Algorithm method for generalized derivations of two dimensional associative algebras

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Abstract

The concept of generalized derivations of two dimensional associative algebras have been studied and their properties. The algorithm for finding generalized derivations have been given and its subspaces for the associative algebras [1]. The two dimensional associative algebras have 5 isomorphism classes of algebras. Each isomorphism classes have different table of multiplication.

The algorithm;

$$\sum_{t=1}^{n}(\alpha\gamma_{ij}^{t}d_{st}-\beta d_{ti}\gamma_{tj}^{s}-\gamma d_{tj}\gamma_{it}^{s})\ =0$$

In case of i, j, s =1, 2, 3, … n is an easier way to find the result for generalized derivations. In complex associative algebras A, generalized derivations have seven subspaces which are $Der_{(1,1,1)}A$, $Der_{(1,1,0)}A$, $Der_{(1,0,1)}A$, $Der_{(1,0,0)}A$, $Der_{(0,1,1)}A$, $Der_{(0,0,1)}A$ and $Der_{(0,1,0)}A$.

Keywords: two dimensional associative algebras, Algorithm method, generalized derivations.

$$\sum_{t=1}^{n}(\alpha\gamma_{ij}^{t}d_{st}-\beta d_{ti}\gamma_{tj}^{s}-\gamma d_{tj}\gamma_{it}^{s})\ =0$$

لكل i، j، s = 1، 2، 3، ...، n هو أسهل طريقة للعثور على نتيجة الاشتقاقات العامة. أما في الجبر الترابطي المعقد A، فان الاشتقاقات العامة لها سبعة اجزاء فرعية هي

$Der_{(1,1,1)}A$, $Der_{(1,1,0)}A$, $Der_{(1,0,1)}A$, $Der_{(1,0,0)}A$, $Der_{(0,1,1)}A$, $Der_{(0,0,1)}A$ and $Der_{(0,1,0)}A$.

1. INTRODUCTION

Associative algebra is one of the classical algebra that have extensively been studied and found to be related to other classical algebra like Lie and Jordan algebras. The theory of finite-dimensional associative algebra is one of the ancient area of the modern algebra. It originates primarily from works of Hamilton [2], who discovered the famous quaternions. The classifications of low-dimensional associative algebras were believed to have been first investigated by Pierce [3]. Most classification problems of finite dimensional associative algebras have been studied for unitary associative while the complete classification of associative algebras in general is still an open problem. Novotny and Hrivnák [4] studied (α, β, γ)-derivations of lie algebra and introduce the invariant functions corresponding to (α, β, γ)-derivations. Form that, all possible intersections of spaces having these derivations are probed and invariant functions corresponding to generalized derivations also introduced. The results on classification of 2 dimensional associative algebras and the basic of generalized derivations have used to achieve our goal on its generalized derivations of low-dimensional associative algebras. This research carries two objectives, to study of (α, β, γ)-generalized derivations in 2-dimensional associative algebras and to find out results of (α, β, γ)-generalized derivation in 2-dimensional associative algebras which could be useful in future research.

Hamilton [2] came up the idea of quaternions like $i^2 = j^2 = k^2 = ijk = -1$. He carved the multiplication formulae with his knife on the stone of Brougham Bridge (nowadays as Broombridge) in Dublin. Any quaternions may be written as $(a, b, c, d) = a + bi + cj + dk$. Unlike the complex numbers, the quaternions are an example of a non-commutative algebra for instance, $(0,1,0,0).(0,0,1,0) = (0,0,0,1)$ but $(0,0,1,0).(0,1,0,0) = (0,0,0,-1)$ Lewis [5]. In 1858, Cayley [6] published a journal about a memoir the theory of matrices. He also explains how to do basic arithmetic with matrices. The law of the addition of matrices is precisely similar to that for the addition of ordinary algebraical quantities. He obtain the remarkable theorem that any matrix whatever satisfies an algebrical equation of its own order, the coefficient of the highest power being unity, and those of the other powers function of the terms of the matrix, the last coefficient being in fact the determinant Cayley [6]. The classification of associative algebras is an old and often recurring problem. The first investigation into it was perhaps done by Pierce [3]. Many other publications related to the problem have appeared. He memoir linear associative algebras had formed the subject of several successive communication to the national academy of science, and is designed to be printed as part of the memoirs of that scientific but impecunious body, whenever means for publication become available. The classification of low-dimensional associative algebras was believed to have been first investigated also by Pierce [3].

A number of approaches were adopted to investigate the generalized derivation on Lie Algebras for instance Hartwig and his group [7] defined

L as a linear operator for the generalized derivation of a Lie algebra A satisfying property $A[x,y]=[Ax,\tau y]+[\sigma x,Ay]$, where the fixed elements are τ,σ for the *End L*. (σ,τ) - derivations have been named as derivations of L. a linear transformation A of a lie algebra L is said to be a generalized derivation if there exists $B\in Der\ L$ such that for all $x,y\ \in L$, the condition $A[x,y]=[Ax,y]+[x,By]$ holds, this generalization were considered by Bresar [8].

Novotnỳ and his colleague [4] presented a new form of a generalization of lie algebras derivations. They used this new version for the problems of low-dimensional cases in algebraic and geometric classification.

1.1 SOME BASIC DEFINATIONS

Definition 1.1.1 pierce [3] *An algebra over a field F is a vector space A over a field F with a bilinear binary operation $f: A\times A\to A$, satisfying the following conditions:*

$$f(x,y+z)=f(x,y)+f(x,z), f(x+y,z)=f(x,z)+f(y,z),$$
$$f(ax,z)=a[f(x,z)], and\ f(x,az)=a[f(x,z)].$$

For all $a\in F$ and all $x,y,z\ \in A$

As usual a bijective homomorphism is called isomorphism.

Definition 1.1.2 Pierce [3] *An associative algebra A over a field $\mathbb{F}$ is a vector space over a field F equipped with a bilinear map $f: A\times A\to A$, satisfying the associative law i.e.*

$$f(f(x,y),z)=f\big(x,f(y,z)\big) for\ all\ x,y,z\ \in A.$$

Further the notation $x\cdot y$ $(even\ just\ xy)$will be used for $f(x,y)$.

Example 1.1.3 *The vector space $M_n(F)of\ n\times n$ matrices with entries in field F is an associative (but not commutative) algebra over field F, of dimensionn^2.*

Example 1.1.4 *the category **Ass** of associative algebras. An associative algebra is a k-vector space A with a bilinear product $A\otimes A\to A$ satisfying:*

$a(bc)=(ab)c,$ *for all $a,b,c\ \in A$.*

Show that at this instant we do not adopt the existence of a unit 1$\in A$.

Example 1.1.5 *the category **Com** of commutative associative algebras in this case left modules, right modules and bimodules coincide. In addition to the axioms in **Ass** we require the commutative*

$$ab=ba \quad for\ all \quad a,b\ \in A$$

And for a module

$$ma=am,\ for\ all \quad m\ \in M, a\in A.$$

Definition 1.1.6 the derivation of associative algebra A is a linear transformation d: $A\times A$ *such that $d(xy)=d(x)y+xd(y)$ holds for all*

$x, y \in A$. The set of all derivations of an associative algebra A is denoted by Der(A).

1.2 REPRESENTATION OF ALGEBRAS

Representation theory was born in 1896 in this work of the representation theory by Curtis [9]. Frobenius created representation theory of finite groups Conrad [10]. A sub representation of V is a subspace $U \subset V$ which is invariant under all operator $\rho(a),\ a \in A$.

If V_1 and V_2 are two representation of A then the direct sum $V_1 \oplus V_2$has an obvious structure of a representation of A.

A nonzero representation V of A is said to be irreducible if its only sub representations are 0 and V itself, and indecomposable if it cannot be written as a direct sum of two nonzero sub representations. Obviously, irreducible implies indecomposable, but not vice versa.

Definition 1.2.1 *Bokut' [11] Associative algebra A is said to be nilpotent if there exists $s \in N$ such that $A^s = 0$*

Example 1.2.2 *Two dimension algebra with multiplication table $e_1 e_1 = e_2$ on a basis $\{e_1, e_2\}$ is nilpotent associative dialgebra.*

An ideal I of associative algebra A is said to be nilpotent if it is nilpotent as subalgebra of D. It is observed that the sum $I_1 + I_2 = \{z \in A | z = x_1 + x_2, x_1 \ \in I_1\ and\ x_2 \in I_2\}$ of two nilpotent ideals I_1, I_2 of A is nilpotent. Therefore there exists unique maximal nilpotent ideal of A called nilradical. The nilradical plays an important role in the classification problem of algebras.

Definition 1.2.3 *Let V_1, V_2 be two representations of an algebra A. A homomorphism (or intertwining operator) $\emptyset : V_1 \to V_2$* is a linear operator which *commutes with the action of A, i. e., $\emptyset(av) = a\emptyset(v) for\ any\ u \in V_1$. A homomorphism $\emptyset$ is said to be an isomorphism of representation if it is an isomorphism of vector spaces. The set (space) of all homeomorphisms of representations $V_1 \to V_2$ is denoted by $Hom_A(V_1 V_2)$.*

1.3 ON (α, β,γ) - GENERALIZED DERIVATION

In 2008, Novotny and Hrivnak studied the (α, β, γ) - derivations of Lie algebras and equivalent invariant functions. They considered finite dimensional Lie algebras. They also generalized the concept of Lie derivations via complex parameters and also obtain various Lie and Jordan operator algebras as well as two one –parametric sets of linear operators. Using these parametric sets, they introduced complex functions with a fundamental property. The important part of Lie theory is a theory of finite dimensional complex Lie algebras. A theory of finite dimensional complex Lie algebras has many uses to associates to other sides of mathematics and physics. They also generalized the concept of derivation of a lie algebra and introduced (α, β, γ) – derivations and show their pertinent properties. They also introduced two invariant functions corresponding to (α,β,γ) – derivations.

2. METHODS OF GENERALIZED DERIVATION

2.1 PROPERTIES OF GENERALIZED DERIVATIONS OF ASSOCIATIVE ALGEBRAS

Proposition 2.1.1 Let $f{:}\,A_1 \to A_2$be an isomorphism of complex associative algebras $(A_1\,,\ \cdot)$ and $(A_2\,,\ *)$ the mapping $\rho(D)(f){:}\, End\ A_1 \to End\ A_2$ defined by $\rho(D)(f) = fDf^{-1}$ is an isomorphism of vector spaces $Der_{(\alpha,\beta,\gamma)}A_1$ and $Der_{(\alpha,\beta,\gamma)}A_2$. [1].

Proof. Due to the isomorphism relation we have $x * y = f(f^{-1}(x) \cdot (f^{-1}(y))$. Let $d \in End\ A_1$ such that:

$$\alpha d(x \cdot y) = \beta d(x) \cdot y + \gamma x \cdot d(y)$$

For all $x, y \in A$. We show that $\rho(d) \in Der_{(\alpha,\beta,\gamma)}A_2$. Indeed,

$$\alpha\rho(d)(x * y) = \alpha\rho(d)f(f^{-1}(x) \cdot (f^{-1}(y))$$
$$= \alpha(f \circ d \circ f^{-1})(f \circ f^{-1}(x) \cdot f^{-1}(y))$$
$$= \alpha(f \circ d)(f^{-1}(x) \cdot f^{-1}(y))$$
$$= f(\alpha d(f^{-1}(x) \cdot f^{-1}(y))$$
$$= f(\beta d\big(f^{-1}(x)\big) \cdot f^{-1}(y) + \gamma f^{-1}(x) \cdot d\big(f^{-1}(y)\big))$$
$$= \beta f(d\big(f^{-1}(x)\big) \cdot f^{-1}(y) + \gamma f(f^{-1}(x) \cdot d\big(f^{-1}(y)\big))$$
$$= \beta\rho(d)(x) * (y) + \gamma x * \rho(d)(y)$$

Let us now classify the possible values of $(\alpha,\beta,\gamma) \in \mathbb{C}$for a linear transformation $d{:}\,A \to A$ to be (α,β,γ)- derivation of A.

Proposition 2.1.2 Let A be a complex associative algebra and $\alpha,\beta,\gamma \in \mathbb{C}$. Then for $Der_{(\alpha,\beta,\gamma)}A$, the values of α,β,γ are prorate as follows [1]:

$Der_{(1,1,1)}A = DerA$;

$Der_{(1,1,0)}A = \{d \in EndA | d(xy) = d(x)y\}$;

$Der_{(1,0,1)}A = \{d \in EndA | d(xy) = xd(y)\}$;

$Der_{(1,0,0)}A = \{d \in EndA | d(xy) = 0\}$;

$Der_{(0,1,1)}A = \{d \in EndA | d(x)y = -xd(y)\}$;

$Der_{(0,0,1)}A = \{d \in EndA | xd(y) = 0\}$;

$Der_{(0,1,0)}A = \{d \in EndA | d(x)y = 0\}$.

Proof. Let $\alpha \neq 0$ appling the operator d to the identity $\alpha(xy)z = \alpha x(zy)$ and using the fact that d is an (α,β,γ) - derivation of A, the system of equations have obtained [1]:

$$\beta(\beta/\alpha - 1) = 0$$

And

$$\gamma(\gamma/\alpha - 1) = 0$$

Considering case by we obtain the following possible values of (α,β,γ) : (α,α,α), $(\alpha,\alpha,0)$, $(\alpha,0,\alpha)$ and $(\alpha,0,0)$, with considering the fact that the

$$Der_{(\alpha,\beta,\gamma)}A = Der_{(1,\beta/\alpha,\gamma/\alpha)}A.$$

We get required $Der_{(\alpha,\beta,\gamma)}A$ in $Der_{(1,1,1)}A$, $Der_{(1,1,0)}A$, $Der_{(1,0,1)}A$, $Der_{(1,0,0)}A$. Let us now consider the case α=0. Again a case by case consideration shows that for the probable values of (α,β,γ) the following may occur:

$(0,\beta,\beta),(0,\beta,0);\ \beta \neq 0$

And

$(0,0,\gamma);\ \beta = 0$

Then thanks to

$Der_{(0,\beta,\gamma)}A = Der_{(0,1,\gamma/\beta)}A;\ \beta \neq 0$

And

$Der_{(0,0,\gamma)}A = Der_{(0,0,1)}A;\ \gamma \neq 0$

Along with $\gamma = 0$ we obtain the required result

Remark 2.1.3 *for any $\alpha,\beta,\gamma \in \mathbb{C}$ the dimension of the vector space $Der_{(\alpha,\beta,\gamma)}A$ is an isomorphism invariant of associative algebras [1].*

2.2 AN EXPLICIT DESCRIPTION IN LOW-DIMENSIONAL CASES

To describe the generalized derivations of 2-dimensional complex associative algebras have first an algorithm to find generalized derivations. Let $\{e_1,e_2,e_3,\dots\dots\dots e_n\}$ be basis of an n-dimensional associative algebras A. then

$$e_ie_j = \sum_{k=1}^{n} \gamma_{ij}^k e_k,\ i,j = 1,2\dots\dots n$$

The coefficient $\{\gamma_{ij}^k\} \in \mathbb{C}^{n^3}$ of the above linear combinations is called the structure constant of A on the basis $\{e_1,e_2,e_3,\dots\dots\dots e_n\}$. An element d of $Der_{(\alpha,\beta,\gamma)}A$ becoming a linear transformation of the vector space A which is expressed in matrix form $[d_{ij}]_{i,j} = 1,2,3,\dots.,n$, i.e.

$$d(e_i) = \sum_{j=1}^{n} d_{ji}e_j, \quad i = 1,2,3,\dots,n$$

According to the definition of generalized derivation we have

$$\alpha d(e_ie_j) = \beta d(e_i)e_j + \gamma e_i d(e_j), \qquad i,j = 1,2,3,\dots n$$

Therefore the algorithm

$$\sum_{t=1}^{n}(\alpha\gamma_{ij}^t\, d_{st} - \beta d_{ti}\gamma_{tj}^s - \gamma d_{tj}\gamma_{it}^s) = 0$$

In case of i, j, s = 1, 2, 3, …, n. the system includes n^2+3 variables which are $\{\alpha,\beta,\gamma,d_{ij}\}$ where i,j =1,2,3,…n. the solution to the system gives the description of generalized derivations of A on the basis $\{e_1,e_2,e_3,\dots\dots\dots e_n\}$. We implement the algorithm to two-dimentional complex associative algebras along with a classification.

2.3 TWO- DIMENSIONAL ASSOCIATIVE ALGEBRAS

Theorem 2.3.1 *Rakhimov [12] let A be a 2-dimentional complex associative algebras. Then it is isomorphic to one of the following pairwise non-isomorphic associative algebras:*

$As_2^1: e_1e_1=e_2;$

$As_2^2: e_1e_1=e_1, e_1e_2=e_2;$

$As_2^3: e_1e_1=e_1, e_2e_1=e_2;$

$As_2^4: e_1e_1=e_1, e_2e_2=e_2;$

$As_2^5: e_1e_1=e_1, e_1e_2=e_2, e_2e_1=e_2.$

2.4 APPLICATION TO ASSOCIATIVE ALGEBRAS IN TWO DIMENTIONAL CASE

Let us consider the first of associative algebra in 2-dimentional case, As_2^1. The set $\{e_1,e_2\}$ is a basis of As_2^1 and from the table of multiplication, we have $As_2^1: e_1e_1=e_2$.

The structure constants can find by using

$$e_ie_j=\sum_{k=1}^{n}\gamma_{ij}^k e_k, i,j=1,2\dots\dots n$$

When n is the dimension of the associative algebra. From the table of multiplication, is

$$e_1e_1=\sum_{k=1}^{2}\gamma_{11}^k e_k=\gamma_{11}^1e_1+\gamma_{11}^2e_2=e_2$$

The structure constants are $\gamma_{11}^1=0$ and $\gamma_{11}^2=1$. Then using the algorithm to find $Der_{(\alpha,\beta,\gamma)}A$ have

$$\sum_{t=1}^{2}(\alpha\gamma_{ij}^t d_{st}-\beta d_{ti}\gamma_{tj}^s-\gamma d_{tj}\gamma_{it}^s)=0$$

For i ,j ,s =1,2.

For the case $Der_{(1,1,1)}A$, just substitute $\alpha=1,\beta=1$and $\gamma=1$ into the algorithm then find

$$\sum_{t=1}^{2}(\gamma_{ij}^t d_{st}-d_{ti}\gamma_{tj}^s-d_{tj}\gamma_{it}^s)=0$$

The implementation of the algorithm to two dimensional associative algebras for i, j, s =1 , 2 to find derivation case results as follow:
At i=1 ,j=1, s=1 ; Only $\gamma_{11}^{2}=1$, then $\gamma_{11}^{2}d_{12}=0$, then $d_{12}=0$
At i=1 , j=1, s=2; $\gamma_{11}^{2}d_{22}-d_{11}\gamma_{11}^{2}-d_{11}\gamma_{11}^{2}=0$, then $d_{22}=2d_{11}$
At i=1 , j=2, s=1; $\gamma_{12}^{1}d_{11}-d_{11}\gamma_{12}^{1}-d_{12}\gamma_{11}^{1}+\gamma_{12}^{2}d_{12}-d_{21}\gamma_{22}^{1}-d_{22}\gamma_{12}^{1}=0$
At i=1 , j=2, s=2; $-d_{12}\gamma_{11}^{2}=0$, then $d_{12}=0$
At i=2 , j=1, s=1; $\gamma_{21}^{1}d_{11}-d_{12}\gamma_{11}^{1}-d_{11}\gamma_{21}^{1}+\gamma_{21}^{2}d_{12}-d_{22}\gamma_{21}^{1}-d_{21}\gamma_{22}^{1}=0$
At i=2 , j=1, s=2; $-d_{12}\gamma_{11}^{2}=0$, then $d_{12}=0$
At i=2 , j=2, s=1; $\gamma_{22}^{1}d_{11}-d_{12}\gamma_{12}^{1}-d_{12}\gamma_{21}^{1}+\gamma_{22}^{2}d_{12}-d_{22}\gamma_{22}^{1}-d_{22}\gamma_{22}^{1}=0$
At i=2 , j=2, s=2; $\gamma_{22}^{1}d_{21}-d_{12}\gamma_{12}^{2}-d_{12}\gamma_{21}^{2}+\gamma_{22}^{2}d_{22}-d_{22}\gamma_{22}^{2}-d_{22}\gamma_{22}^{2}=0$
Finally can get $Der_{(1,1,1)}A$ for As_{2}^{1} is $\begin{pmatrix} d_{11} & 0 \\ d_{21} & 2d_{11} \end{pmatrix}$
By using the algorithm to find another case, just change the number of Derivation-(α,β,γ) to find $Der_{(1,1,0)}A$, $Der_{(1,0,1)}A$, $Der_{(1,0,0)}A$, $Der_{(0,1,1)}A$, $Der_{(0,0,1)}A$ and $Der_{(0,1,0)}A$ which are listed in the Table 3.1.
For the second class can be continued again of associative algebras in 2-dimensional case, As_{2}^{2} by using the same way and use also the same way to find for all another cases, we just change the number of dimension in the algorithm only.

3. Results

3.1 (α,β,γ)- Derivations of two dimentional associative al-gebras

Table 3.1 the description in(α,β,γ)- Derivations of two dimensional associative algebra

Isomorphism class	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}A$	Dim.
As_{2}^{1}	(1,1,1)	$\begin{pmatrix} d_{11} & 0 \\ d_{21} & d_{11} \end{pmatrix}$	2
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 \\ d_{21} & d_{11} \end{pmatrix}$	2
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 \\ d_{21} & d_{11} \end{pmatrix}$	2
	(1,0,0)	$\begin{pmatrix} d_{11} & 0 \\ d_{21} & 0 \end{pmatrix}$	2

	(0,1,1)	$\begin{pmatrix} 0 & 0 \\ d_{21} & d_{22} \end{pmatrix}$	2
	(0,0,1)	$\begin{pmatrix} 0 & 0 \\ d_{21} & d_{22} \end{pmatrix}$	2
As_2^2	(0,1,0)	$\begin{pmatrix} 0 & 0 \\ d_{21} & d_{22} \end{pmatrix}$	2
	(1,1,1)	$\begin{pmatrix} 0 & 0 \\ d_{21} & d_{22} \end{pmatrix}$	2
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 \\ 0 & d_{11} \end{pmatrix}$	1
	(1,0,1)	$\begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix}$	4

Isomorphism class	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}A$	Dim.
As_2^2	(1,0,0)	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	0
	(0,1,1)	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	0
	(0,10,1)	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	0
	(0,1,0)	$\begin{pmatrix} 0 & 0 \\ d_{21} & d_{22} \end{pmatrix}$	2
As_2^3	(1,1,1)	$\begin{pmatrix} 0 & 0 \\ d_{21} & d_{22} \end{pmatrix}$	2
	(1,1,0)	$\begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix}$	4
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 \\ 0 & d_{11} \end{pmatrix}$	1
	(1,0,0)	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	0
	(0,1,1)	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	0
	(0,0,1)	$\begin{pmatrix} 0 & 0 \\ d_{21} & d_{22} \end{pmatrix}$	2
	(0,1,0)	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	0
As_2^4	(1,1,1)	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	0
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 \\ 0 & d_{22} \end{pmatrix}$	2

	(1,0,1)	$\begin{pmatrix} d_{11} & 0 \\ 0 & d_{22} \end{pmatrix}$	2
	(1,0,0)	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	0
	(0,1,1)	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	0
	(0,0,1)	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	0
	(0,1,0)	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	0

Isomorphism class	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}A$	Dim.
As_2^5	(1,1,1)	$\begin{pmatrix} 0 & 0 \\ 0 & d_{22} \end{pmatrix}$	1
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 \\ d_{21} & d_{11} \end{pmatrix}$	2
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 \\ d_{21} & d_{11} \end{pmatrix}$	2
	(1,0,0)	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	0
	(0,1,1)	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	0
	(0,0,1)	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	0
	(0,1,0)	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	0

Conclusion

Some basic definitions and concepts of algebras have been discussed briefly such algebras, associative algebras, derivations of associative algebras and also the definition generalized derivations of associative algebras. The generalized derivations of associative algebras in low-dimensional cases have been studied actually in two dimensional only. The algorithm have been applied to find the generalized derivations of associative algebras based on the observation from the scheme of Novotn`y, P. & Hrivn´ak, J. [4].

The derivation scheme has been used mainly as discussed in the method. The algorithm has been proved as used in the calculation and applied it to find the (α,β,γ) –derivations of associative algebras. The results of generalized derivations of associative algebras have been presented in the tables.

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طريقة الخوارزمية للاشتقاقات العامة من الجبر الارتباطية (التجميعية) ثنائية الابعاد

الخلاصة

تم دراسة مفهوم الاشتقاقات العامة من الجبر الارتباطي (التجميعي) ثنائي الأبعاد وخصائصها. وقد أعطيت خوارزمية للعثور على الاشتقاقات العامة واجزائها الفرعية للجبر الارتباطي (التجميعي)[1]. الجبر الارتباطية (التجميعية) ثنائية الأبعاد لها 5 فئات متماثلة من الجبر. لكل فئة من الفئات المتماثلة لديها جدول ضرب مختلف.