On The Analytical and Numerical Solutions of Heat Equation

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Abstract

In this paper, we give first the general meaning and importance of partial differential equations, with stating some known types of the second-order linear partial differential. Furthermore, we show the classification to the associated types of initial-boundary conditions.
Next, we will use separation of variables method to find a formula for the analytical solution of the heat equation in one-dimensional space. Finally, we compute the numerical solution of an initial-boundary problem of the heat equation in one-dimensional space, using Euler Explicit and Implicit finite different methods.

1- Introduction and Background

Many physical and engineering problems, mathematically, can be modeled in the form of partial differential equations. Partial differential equation describes practically a useful phenomenon such as transport-chemistry problems of the direction-diffusion-reaction type. Also, such types of PDE play an important role in the modeling of the atmosphere, ground water and surface water.
Most physical phenomena in the domain of fluid dynamics, electricity, mechanic, optics, and heat flow can be modeled in general as linear, nonlinear partial differential equations.
Next, we will give a definition to partial differential equations, and we will state some known types of second order partial differential equations. Furthermore, we will give a classification to the types of initial-boundary conditions.

Definition (1 – 1) [2]

Differential equation is an equation involving a function and some of its derivatives from which this function is to be determined. Differential equation which involves functions defending only on one variable is called ordinary differential equation (O D E), while the differential equation which involves a function that depends on several variables is called partial differential equation (P D E), see [1,2,3,4,5].

A few well – known second order linear partial differential equations

The general form of second order linear partial differential equations in n-dimensional-space with constant coefficients takes the forms:

$$\sum_{i,j=1}^{n} a_{ij} \frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{i=1}^{n} b_i \frac{\partial^2 u}{\partial x_i \partial t} + \sum_{i=1}^{n} c_i \frac{\partial^2 u}{\partial t \partial x_i} + c \frac{\partial u}{\partial t} + d \frac{\partial^2 u}{\partial t^2} \quad \dots\dots\dots(1-1)$$

For notational simplicity we may refer to the partial derivatives as follows:

$$u_t = \frac{\partial u}{\partial t} , \; u_x = \frac{\partial u}{\partial x} , \quad u_{xx} = \frac{\partial^2 u}{\partial x^2} \; \dots$$

Examples:

(1) Heat equation in one dimension: $u_t = k u_{xx}$
(2) Heat equation in two dimensions: $u_t = u_{xx} + u_{yy}$
(3) Laplace's equation in polar coordinates $u_{rr} + \frac{1}{r} u_r + \frac{1}{r2} u_{\theta\theta} = 0$
(4) Ware equation in three dimensions: $u_{tt} = u_{xx} + u_{yy} + u_{zz} = 0$
(5) Telegraph equation: $u_{tt} = u_{xx} + \alpha \, u_t + B$

Types of Initial and boundary condition:

In order to obtain a unique solution to equation (1 – 1) we need to impose certain conditions associated with equation (1 – 1). Equation (1 – 1) with certain conditions may be further classified as initial valve or boundary value problems. In the first case, at least one of the independent variables is defined in an open region. And in the second case the region is closed for all independent variables and conditions are specified at all boundaries.

In general initial – boundary conditions for PDE are divided into three types, [2].

(1) Dirichlet conditions:

The value of the dependent variable u is given at fixed values of the independent variables, that is $u(x,t) = g(x,t) on \, \partial D,$ where g are given function.

(2) Neumann conditions:

In this case the derivative of the dependent variable is given as a constant or as a function, of the independent variables, that is

$$\frac{\partial u(x,t)}{\partial t} = G(x,t) \; on \; \partial D$$

(3) Cauchy conditions:

A parabolic partial differential equation that has a combination of both Dirichlet and Neumann conditions on the boundary of D, ∂ D, is said to have a Cauchy conditions.

2- Analytical Solutions of Heat Equation

The heat equation with n space variables is the following

$\frac{\partial u(x,t)}{\partial t} = C^2\Delta u(x,t) + h(x,t), \ (t > 0, x \in R^n)$

Heat equation is a special case of a class of equations called parabolic type equation. They model heat conduction, etc.

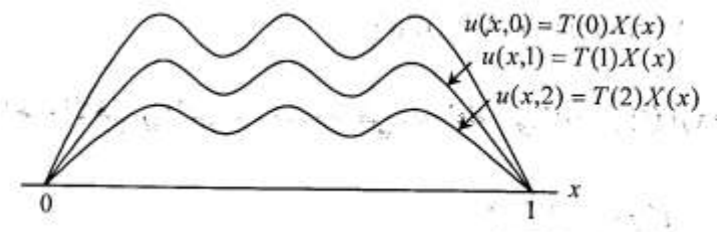
In this section, we will study the solution of the case when $n = 1$, using separation of variables method.

Overview of separation of variables:

Separation of variables looks for simple type solution of the partial differential equation of the from

$$u(x,t) = X(x)T(t),$$

where X (x) is some function of *x* and *T(t)* is some function of t . The solutions are simple because any temperature $u(x,t)$ of this form will retain its basic shape for different values of time t . see the following figure (Graph of $X(x)T(t)$ for different value of *t*).



The general idea is that it is possible to find an infinite number of these solutions to the partial differential equation (which, at the same time, also satisfy the boundary conditions). These simple functions

$u_n(x,t) = X_n(x)T_n(t)$ (Called fundamental solution) ,

are the building blocks of our problem, and the solution *u(x, t)* we are looking for, is found by adding the simple fundamental solutions $X_n(x)T_n(t)$ in such a way that the resulting sum

$\sum_{n=1}^{\infty} A_n X_n(x)T_n(t)$, and satisfies the initial conditions.

In as much as this sum still satisfies the partial differential equation and the boundary conditions, we now have the solution to our problem.

Using separation of variables to solve heat equation:

STEP 1 (finding elementary solution to the partial differential equation)

We wish to find the function $u(x,t)$ that satisfies the following four conditions:

Partial differential equation

$$u_t = \alpha^2 u_{xx} \quad , \quad 0 < x < 1 \quad 0 < t < \infty$$

Boundary conditions

$$\begin{cases} u(0,t) = 0 \\ u(1,t) = 0 \end{cases}$$

where $0 < t < \infty$

Primary requirement

$u(x,0) = \emptyset\,(x)\,, \qquad 0 \le x \le 1$

To begin , we look for solutions of the form $u\,(x,t) = X\,(x)\,T\,(t)$ by substituting X (x) T (t) into the partial differential equation and solving for X (x) T (t) .

Making this substitution gives

$$X\,(x)\,T'(t) = \alpha^2\,X''(x)\,T\,(t)$$

Now, here is the part that makes all this work: if we divide each side of this

$$\frac{T'(t)}{\alpha^2 T\,(t)} = \frac{X''(x)}{X\,(x)}$$

And obtain what is called separated variables, that is, the left side of the equation depends only on T and the right side, only on X. As much as x and t are independent of each other, each side must be a fixed constant (say k); we can write

$$\frac{T'(t)}{\alpha^2 T\,(t)} = \frac{X''(x)}{X\,(x)} = k$$

or

$$T' - k\alpha^2 T = 0$$
$$X'' - kX = 0$$

So now we can solve each of these to OPEs, multiply them together to get a solution to the partial differential equation (note that we have essentially changed a second – order partial differential equation to two ODEs). However, we now make or important observation, namely, that we want the separation constant, *k* to be negative (or else the *T(t)* factor doesn't go to Zero as t <∞) . With this in mind, it is general practice to rename $k = -\lambda^2$, where is non-zero $-\lambda^2$ is guaranteed to be negative), calling our separation constant by its new mean, we can now write the two ODEs as follows:

$$T' + \alpha^2\lambda^2 T = 0$$
$$X'' + \lambda^2 X = 0$$

We will now solve these equations. Both equations are standard type ODEs and have solution

$$T\,(t) = Ae^{\lambda^2\alpha^2 t} \quad \text{(A an arbitrary constant)}$$

$$X(x) = A\sin(\lambda x) + B\cos(\lambda x) \qquad \text{(A, B arbitrary)}$$

And hence all functions.

$$u(x,t) = \exp(\ -\lambda^2\alpha^2 t\ )[A\ sin\ (\lambda x) +\ B\ Cos\ (\lambda x)]$$

(with A, B and λ arbitrary) will satisfy the partial differential equation.

$$u_t\ = \alpha^2\ u_{xx}\quad ,\quad 0 < x < 1 \quad 0 < t < \infty$$

At this point, we have an infinite number of functions that satisfy the partial differential equation.

STEP 2 (finding solutions to the partial and the Boundary conditions).

We are now to the point where we have many solutions to the partial differential equation but not all of them satisfy the BCs or the priming requirement. The next step is to choose a certain subset of our current top of solutions.

$$u(x,t) = \exp(\ -\lambda^2\ \alpha^2 t\ )[A\ sin\ (\lambda x) +\ B\ Cos\ (\lambda x)]$$

That satisfies the boundary conditions

u (0, t) =0 , *u (1, t) =0*

To do this, we substitute our solutions into these BCs, getting

$u\ (0,t)\ = Be^{-\lambda^2\alpha^2 t}\ = 0 \quad \Rightarrow B = 0$

$u\ (1,t)\ = Ae^{-\lambda^2\alpha^2 t}\ sin\,\lambda\ = 0 \quad \Rightarrow sin\lambda\ = 0$

This last B.Cs. restricts the separation constant from being any non-zero number, it must be a root of the equation sin λ=0 .

In other words, in order that $u\ (1,t) = 0$ it is necessary to pick

$\lambda = \ \pm\pi, \pm 2\pi \pm\ 3\pi$ ...

$\lambda_n = \pm n\pi\ \ n = 1, 2, 3$ ...

3- Numerical Solutions of Heat equation

If we use finite difference operator to approximate the partial derivatives, space or time, then the continuous formulation of the PDE is transformed to a discrete formulation and such process will introduce or error called the truncation error. This, applying finite difference approximation to the one and two dimensional PDE, will establish one and two dimensional network girls.

In this section, we will use the finite difference methods (explicit, implicit) to find the numerical solution of an initial-boundary problem of heat equation.

A finite deference scheme, to a given PDE, is said to be convergence if the numerical solution tends to the exact solution as the discretization of the space and time steps tend to zero.

Finite difference methods for finding the numerical solution of one dimension linear second order parabolic PDE:

The general from of second order linear parabolic partial differential equation, in one dimensional, takes the from:

$$\frac{\partial u}{\partial t} = a\ (x, t)\ \frac{\partial^2 u}{\partial x^2} + b\ (x, t)\ \frac{\partial u}{\partial x} + c\ (x, t)\ u, \qquad (3\text{-}1)$$

where $a(x,t) > 0$ $\forall (x,t) \in R \, x \, [0,T]$, and $c(x,t) \le 0$.

If we put a(x, t) =λ where λ> 0,

and $b(x,t) = c(x,t) = 0 \quad \forall x \in R, \quad \forall t \in (0,T)$

Then we will consider the one-dimensional PDE (Heat Equation) which has the form

$$\frac{\partial u(x,t)}{\partial t} = \lambda \frac{\partial^2 u(x,t)}{\partial x^2} \qquad (3\text{-}2)$$

In order to use finite difference replacement of equation (3-2), the region to be examined is covered by a rectilinear grid with sides parallel to x – axes and t – axes , with h and k being the grid spacing in the x and t directions respectively and the grid points (x_i , t_n) are given by x_i = ih , t_n = nK where i , n are integers such that nK ≤ T and i = n = 0 at the origin .

Also, the functions satisfying the difference and differential equations at the grid points X_i = ih, t_n = nK are denoted by U_i^n and u_i^n respectively.

If we assume that $u(x,t)$ is continuous with sufficient continuous derivatives, then we can use Taylor's expansion to compute the first and second space derivatives of u at (x_i, t_n) and thus we obtain different types of approximation (see [2]]) which are:

(1) Forward difference formula

$$\frac{\partial u}{\partial x}\Big|_i^n = \frac{1}{h}(u_{i+1}^n - u_i^n) + T_i^n, \qquad (3\text{-}3)$$

where

$$T_i^n = -\frac{h}{2}\frac{\partial^2 u(x,t)}{\partial x^2} = 0(h) \quad x_i \le \theta \le x_i + h$$

Is a local truncation error. Note that we say that this error is of order h, E = 0 (h), means │E │≤ gh where g is appositive real number.

(2) Backward difference formula

$$\frac{\partial u}{\partial x}\Big|_i^n = \frac{1}{h}(u_i^n - u_{i-1}^n) + T_i^n \qquad (3\text{-}4)$$

where

$$T_i^n = \frac{h}{2}\frac{\partial^2 u(\theta + t_n)}{\partial x^2} = 0(h) \quad x_i \le \theta \le x_i + h$$

(3) Centered difference formula

From equation (2 – 3) and (2 – 4) we get

$$\frac{\partial u}{\partial x}\Big|_i^n = \frac{1}{2h}(u_{i+1}^n - u_{i-1}^n) + T_i^n \qquad (3\text{-}5)$$

where $T_i^n = -\frac{h^2}{6}\frac{\partial^3 u(x,t_n)}{\partial x^3} = 0(h^2) \qquad x_i - h \le \theta \le x_i + h$

The finite difference approximation of second space derivative of u at (x_i, t_n) is given by

$$\frac{\partial^2 u}{\partial x^2}\Big|_i^n = \frac{1}{h^2}[u_{i+1}^n - 2u_i^n + u_{i-1}^n] + T_i^n \qquad (3\text{-}6)$$

where $T_i^n = -\frac{h^2}{12}\frac{\partial^4 u(x,t_n)}{\partial x^4} = 0(h^2)$ $\quad x_i - h \le \theta \le x_i + h$

we will consider the numerical solution of the PPDE, equation (3-2), by using three types, equation (3-2), by using three types of finite difference method which are explicit method, implicit method.

Euler Explicit method

An explicit finite difference method involves only grid point at the advanced time level.

In order to drive a formula for this method we will use Taylor expansion to $u(x, t + k)$ about u (x, t) and thus we get.

$$u(x, t + k) = (1 + k\frac{\partial}{\partial t} + \frac{1}{2}k^2\frac{\partial^2}{\partial t^2} \quad + \ldots)$$

$$= exp(k\frac{\partial}{\partial t})u(x, t) \qquad (3\text{-}7)$$

From equation (3-3), $\frac{\partial u}{\partial t} = \lambda\frac{\partial^2 u}{\partial x^2}$, we have

$$u(x, t + k) = \exp\left(\lambda k\frac{\partial^2}{\partial x^2}\right)u(x, t)$$

If we put x = ih and t = nk and u (x, t) $= u_i^n$, then we gave

$$u_i^{n+1} = \exp(\lambda k\frac{\partial^2}{\partial x^2})u_i^n \qquad (3\text{-}8)$$

we approximate the second space derivative, in the last equation, by using center difference formula, we have.

$$u_i^{n+1} = [1 + \lambda r s_x^2]u_i^n + O(k^2), \ \ where\ r = \frac{k}{h^2}$$

Using U_i^n as an approximation value to u_i^n , we get

$$U_i^{n+1} = (1 - 2\lambda r)U_i^n + \lambda r(U_{i+1}^n + U_{i-1}^n) \qquad (3\text{-}9)$$

$\forall_i$=1, 2, 3…m-1 and m=$\frac{1}{h}$.

The system of linear equation (3 – 9) can be put in matrix from as:

$$U_i^{n+1} = (1 + \lambda r H)U^n + z\ , \qquad (3-10)$$

Where $U^n = [U_i^n, U_2^n, \ldots, U_{m-1}^n]^T$ is the vector of numerical solution of equation of (3-2) at time t = nk, and I is the identity matrix of order (m-1), H is the tri-diagonal $(m-1)x(m-1)$ matrix given by

$$H = \begin{bmatrix} -2 & 1 & & & 0 \\ 1 & -2 & 1 & & \\ & & & \ddots & \\ 0 & & & 1 & -2 \end{bmatrix}$$

And Z is vector of boundary condition of order (m-1) x 1, its elements are

$Z = \lambda r[U_0, 0, \ldots 0, U_m]$

Euler Implicit method

Although the use of explicit method computationally easy to use the time step k in necessarily to be very small and mast satisfy the stability condition.

$$\mathbf{0} \leq \boldsymbol{\lambda r} \leq \frac{1}{2}$$

Therefore we need to use implicit method which is stable for infinite values of r.

From equation (3 – 8) we have

$\exp(-\lambda k\frac{\partial^2}{\partial x^2})u_i^{n+1} = u_i^n$

If we approximate the second derivative in the last equation by using center difference formula, and U_i^n as an approximate value to u_i^n , then we get

$[1 - \lambda r s_x^2]U_i^{n+1} = U_i^n \qquad i = 1,2,..m-1, \quad m = \frac{1}{h}$

Thus $(1 + 2\lambda r)U_i^{n+1} - \lambda r(U_{i+1}^{n+1} + U_{i-1}^{n+1}) = U_1^n$ (3 – 11)

The last formula is called the backward difference scheme or (implicit method). The system of equation in (3 – 11) can be put in matrix from as:

$(1 - \lambda r H)U^{n+1} = U^n + z$ (3 – 12)

Where the two matrices I, H and the two vectors and are given in equation (3 – 10).

4- Numerical Results and Discussion

In this section, we will briefly discuss the numerical results of using: different types of finite difference method to find the numerical solution of Heat equation in one dimensional-space, these F.D. methods are:

1-Explicit method.

2-Implicit method

Moreover, we will point out some conclusions based on our analytical and numerical results.

Numerical solution of a given problem (P1):

Consider the following initial-boundary problem of heat equation (3-2) in one dimsensional-space:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \qquad 0 \leq x \leq 1,\ t > 0 \quad, \lambda > 0$$

where the initial and boundary conditions are given as

$u(0,t) = u(1,t) = 0 , \qquad t > 0$

$u(x,0) = \sin \pi x$

by the results of section 2, The exact solution of the above problem has the form $u(x, t) = e^{-\lambda\pi 2t} \sin \pi x$ (4-1)

To find the numerical solution of the above problem, we will using Euler explicit and Euler implicit finite difference methods which were illustrated in section 3.

We will consider the problem with different value of the grid space in x – Direction h, gird time k, and λ.

It is clear that the exact solution, u (x, t), will approach zero as λ and t increase. Thus it is difficult to compute the solution (4 – 1).

In tables (1) and (2) we present the numerical solution (with different values of h , k and λ) for the problem (p_1) .

It is clear that as λ and k increase, with decreasing h, the salability condition $0 < \lambda\, r \leq \frac{1}{2}$

for the explicit method is violated and thus we have an inaccurate numerical solution.

However, this is not the case for implicit methods where such methods are classified as unconditionally stable.

In table (3) we present the absolute error bounds, with different values of h , k and λ, for the above problem (p_1), compering the exact solutions with the numerical solutions . Also we conclude that as λ and k increase with decreasing h, the error bounds increase for all the above methods, which is an indication for a stability problem.

All the numerical solutions have been computed with using Matlab codes. Next, we state the Matlab codes of Euler Explicit and implicit methods.

Euler explicit Method Code

```
numx = 41;      %number of grid points in x
numt = 2000;    %number of time steps to be iterated over
dx = 1/(numx-1) ; t (1)=0;
u = zeros ([numx, numt]); uex = zeros ([numx, numt]);
x = 0: dx: 1; %vector of x values,   to be used for plotting
%t =0: dt: dt*(numt);
y=10*sin (pi*x);    %B=zeros ([numt-1, numt-1]); u (:, 1) =y';
u (1, : )=0  ;u(numx, : ) = 0;
for j = 1: numt-1
 dt=min((dx^2)/2,dx^(1/10 0)/norm(u ( : , j) ) ) ;
r=dt/ (dx^2);
E (j) =r;
t (j + l)=t(j)+dt;
for i=2: numx-1
u(i,j+l)=(l-2*r)*u(i,j)+r*u(i+l,j)+r*u(i-l,j);
end
end
u; x;
```

Euler Implicit Method Code

```
numx = 21;      %number of grid points in x
numt =120;    %number of time steps to be iterated over
dx = 1/ (numx-1);
u = zeros ([numx-2, numt]);
x = dx: dx: l-dx;       %vector of x values,   to be used for plotting
%t =0: dt: dt*(numt);
y=20*sin (pi*x);
t(1)=0; u(:,l)=y'
for j =1:numt-1
dt=min((dx^2)/2,dx6(1/100)/norm(u(:,j)));
r=dt/ (dx^2);
E (j) =r;
t(j+1)=t(j)+dt;
 u(l, j+l) = (l-2*r)*u(l,j)+r*u(2,j);
 for i=2:numx-3
u(i,j+1) = (l-2*r)*u(i,j)+r*u (i + 1,j)+r*u (i-1,j);
end
u(numx-2,j+l)=(l-2*r)*u(numx-2,j)+r*u(numx-3,j); end
A=zeros ([numx-2, numx-2]);         for i=l: numx-2
For 1=1: numx-2 if i==j
A (i, j) =l+2*r;
if j< numx-2 A(i,j+1)=-r; end
if j> 1
 A(i,j-1)=-r;
end
end
end
end
uimp (:, 1)=u (:,1); for j =2:numt
z=uimp (:,j-1);  x0=u(: , j ) ; uim=A\x0; uimp (:,j)=uim';
end
```

Table (1): Comparison between the numerical solutions of problem (PI), with λ=l, using Euler explicit and implicit method

Step Length and λ	Time t	U(0.25,t)		Time t	u(0.5,t)	
		Euler explicit	Euler implicit		Euler explicit	Euler implicit
h=0.25 k=0.0104 λ= *1*	0.0104	0.6410	0.6396	0.0104	0.9062	0.9045
λr=0.1664	0.0208	0.5769	0.5766	0.0208	0.8152	0.8149
h=0.25 k=0.0052 λ= *1*	0.0104	0.6391	0.6387	0.0104	0.9036	0.9028

λr=0.083	0.0208	0.5764	0.5762	0.0208	0.8148	0.8146
h=0.125 k=0.0104 λ= *1*	0.0104	0.6416	0.6409	0.0104	0.9104	0.9098
λr=0.6656	0.0208	0.5807	0.5788	0.0208	0.8198	0.8187
Exact solution λ=*1*	0.0104	0.6381		0.0104	0.9024	
	0.0208	0.5758		0.0208	0.8144	

Table (2): Comparison between the numerical solutions of problem (P1), with λ, =5, using Euler explicit & implicit methods

Step Length and λ	Time t	u(0.25,t)		Time t	u(0.5,t)	
		Euler explicit	Euler implicit		Euler explicit	Euler implicit
h=0.25 k=0.0104 $\lambda=5$	0.0104	0.4297	0.4244	0.0104	0.6056	0.6040
λ r=0.8300	0.0208	0.2617	0.2551	0.0208	0.3691	0.3603
h=0.25 k=0.0052 $\lambda=5$	0.0104	0.4261	0.4239	0.0104	0.6010	0.5995
λr=0.4160	0.0208	0.2587	0.2545	0.0208	0.3609	0.3599
h=0.125 k=0.0104 $\lambda=5$ λr=3.328	0.0104	0.4322	0.4288	0.0104	0.6087	0.6008
	0.0208	0.2720	0.2597	0.0208	0.3702	0.3641
Exact solution λ=*5*	0.0104	0.4232		0.0104	0.5985	
	0.0208	0.2533		0.0208	0.3582	

Table (3)

Comparison between the errors in numerical results of problem (P1)

Step Length and λ	Time t	Error $=\sum_{i=1}^{2} \mid u(x_i,t)-U(x_i,t)\mid$, where U is approximate value of u, $x_1 = 0.25$, $x_2 = 0.5$	
		Euler explicit	Euler implicit
h=0.25 k=0.0104 λ=1	0.0104	*0.0029*	0.0015
	0.0208	0.0011	0.1812
h=0.25 k=0.0052 λ=*1*	0.0104	0.001	0.0006
	0.0208	0.0006	0.0004
h=0.125 k=0.0104 λ=*1*	0.0104	*0.0035*	*0.0651*
	0.0208	0.0049	0.003
h=0.25 k=0.0104 λ=*5*	0.0104	*0.0065*	0.0012
	0.0208	0.0084	0.0018
h=0.25 k=0.0052 λ=5	0.0104	0.0029	0.1013
	0.0208	0.0054	0.0012
h=0.125 k=0.0104 λ=*5*	0.0104	0.009	0.0056
	0.0208	0.0187	0.0064

\Conclusion and Future work :

From the previous study of the theoretical and numerical solution of Heat equation in one dimensional-space, we can conclude the following:

1- We can easily use separation of variables method to compute the analytical solution of heat equation with homogeneous boundary conditions. For other problems of heat equation with complicated boundary conditions, we may need to use another technique.

2- Since explicit finite difference method has a small region of stability so, we cannot always use it especially with small mesh grids thus we have to use implicit methods which have a bigger region of stability and unconditionally stable and our numerical results for problem P_1 confirm this claim.

REFERENCES

[1] Peter Giesl, "partial differential equation". Lecture notes, university of Sussex, 2010.

[2] Maan A. Resheed, "On numerical solutions of second order parabolic partial differential equations using finite difference methods". MSc. thesis, university of Baghdad, 2005.

[3] Stanley J. Farlow, "partial differential equation for scientists and Engineers", 1993.

[4] Erich Miersemann, , “ partial differential equations," Department of mathematics, Leipzig university 2012.

[5] Water A. Strauss, "partial differential equations," John Wiley & Sons, USA, 2007.

حول الحلول التحليلية والعددية لمعادلة الحرارة

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الملخص

في هذا البحث, نعطي اولا فكرة عامة عن المعادلات التفاضلية الجزئية واهميتها مع ذكر بعض من المعادلات التفاضلية الجزئية المعروفة الخطية ذات الرتبية الثانية. اكثر من هذا ,سوف نوضح تصنيف لانواع الشروط الابتدائية–الحدودية المرتبطة مع المعادلات التفاضلية الجزئية. وبعد هذا, سوف نستخدم طريقة فصل المتغيرات لايجاد صيغة للحل التحليلي لمعادلة الحرارة المعرفة في الفضاءاحادي البعد مع شروط ابتدائية–حدودية. اخيرا ,سوف نقوم بحساب الحل العددي لمساله قيمية ابتدائية–حدودية لمعادلة الحرارة في الفضاء احادي البعد باستخدام طريقتي اويلر الواضحة والضمنية والمعتمدتان على الفروقات المنتهية.