i \Rightarrow \underset{1\leq i\leq n}{\otimes} M_i = \underset{1\leq i\leq n}{\otimes} q_i$$

$\underset{1\leq i\leq n}{\otimes}$ Ker f_i be small topological subgroup of $\underset{1\leq i\leq n}{\otimes} q_i$

$f : \underset{1\leq i\leq n}{\otimes} q_i \longrightarrow \underset{1\leq i\leq n}{\otimes} G_i$ be topological group homomorphism small and surjective.As f be topological projective cover of $\underset{1\leq i\leq n}{\otimes} G_i$.

<u>Theorem (4.6)</u>:

If $f = \underset{1\leq i\leq n}{\oplus} f_i : \underset{1\leq i\leq n}{\oplus} q_i \longrightarrow \underset{1\leq i\leq n}{\oplus} G_i$ is represented topological group cover of $\underset{1\leq i\leq n}{\oplus} G_i$ and $g = \underset{1\leq i\leq n}{\otimes} g_i : \underset{1\leq i\leq n}{\otimes} E_i \longrightarrow \underset{1\leq i\leq n}{\otimes} G'_i$ be represented topological group cover of $\underset{1\leq i\leq n}{\otimes} G'_i$, then $J= g \oplus f$ be represented topological group cover of $\underset{1\leq i\leq n}{\otimes} G'_i \oplus \underset{1\leq i\leq n}{\oplus} G_i$.

<u>Proof:</u>

Let $J = g \otimes f$ be topological group of $\underset{1\leq i\leq n}{\otimes} G'_i \oplus \underset{1\leq i\leq n}{\oplus} G_i$.

We prove that $J = g\otimes f$ is a topological group cover of $\underset{1\leq i\leq n}{\otimes} G'_i \oplus \underset{1\leq i\leq n}{\oplus} G_i$.

$J= g\otimes f$ is surjective and small topological group homomorphism. By theorem (4.2) f is toplogical group cover of $\underset{1\leq i\leq n}{\oplus} G_i$ and g is toplogical group cover of $\underset{1\leq i\leq n}{\oplus} G'_i$., thus Ker f is small topological subgroup of $\underset{1\leq i\leq n}{\oplus} q_i$ and Ker g is small topological subgroup of $\underset{1\leq i\leq n}{\otimes} E_i$, then:

Ker $f_i + M_i = q_i \Rightarrow M_i = q_i$
Ker $g_i + N_i = E_i \Rightarrow N_i = E_i$

Ker J = Ker($g_i \otimes f_i$)
Ker($g_i \otimes f_i$) + ($N_i \otimes M_i$) = $E_i \otimes q_i$
Thus :
$N_i \otimes M_i = E_i \otimes q_i$, $1 \le i \le n$.
That is $J = g \otimes f$ is surjective and small topological group homomorphism of $\underset{1\le i\le n}{\oplus} G'_i \oplus \underset{1\le i\le n}{\oplus} G_i$, $1 \le i \le n$.
Thus
$(\underset{1\le i\le n}{\oplus} N_i \oplus \underset{1\le i\le n}{\otimes} M_i) = \underset{1\le i\le n}{\otimes} E_i \oplus \underset{1\le i\le n}{\oplus} q_i$
M = q
Where $M = (\underset{1\le i\le n}{\oplus} M_i \oplus \underset{1\le i\le n}{\otimes} N_i)$.
J be topological homomorphism surjective and small.

Theorem (4.7):

If $f = \underset{1\le i\le n}{\oplus} f_i : \underset{1\le i\le n}{\oplus} q_i \longrightarrow \underset{1\le i\le n}{\oplus} G_i$ be topological group cover of $\underset{1\le i\le n}{\oplus} G_i$ and $g = \underset{1\le i\le n}{\otimes} g_i : \underset{1\le i\le n}{\otimes} E_i \longrightarrow \underset{1\le i\le n}{\otimes} G'_i$ be topological group cover of $\underset{1\le i\le n}{\otimes} G'_i$,
then $J = g \otimes f$ is a topological group cover of $\underset{1\le i\le n}{\otimes} G'_i \otimes \underset{1\le i\le n}{\oplus} G_i$.

Proof:

Let $J = g \otimes f$ be topological group to $\underset{1\le i\le n}{\otimes} G'_i \otimes \underset{1\le i\le n}{\oplus} G_i$.

We prove that $J = g \otimes f$ is a topological group cover to $\underset{1\le i\le n}{\otimes} G'_i \otimes \underset{1\le i\le n}{\oplus} G_i$.

$J = g \otimes f$ is surjective and small topological group homomorphism. By theorem (4.2) f is toplogical group cover of $\underset{1\le i\le n}{\oplus} G_i$ and g is toplogical group cover of $\underset{1\le i\le n}{\oplus} G'_i$., thus Ker f is small topological subgroup of $\underset{1\le i\le n}{\oplus} q_i$ and Ker g is small topological subgroup of $\underset{1\le i\le n}{\otimes} E_i$, then:

Ker $f_i + M_i = q_i \Rightarrow M_i = q_i$
Ker $g_i + N_i = E_i \Rightarrow N_i = E_i$
Ker J = Ker($g_i \otimes f_i$)
Ker($g_i \otimes f_i$) + ($N_i \otimes M_i$) = $E_i \otimes q_i$
Thus :
$N_i \otimes M_i = E_i \otimes q_i$, $1 \le i \le n$.
That is $J = g \otimes f$ is surjective and small topological group homomorphism of $\underset{1\le i\le n}{\oplus} G'_i \otimes \underset{1\le i\le n}{\oplus} G_i$, $1 \le i \le n$.
Thus
$(\underset{1\le i\le n}{\oplus} N_i \oplus \underset{1\le i\le n}{\otimes} M_i) = \underset{1\le i\le n}{\oplus} E_i \otimes \underset{1\le i\le n}{\oplus} q_i$
M = q

Where $M = (\underset{1\le i\le n}{\oplus} N_i \oplus \underset{1\le i\le n}{\otimes} M_i)$.

J be topological homomorphism surjective and small.

References

1. boschi, E.;Essential subgroup of topological Groups, Comm. In Algebra28(10) 4941-4970(2000).
2. Bourbaki, N.;Elements of mathematics, General Topology, Chapter (1-4), Springer Verlage, Berline, Heidelberg, New York, Paris, London, Tokyo, 2nd Edition, (1989).
3. Dikranjan, D.; Compactness and Connectedness in Topological Groups, Topology and its Applications, 84,227-252 (1998).
4. Dikranjan, D., Recent Advances in Minimal Topological Groups, Topology and Applications; 85, 53-91 (1998).
5. Pontryajin, L.S.;Topological Groups, Gordon and Bread science publishers, Inc.New York 2nd edition(1990).
6. Prosmans, F.; Derived Projective Limits of Topological Albelian Grooups; Journal of Functional Analysis, 162, 135-177 (1999).
7. Thacenko, M. and Dikranjan, D.; Sequentially Complete Groups; Dimension and Minimality, J. Pure and Applied Algebra, 157 (215-239), (2001)

الأغطية الاسقاطية التبولوجية للزمر التبولوجية

م.م. محمد جبار حسين

الجامعة المستنصرية/ كلية التربية الاساسية– قسم الرياضيات

المستخلص

الهدف الرئيسي من هذا البحث هو حساب الضرب التنسوري لأغطية اسقاطية تبولوجية للزمر التبولوجية بعد أن وضحنا ليس كل زمرة تبولوجية تمتلك غطاء اسقاطي تبولوجي معتمدين بذلك على صفات الضرب التنسوري للزمر التبولوجية الجبرية وعلى ادخال الصفات التبولوجية التي تتناسب مع التركيبة الجبرية تبولوجياً من خلال تعريف الزمر التبولوجية ثم الاسقاط على تلك الزمر التبولوجية حيث أن الزمرة التبولوجية q تسمى زمرة اسقاطيةً تبولوجية اذا كان لكل تشاكل زمري تبولوجي شامل ولكل تشاكل زمري تبولوجي $B \longleftarrow f: q$ يوجد تشاكل تبولوجي $A \longrightarrow f': q$ بحيث يجعل المبيان الاتي ابدالياً

$$\begin{tikzcd} & q \arrow[dl, "f'"'] \arrow[d, "f"] \\ A & B \arrow[l] \end{tikzcd}$$

والاكثر من ذلك قد تم اعطاء نظريات جديدة عند نهاية البحث.