On Contra –rgα-Continuous Functions Types And Almost Contra –rgα-Continuous Function

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Abstract :

The main aim of this paper is to study some new classes of contra – continuous funtions which are (contra –rgα-continuous function ,contra –rgα-continuous function and contra–rgα**-continuous function) in topological space and introduce some of their properties and relation among them .Also we define and study other type of contra- rgα-continuous function called almost contra –rgα- continuous function .*

١-*Introduction :*

In ١٩٩٦, Dontchev [١] introduced the notion of contra continuity and obtained some results concerning compactness. S-closedness and strong S-closedness. Recently anew weaker from of this class of function.
called contra-semi continuous functions is introduced And investigated by Dontchev and Noiri[٣] .Also ,in [٤] (Jafari .S And Noiri.T in ٢٠٠١) introduced and studied new definition of contra-continuous is said to be contra – α-continuous function .
The concepts (rgα-closed , rgα-open and rgα-continuous functions types) was introduced by (vadivel .A and vairamanickam.K in [١٠] , [١١]).

This paper is to introduce and investigate anew class of functions namely (contra- rgα-continuous ,contra- rgα*-continuous and contra- rgα**- continuous) functions and given the relation between these functions. Also we obtain several basic properties and study the notion almost contra - rgα-continuous function .
Throughtout the paper X ,Y and Z denote the topological space (X ,τ)
(Y ,σ) and (Z,μ) respectively and on which no seperation axioms are assumed unless otherwise explicitely stated .For any subset *A* of a space (X τ ,) the closure of *A*. interior of *A* and the complement of *A* are denoted by *cL (A)* and A^c or *X-A* respectively.

٢-*Preliminaries:*

Some definition and basic concepts have been given in this section .

Definition (٢- ١): [٥], [٩]

Subset *A* of a space (X,τ) is said to be

١- ***α-open set*** if $A\subseteq int(cL(int(A))$ ***and α-closed set*** if $cL(int(cL(A)) \subseteq A$
٢- ***regular open se***t if $A=int(cL(A))$ and ***regular closed set*** if
$A=cL(int(A))$
The intersection of all α-closed subset of (X ,τ) containing *A i*s called the ***α-closure*** of A and is denoted by ***αcL(A***) .

<u>Definition(٢- ٢): [١٠]</u>

A subset ***A*** of a space (X, τ) is said to be
١- ***regular α-open se***t (briefly rα-open) if there is a regular open set *U* such that $U\subset A\subset \alpha cL(U)$.The family of all regular α-open sets of X is denoted by $\boldsymbol{R_{\alpha}}$ ***٠(X).***
٢- ***regular generalized α-closed set*** (briefly, rgα-closed) If $\alpha cL(A) \subset U$ whenever $A\subset U$ and *U* is regular α- open in X. The set of all rgα-closed set in *X* denoted by ***RG αC(X) .***
٣-***regular generalized α-open*** (briefly, rgα-open) in *X* If A^c is rgα-closed set in *X* .The family of all rgα-open sets in *X* denoted by ***RG αO(X).***

<u>Definition(٢- ٣): [٦]</u>

A subset A of a topological space(X, τ) is said to be ***Regular generalized closed*** (briefly, rg-closed)if $cL(A) \subseteq U$ whenever $A\subseteq U$ and *U* is regular open set in X.

<u>Remark(٢- ٤): [٩]</u>

In any topological space (X, τ)
١-Every open (resp.closed) set in X is rgα-open (resp.rgα-closed) set.
٢- Every rgα-closed (resp.rgα-open) set in X is rg-closed (resp.rg-open)set.

The converse of the above Remark need not be true ,as seen from the following example.

<u>Example(٢- ١):</u>

Let X=*{a,b,c,d,e}* with topology
τ ={X,ø ,{a} , {d} , {e} ,{a,d},{a,e} ,{d,e} ,{a,d,e}} . Then the set *A={b}* is a rgα-closed set but is not closed set in X ,and the set *B={a,b}* is rg-closed set but not rgα-closed set in X.also A^c=,{a,c,d} is a rgα-open set but is not open in *X*,and B^c*={c,d}* is a rg –open set but is not rgα-open.

<u>Remark(٢- ٥): [٥] , [١٠]</u>

For any topological space *(*X, τ).
١- Every open (resp.closed) set in α-open (resp. α-closed) set in X.
٢- Every α-open (resp. α-closed) set is rgα-open (resp.rgα-closed) set in X.
٣-The union of two rgα-closed subset of X is also rgα-closed subset of X. But the intersection of two rgα-closed set in X is generally not rgα-closed set in X .

But, the converse of Remark (٢- ٥) need not be true, as seen from the following example

Example(٢-٢):

Let X=*{a,b,c }* with topology *τ ={*X,ø *,{a} ,{a,c }}.*

Then the set *A={a,b}* is α-open set but is not open set in X and A^c*={c}* is a α-closed set in X but is not closed set in X.

Also ,the set *B={b,c }*is a rgα-open set but is not α-open set in X and B^c*={a}* is a rgα-closed set in X but is not α- closed .

Definition(٢-٦): [٥] ,[٨], [١١]

A function f: (X ,τ) →(Y, σ) is said to be

١-***continuous*** if the inverse image of every open (closed) set in Y is an open (closed)set in X.

٢-***α-continuous*** if the inverse image of every open (closed) set in Y is an α-open (α-closed) set in X.

٣- ***rgα-continuous*** if the inverse image of every open (closed)set in Y is an rgα-open (rgα-closed)set in X.

٤- ***rgα-irresolute*** if the inverse image of every rgα-open (rgα-closed) set in Y is an rgα-open(rgα-closed) set in X.

٥-***Strongly rgα-continuous*** if the inverse image of every rgα-open(rgα-closed) set in Y is an open (closed)set in X.

٦-***almost continuous*** if the inverse image of every regular open set in Yis an open set in X.

Definition(٢-٧): [٦] :

A space (X,τ) is said to be a ***T^*½-space*** if every rg-closed set in X is closed.

Definition(٢-٨): [٢],[٨]

A space(X,τ)is said to be ***locally indiscrete*** if every open subset of X is closed.

Definition(٢-٩): [١] , [٤]

A function f: (X,τ) →(Y,σ) *i*s said to be

١-***contra-continuou***s if f^{-1}(*V*) is closed in X for every open set *V* in Y.

٢- ***contra- α -continuous*** if f^{-1}(*V*) is α – closed set in X for every open set *V* in Y.

Remark(٢-١٠): [٤]

Every contra-continuous function is contra- α –continuous, but not conversely as see from the following example.

Example(٢-٣):

Consider X=*{a,b,c }* with topology *τ ={*X,ø *,{a} ,{a,c }}.*

let the function f: (X, τ) →(X, τ) be defined by f(a)=b,

f (b)=c and f(c)=a. It is clear that f is contra- α –continuous but is not contra-continuous function .since the set {a} is an open set in X.

But f^{-1}(a)=c is α – closed but is not closed set in (X, τ) .

٣-On Contra-rgα-Continuous Functions Types:

In this section we introduce and study new class of contra-continuous function namely (contra- rgα-continuous , contra- rgα*-continuous and contra- rgα**-

continuous)function and discussion the relations between them. Also, we give some proposition and results about the composition of these functions.

Definition(٣- ١):

A function $f: (X, \tau) \rightarrow (Y,\sigma)$ is said to be ***contra-rgα-continuous*** if $f^{-1}(A)$ is rgα-open set in (X,τ) for every closed set A of (Y,σ).

Proposition(٣- ٢):

A function $f: (X,\tau) \rightarrow (Y,\sigma)$ is a contra- rgα- continuous if and only if for each open set A in (Y,σ) ,then $f^{-1}(A)$is rgα-closed set in (X,τ).

Proof:

Assume that f is contra- rgα-continuous function . Let A be any open set in (Y,σ).then A^c is a closed set in Y.since f is contra- rgα- continuous. Thus $f^{-1}(A^c)$is an rgα-open set in X . But $f^{-1}(A^c)=X-f^{-1}(A)=(f^{-1}(A))^c$ and so $f^{-1}(A)$is a rgα-closed set in (X,τ).

Conversely . let A is closed set in (Y,σ). Then A^c is an open set in (Y,σ).By assumption we get $f^{-1}(A^c)$ is rgα-closed set in (X,τ).

But $f^{-1}(A^c)=X-f^{-1}(A)=(f^{-1}(A))^c$. Thus, $f^{-1}(A)$ is an rgα-open set in (X,τ). Hence a function $f: (X,\tau) \rightarrow (Y,\sigma)$ is contra- rgα- continuous

Proposition(٣- ٣):

Every contra continuous (resp.contra- α- continuous) function is contra- rgα-continuous.

Proof:

It follows from the definition (٢-٦) and the fact that every closed (resp. α-closed) set is rgα-closed set.

Converse of the above proposition need not be true, as seen from the following example.

Example(٣- ١):

Let X=Y={a,b,c } with topology $\tau=\{X ,\emptyset ,\{a\} \}$and $\sigma=\{Y ,\emptyset ,\{a\} , \{b \},\{a,b \}\}$.Define $f: (X,\tau) \rightarrow (Y,\sigma)$ by f(a)=a , f(b)=b and f(c)=c. Then this function is contra- rgα- continuous but is not contra –continuous(resp.contra - α- continuous). Since the set A={a} is an open set in (Y,σ) but $f^{-1}(A)= f^{-1}(\{a\})= \{a\}$is not closed (resp. α-closed) in (X,τ)

Here, we define and introduce some new notions and results, that shall needed in this work.

Definition(٣- ٤):

A space $(X,\tau)$is said to be ***rgα-locall indiscrete*** if every rgα-open set in X is closed.

Example(٣- ٢): Let X= {a,b } with topology$\tau=\{X ,\emptyset ,\{a\}, \{b \}\}$ it easy seen that a space(X,τ) is a rgα-locally indiscrete.

Notes, the following proposition and results given the necessarily condition in order to every rgα-closed (resp. rgα-open) set is a closed (resp.open) set.

Proposition(٣- ٥): If a space(X,τ) is a *T^*½-space*.Then every rgα-closed set in X is a closed set .

Proof:

It follows from the Remark (٢-٤)- step-٢- and definition(٢-٧).

Corollary(٣- ٦): If a space(X,τ) is a *T^*½-space*,Then every rgα-open set in X is an open set.

Proof:

Let A be an rgα-open set in (X,τ).Then A^c is a rgα-closed set in (X,τ), Since X is *a T^*½-space*.Then A^c is a closed set in X .Hence *A* is an open set in X.

Proposition(٣- ٧):

If a function f: (X,τ) →(Y,σ) is rgα- continuous and (X, τ) is rgα-locally indiscrete space, then f is contra-continuous.

Proof:

Let *A* be an open set of (Y,σ).thus $f^{-١}$(*A*) is a rgα-open set in (X, τ).since (X, τ) is a rgα-locally indiscrete. then $f^{-١}$(*A*) is closed set in (X, τ).Hence f is contra –continuous.

Corollary(٣- ٨):

If a function f: f: (X, τ) →(Y ,σ) is rgα- continuous and (X, τ) is rgα-locally indiscrete space, then f is contra-α-continuous.

Proof:

It follows from the proposition(٣-٧) and the fact that every contra-continuous function is contra-α-continuous.

Proposition(٣- ٩): If f: (X, τ) →(Y ,σ) is a rgα- continuous function and let X is rgα-locally indiscrete. then f is contra- rgα- continuous.

Proof:

Let *A* is an open set in Y. Thus $f^{-١}$(*A*) is a rgα- open set in X .Since X is rgα-locally indiscrete and by using definition (٣-٤) we get $f^{-١}$(*A*) is a closed set in X and by Remark (٢-٤) we have $f^{-١}$(*A*) is a rgα-closed set in X. Hence ,f is contra-rgα- continuous.

Similarly, we prove the following proposition_

Corollary(٣- ١٠): If f: (X, τ) →(Y ,σ) is a continuous function and let X is locally indiscrete. Then f is contra- rgα- continuous.

Now, the following proposition and corollary given the condition to make the converse of a proposition (٣- ٣) true.

Proposition(٣- ١١):

If a function f: (X, τ) →(Y, σ) is contra-rgα- continuous and X is *T^*½ space* ,then f is contra-continuous.

Proof:

Let *A* be an open set in Y. Thus, $f^{-١}$(*A*) is a rgα- closed set in X .since X is a *T^*½ space* and by using proposition (٣-٥) we get $f^{-١}$(*A*) is a closed set in X .Hence f is contra –continuous function.

Corollary(٣-١٢):

If f: (X, τ) →(Y,σ) is a contra-rgα- continuous function and X is *T^*½ space* ,then f is contra-α- continuous.

Proof:

It follows from the proposition (٣-١١) and the fact that every contra-continuous function is contra-α- continuous.

Next, other type of contra-rgα- continuous is called contra-rgα*- continuous are given by following definition.

Definition(٣-١٣) :

A function f: (X, τ) →(Y ,σ) is said to be ***contra-rgα*- continuous*** if $f^{-1}(A)$ is an open set in(X, τ) for every rgα-closed set *A* of (Y,σ).

Proposition(٣-١٤):

A function f: (X, τ) →(Y ,σ) is contra-rgα*- continuous if and only if for each rgα-open set *A* of (Y,σ),then $f^{-1}(A)$ is a closed set in (X, τ) .

Proof:

Assume that f is contra-rgα*- continuous. Let *A* be any rgα-open set in (Y,σ). Then A^c is a rgα-closed set in (Y,σ). Since f is contra-rgα*- continuous function. Thus, $f^{-1}(A^c)$ is an open set in (X, τ).

But $f^{-1}(A^c)=X-f^{-1}(A) = (f^{-1}(A))^c$ then , $f^{-1}(A)$ is closed set in (X, τ).

Conversely, let *A* is a rgα-closed set in(Y,σ). Then $A^c$is an rgα-open set in(Y,σ). By assumption we get $f^{-1}(A^c)$ is a closed set in(X, τ).But $f^{-1}(A^c)=X-f^{-1}(A) = (f^{1}(A))^c$ Hence $f^{-1}(A)$ is an open set in (X, τ) .Therefore f is contra-rgα*- continuous function .

Proposition(٣-١٥):

Every contra- rgα*- continuous function is contra-continuous.

Proof:

Let f: (X τ) →(Y ,σ) be a contra-rgα*- continuous function and let *A* be an open set in Y .By Remark(٢-٤) we get *A* is rgα-open set in Y. Thus $f^{-1}(A)$ is *A* closed set in X.Hence a function f is a contra –continuous.

Corollary(٣-١٦):

If function f: (X, τ) →(Y ,σ) is a contra-rgα*- continuous
Then f is

i- contra –α- continuous.

ii-contra-rgα-continuous.

Proof:

(i) It follows from proposition(٣-١٥) and the fact that every contra- continuous function is contra –α- continuous.

(ii) It follows from proposition(٣-١٥) and proposition(٣-٣).

The converse of proposition (٣-١٥) and corollary (٣-١٦) may not be true. To illustrate that consider the following example.

Example(٣-٣):

Let X= {a ,b, c,d } with topology
τ={X ,ø ,{a}, {b},{a,b},{b,c},{a,b,c},{a,b,d}}.
Define f: (X ,τ) →(X τ) by f(c)=a , f(b)=d , f(a)=c and f(d)=b . It is observe that f is contra –continuous (contra –α- continuous and contra-rgα- continuous) function.
But is not contra-rgα^*- continuous, since the set {c,d }is rgα^*-open set in (X,τ) but f^{-1}({c,d })={a,b } is not closed set in (X,τ).

Here, in the following proposition and result given the condition to make the converse of a proposition(٣-١٥) and corollary(٣-١٦) true.

Proposition(٣-١٧):

If f: (X ,τ)→(Y,σ) is contra - continuous function and let Y is $T^*½$ *space*
Then f is contra-rgα^*- continuous .

Proof:

Let *A* be an rgα-open set in Y. since Y is $T^*½$*space* and by using corollary(٣-٦) we get *A* is an open set in Y.Since f is a contra-continuous function.Thus, $f^{-1}(A^c)$
is a closed set in (X,τ).Hence, a function f is a contra-rgα^*- continuous.

Similarly ,we prove the following corollary.

Corollary(٣-١٨):

Let f: (Xτ) →(Y,σ)be any function and let X,Y are both $T^*½$ space . Then f is contra-rgα^*- continuous function if

i) f is contra –α- continuous function.*(*

(ii) f is contra-rgα- continuous function.

Proposition(٣-١٩):

If f: (X, τ) →(Y ,σ) is a strongly - rgα- continuous function and let X is a locally indiscrete. Then f is contra-rgα^*- continuous.

Proof:

Let *A* be an rgα-open set in Y. Thus $f^{-1}(A)$ is an open set in X .Since X is a locally indiscrete. Then $f^{-1}(A)$ is a closed set in X .Hence, f is contra – rgα^*-continuous function.

Proposition(٣-٢٠):

If f: (X,τ) →(Y,σ) is a rgα- continuous function .let Y is a $T^*½$ *space* and X is rgα- locally indiscrete, Then f is contra – rgα^*- continuous function.

Proof:

Let *A* be an rgα-open set in Y. Since Y is $T^*½$ *space* then *A* is an open set in Y and also since f is a rgα- continuous function. Thus, $f^{-1}(A)$ is a rgα-open set in X. But X is a rgα- locally indiscrete. Then $f^{-1}(A)$ is a closed set in X. Hence, f is contra – rgα^*- continuous function.

The proof of the following corollary it is easy. Thus it is omitted.

Corollary(٣-٢١):

If f: (X,τ) →(Y,σ) is continuous function and X is a locally indiscre, Y is a $T^{*}½$ *space* . Then f is a contra – rgα*- continuous function.

The following definition given other type of contra – rgα- continuous function is said to be contra – rgα**- continuous function.

Definition(٣-٢٢):

A function f: (X,τ) →(Y,σ) is said to be ***contra-rgα**- continuous*** if $f^{-1}(A)$ is a rgα- open set in(X, ,τ) for every rgα-closed set in (Y, σ).

Proposition(٣-٢٣):

A function f: (X, ,τ) →(Y,σ) is a contra-rgα**- continuous if and only if for each rgα- open set *A* in (Y,σ), then $f^{-1}(A)$ is a rgα-closed set in (X ,τ)

Proof:

Suppose that f is contra-rgα**-continuous function and let *A* be any rgα- open set in (Y,σ). Then A^c is a rgα-closed set in (Y,σ).Thus, $f^{-1}(A^c)$ is a rgα- open set in(X, ,τ). But $f^{-1}(A^c)=X-f^{-1}(A) = (f^{1}(A))^c$ .Hence $f^{-1}(A)$ is a rgα-closed set in (X, ,τ).

Conversely, let *A* is a rgα-closed set in (Y,σ). then A^c is an rgα-open set in(Y,σ). By assumption $f^{-1}(A^c)$ is a rgα-closed set in (X, ,τ).

Since $f^{-1}(A^c)=X-f^{-1}(A) = (f^{1}(A))^c$ .Then $f^{-1}(A)$ is a rgα-open set in (X, τ).Hence f is contra-rgα**-continuous function.

Proposition(٣-٢٤):

Every contra-rgα**-continuous function is contra-rgα-continuous.

Proof:

Let f: (X,τ) →(Y,σ) be a contra-rgα**- continuous function and let *A* is a closed set in Y and by using Remark(٢-٤) we get *A* is a rgα-closed set inY.Since f is a contra-rgα**-continuous. Then $f^{-1}(A)$ is a rgα-open set in X. Hence ,a function f is a contra-rgα-continuous.

***But , the converse of above proposition need not be true as seen from the following example*.**

Example(٣-٤): Let X= {a ,b, c,d } with topology τ={X ,ø ,{a}, {b },{a,b},{b,c},{a,b,c},{a,b,d}}.Let the function f: (X,τ) →(X,τ) be defined by f(a)= f(c)=a , f(b)=d , and f(d)=b . Then this function is contra – rgα- continuous But is not contra-rgα**- continuous function, since the set {a,b }is rgα-closed set in X but f^{-1}({a,b})={a,c,d } is not rgα-open set in X.

Now, the following proposition show the relation between contra-rgα*-continuous function and contra-rgα**- continuous function.

Proposition(٣-٢٥):

Every contra-rgα*-continuous function is contra-rgα**-continuous.

Proof:

Let f: (X ,τ) →(Y,σ) be a contra-rgα^*- continuous function and let *A* be an rgα-open set in Y. Thus $f^{-١}(A)$ is a closed set in X. and by using Remark(٢-٤) we get $f^{-١}(A)$ is a rgα-closed set in X.. Hence ,a function f is a contra-rgα^{**}-continuous.

The converse of above proposition need not be true, as seen from the following example.

Example(٣-٥): Let X= {a ,b, c, } with topology τ={x ,ø ,{a}, {a, c }}. Define f: (Xτ) →(X,τ) by f(a)=a , f(b)=b, f(c)=c. Then it is easy see that f is contra-rgα^{**}-continuous function .but not contra-rgα^*-continuous. Since the {a,c }is an rgα-open set in X but $f^{١}$({a,c}) ={a,c } is rgα-closed set in X but is not closed set.

Next, in the following proposition addition the necessarily condition in order to the converse of proposition(٣-٢٤), (٣-٢٥) true

Proposition(٣-٢٦):

If f: (X,τ) →(Y,σ) is a contra- rgα- continuous function and let Y be a $T^*½$ *space*. Then f is contra – rgα^{**}- continuous function.

Proof:

Let *A* be a rgα-closed set in Y. Since Y is $T^*½$ *space* and by proposition(٣-٥) we get *A* is a closed set in Y and also since f is a contra- rgα- continuous. Thus, $f^{-١}(A)$ is a rgα-open set in X. Hence, f is contra – rgα^{**}- continuous.

Similarly ,we prove the following corollary.

Proposition(٣-٢٧):

If f: (X,τ) →(Y,σ) is a contra- rgα^{**}- continuous function and let X be a $T^*½$ *space*. Then f is contra – rgα^*- continuous.

Remark(٣-٢٨):

The following example shows that contra-continuous function and contra-α-continuous function are independent of a contra- rgα^{**}- continuous function.

Example(٣-٦): Let X=Y= {a ,b, c, } with topologies τ={X ,ø ,{a},{b }, {a, b}} and σ = {y, ,ø ,{a}}. Define f: (X,τ) →(Y,σ) by f(a)=b , f(b)=c and f(c)=a. It is easy seen that f is a contra-continuous (contra-α-continuous) function, but is not contra- rgα^{**}- continuous function.

Since the set{b }is a rgα-open set in (Y,σ) but $f^{-١}$({b })={a } is not rgα-closed set in (X,τ).

Example(٣-٧): Let X=Y= {a ,b, c, d } with topologies τ={X ,ø ,{a},{b }, {a, b},{a, b, c}} and σ = {Y, ,ø ,{a},{b},{a, b}{b, c},{a, b, c},{a, b, d}} Define f: (X,τ) →(Y,σ) by f(a)= f(b)=a, f(c)=c and f(d)=d. It is observe that f is contra-rgα^{**}-continuous function but is not contra- continuous (resp. contra -α-continuous)function.

Since the set{a }is an open set in (Y,σ) but $f^{-١}$({a})={a ,b } is not closed(resp. α-closed) set in (X,τ).

The following propositions results given the condition to make Remark(٣-٢٨) true:

Proposition(٣-٢٩):

If f: (X,τ) →(Y,σ) is a contra- rgα**- continuous function, and let X be a T^{*}½ *space*. Then f is contra –continuous.

Proof:

Let *A* be an open set in Y .By Remark(٢-٤) we get *A* is a rgα-open set in Y. Since f is contra- rgα**- continuous. Then ,f$^{-١}$(*A*) is a rgα-closed set in X and also since X is a T^{*}½ *space.* Thus, f$^{-١}$(*A*) is closed set in X. Hence, a function f is a contra –continuous.

Corollary(٣-٣٠):

If f: (X,τ)→(Y,σ) is a contra- rgα**- continuous function and let X is a T^{*}½ *space* . Then f is a contra –α- continuous.

Proof:

It follows from proposition(٣-٢٩) and the fact that every contra-continuous function is contra-α-continuous.

Proposition(٣-٣١):

If f: (X,τ) →(Y,σ) is a contra- continuous function and Y is T^{*}½ *space.* Then f is contra – rgα**-continuous.

Proof:

Let *A* be an rgα-open set in Y. Since Y is T^{*}½ *space* and by using Corollary (٣-٦) we get *A* is an open set in Y and also since f is contra- continuous function. Then, f$^{-١}$(*A*) is a closed set in X and by
Remark(٢-٤) we obtain, f$^{-١}$(*A*) is closed set in X.Hence, a function f is a contra-rgα**-continuous.

The proof of the following corollary it is easy. Thus it is omitted.

Corollary(٣-٣٢):

If f: (X,τ) →(Y,σ) is a contra- α- continuous function and Y is T^{*}½*space* . Then f is contra– rgα** -continuous.

Proposition(٣-٣٣):

If f: (X,τ) →(Y,σ) is a rgα-irresolute function and let X be a rgα-locally indiscrete. Then f is

(i) contra- rgα*-continuous.

(ii) contra- rgα**-continuous.

Proof:

(i) Let *A* be an rgα-open set in Y.Thus f$^{-١}$(*A*) is a rgα-open set in X.Since X is a rgα-locally indiscrete.
Then f$^{-١}$(*A*) is closed set in X. Hence, a function f is contra- rgα**-continuous .

(ii)It following from step-i-and proposition(٣-٢٥).

Corollary(٣-٣٤):

If f: (X,τ) →(Y,σ) is a strongly rgα- continuous function and let X is a locally indiscrete. Then f is contra– rgα** - cotinuous function.

Proof:

It follows from the proposition(٣-١٩) and proposition(٣-٢٥).

Corollary(٣-٣٥):

If f: (X,τ) →(Y,σ) is a rgα- continuous function. let X is a rgα- locally indiscrete and Y is $T^*½$ *space.* Then f is contra– rgα** -continuous function.

Proof:

It follows from the proposition(٣-٢٠) and proposition(٣-٢٥).

Now, we introduce the composition of these types of contra- rgα-continuous function with continuous (resp. rgα-continuous, strongly rgα-continuous and rgα-irresolute) function .

Proposition(٣-٣٦):

Let f: (X ,τ) →(Y, σ) and g: (Y, σ)→(Z,μ) be two functions, if g is continuous and f is contra- rgα- continuous .Then gof: (X ,τ) →(Z, μ) is a contra – rgα-continuous.

Proof:

Let *A* be an open set in *Z*. Thus $g^{-1}(A)$ is an open set in Y .Since f is a contra-rgα-continuous.Then

$f^{-1}(g^{-1}(A))$ is a rgα-closed set . But $f^{-1}(g^{-1}(A))= (gof)^{-1}(A)$ Hence,

gof: (X, τ) →(Z ,μ) is a contra– rgα-continuous function.

Similarly , we prove the following corollary.

Corollary(٣-٣٧):

If f: (X ,τ) →(Y ,σ) and g: (Y ,σ)→(Z, μ) be two functions if g is continuous and f is

(i) contra- rgα**-continuous. Then, gof: (X,τ) →(Z,μ) is contra – rgα-continuous.

(ii) contra- rgα**-continuous. Then, gof: (X,τ) →(Z,μ) is a contra – continuous(resp. contra – rgα –continuous) function.

Proposition(٣-٣٨):

Let f: (X ,τ) →(Y ,σ) and g: (Y, σ)→(Z ,μ) be two functions. Then, gof: (X,τ) →(Z,μ) is a contra – rgα-continuous if g is rgα-continuous and

(i) f is a contra- rgα**-continuous.

(ii) f is a contra- rgα*-continuous.

Proof:

(i) Let *A* be a closed set in Z. Thus $g^{-1}(A)$ is a rgα-closed set in Y .Since f is a contra- rgα**-continuous. Then, $f^{-1}(g^{-1}(A))$ is a rgα-open set in X. But $f^{-1}(g^{-1}(A))= (gof)^{-1}(A)$. Hence, gof: (X, τ) →(Z ,μ) is a contra– rgα-continuous function.

(ii)By assumption f is a contra- rgα*-continuous function and by using proposition(٣-٢٥) we get f is a contra- rgα**-continuous function and by step-i- we obtain,

gof: (X ,τ) →(Z , μ) is a contra– rgα-continuous.

Proposition(٣-٣٩):

Let f: (X ,τ) →(Y ,σ) and g: (Y ,σ)→(Z, μ) be two functions. Then, their composition gof: (X ,τ) →(Z ,μ) is a contra – rgα**-continuous if g is rgα-irresolute and

(i)f is a contra- rgα**-continuous.

(ii) f is a contra- rgα*-continuous.

Proof:

(i) Let *A* be a rgα- closed set in Z. Thus, $g^{-1}(A)$ is a rgα-closed set inY. Since f is a contra- rgα**-continuous. Then, $f^{-1}(g^{-1}(A))$ is an rgα-open set in X. But $f^{-1}(g^{-1}(A))= (gof)^{-1}(A)$. Hence, gof: (X ,τ) →(Z, μ) is a contra– rgα**-continuous function.

(ii) By assumption f is a contra- rgα*-continuous function and by proposition(٣-٢٥) we get f is a contra- rgα**-continuous function and by step-i- we obtain, gof: (X ,τ) →(Z, μ) is a contra– rgα-continuous.

Similarly, we prove the following results:

Corollary(٣-٤٠):

Let f: (X ,τ) →(Y ,σ) and g: (Y ,σ)→(Z ,μ) be two functions. Then, their composition gof: (X ,τ) →(Z, μ) is a contra – rgα**-continuous, if g is strongly rgα- continuous and

(i)f is a contra-continuous.

(ii) f is a contra- rgα*-continuous.

Corollary(٣-٤١):

Let f: (X ,τ) →(Y ,σ) and g: (Y ,σ)→(Z ,μ) be two functions. Then, their composition gof: (X, ,τ) →(Z ,μ) is a contra – rgα*-continuous, if g is a strongly rgα- continuous and

(i)f is a contra- rgα-continuous.

(ii) f is a contra- rgα**-continuous.

Proposition(٣-٤٢):

If f: (X τ) →(Y ,σ) is a strongly rgα-continuous function and
g: (Y, σ)→(Z ,μ) is a contra- rgα-continuous
function. Then, their composition gof: (X ,τ) →(Z ,μ) is

(i)contra-continuous.

(ii) contra- α-continuous.

(iii) contra- rgα-continuous.

Proof:

(i) Let *A* be an open set in Z. Since g is contra -rgα-continuous.

Then, $g^{-1}(A)$is a rgα-closed set inY.and also Since f is a strongly rgα-continuous function. Thus, $f^{-1}(g^{-1}(A))$ is a closed set in X.

But $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$. Hence, gof: (X ,τ) →(Z, μ) is a contra-continuous.

(ii)It follows from the step-i- and Remark(٢-١٠).

(iii)It follows from the step –i-and proposition(٣-٣).

The proof of the following corollary it is easy. Thus it is omitted.

Corollary(٣-٤٣):

If f: (X ,τ) →(Y, σ) is a rgα-irresolute and g: (Y, σ))→(Z, μ) is a contra- rgα-continuous function.

Then, gof: (X, τ) →(Z ,μ) is a contra- rgα-continuous function.

Proposition(٣-٤٤):

if f: (X ,τ) →(Y, σ) is a strongly rgα-continuous function and g: (Y,σ)→(Z,μ) is a contra- rgα*-continuous function. Then, their composition gof: (X ,τ) →(Z ,μ) is contra- rgα*-continuous function.

Proof:

Let A be a rgα- closed set in Z. Thus, $g^{-1}(A)$ is an open set in Y. By Remark(٢-٤) we get $g^{-1}(A)$ is a rgα-open set in Y and Since f is a strongly rgα-continuous function. Then, $f^{-1}(g^{-1}(A))$ is an open set in X. But $f^{-1}(g^{-1}(A))= (gof)^{-1}(A)$. Hence, gof: (X ,τ) →(Z ,μ) is a contra- rgα*-continuous function.

Similarly , we prove the following results:

Corollary(٣-٤٥):

If f: (X ,τ) →(Y, σ) is a rgα-continuous (or rgα-irresolute) and g: (Y,σ) →(Z,μ) is a contra- rgα*-continuous. Then, their composition gof: (X ,τ) →(Z, μ) is a contra- rgα**-continuous (resp. contra- rgα-continuous) function.

Corollary(٣-٤٦):

Let f: (X,τ) →(Y,σ) and g: (Y,σ) →(Z,μ) be two function . Then, their composition gof: (X,τ) →(Z,μ) is

(i) contra- rgα*-continuous if f is strongly rgα-continuous and g is contra- rgα**-continuous .

(ii) contra- rgα**-continuous if f is a rgα-continuous and g is contra- rgα**-continuous .

Proposition(٣-٤٧):

If f: (X ,τ) →(Y, σ) and g: (Y ,σ)→(Z ,μ) are both contra- rgα*-continuous function. Then, gof: (X ,τ) →(Z ,μ) is a

(i) rgα-continuous.

(***ii***) rgα-irresolute.

(iii) strongly rgα-continuous.

Proof:

(i)Let *A* be a closed set in Z. By Remark(٢-٤) we get *A* is a rgα-closed set in Z. thus, $g^{-1}(A)$ is an open set in Y,and also by Remark(٢-٤) we get $g^{-1}(A)$ is a rgα-open set in Y. Since f is a contra- rgα*-continuous function. Then, $f^{-1}(g^{-1}(A))$ is a closed set in X .

By Remark(٢-٤) we obtain $f^{-1}(g^{-1}(A))$ is a rgα-closed set in X.

But $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$. Hence, gof: (X ,τ) →(Z ,μ) is a rgα-continuous function.

(ii) Let *A* be a rgα-closed set in Z .Thus, $g^{-1}(A)$ is an open set in Y, By Remark(٢-٤) we get $g^{-1}(A)$ is a rgα-open set in Y. Since f is a contra- rgα*-continuous. Then, $f^{-1}(g^{-1}(A))$ is a closed set in X ,and also by using Remark(٢-٤) we obtain $f^{-1}(g^{-1}(A))$ is a rgα-closed set in X.

But $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$. Hence, gof: (X ,τ) →(Z, μ) is a rgα- irresolute function.

(iii) Let *A* be an rgα-open set in Z. Thus, $g^{-1}(A)$ is a closed set in Y, By Remark(٢-٤) we get $g^{-1}(A)$ is a rgα-closed set in Y. Since f is a contra- rgα*-continuous function. Then, $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is an open set in X. Hence, gof: (X ,τ) →(Z, μ) is a strongly rgα-continuous.

Simillary, we prove the following corollary:

Corollary(٣-٤٨):

If f: (X ,τ) →(Y, σ) and g: (Y ,σ)→(Z ,μ) are both contra- rgα**-continuous functions. Then, their composition gof: (X ,τ) →(Z ,μ) is a

(i) rgα-continuous.

(ii) rgα-irresolute.

Proposition(٣-٤٩):

If f: (X ,τ) →(Y ,σ) and g: (Y, σ)→(Z, μ) are both contra- rgα-continuous function and let Y be a $T^*½$ *space*. Then, gof: (X ,τ) →(Z ,μ) is a their composition

rgα-continuous function.

Proof:

Let *A* be a closed set in Z. Thus, $g^{-1}(A)$ is a rgα- open set in Y. since Y is $T^*½$ *space* and by using corollary (٣-٦) we get, $g^{-1}(A)$ is an open set in Y, and also since f is a contra- rgα-continuous. Then, $f^{-1}(g^{-1}(A))$ is a rgα-closed set in Z. But $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$. Hence, gof: (X,τ) →(Z,μ) is a rgα-continuous.

٤-Almoset Contra- rgα-Continuous Function:

In the section we introduce new closs of almost continuous function called almost contra- rgα-continuous function and we study their basic properties.

Definition(٤-١):

A function f: (X ,τ) →(Y ,σ) is said to be ***almost contra-rgα- continuou***s if $f^{-1}(A)$ is rgα-closed set in (X, τ) for every regular open set *A* in (Y,σ).

Proposition(٤- ٢):

Let RGαO (X, τ) is a closed under arbitrary unions. Then the following statements are equivalent for a function f: (X ,τ) →(Y, σ).

(i) f is almost contra-rgα- continuous.

(ii) $f^{-1}(A)$ is rgα-open set in (X, τ) for every regular closed set A in (Y,σ).

(iii)For every x ∈X and every regular closed set A in Y containing f(x), there exists a rgα-open set B in X containing x such that $f(B) \subseteq A$.

(iv) For every x ∈X and every regular open set G in Y not containing f(x), there exists a rgα-closed set H in X containing x such that $f^{-1}(G) \subseteq H$. not

Proof:

(i)⇒(ii)Let A is a regular closed set in(Y,σ).
Then, A^c is regular open set in(Y,σ). Since f is almost contra-rgα- continuous function. Thus, $f^{-1}(A^c)$ is rgα-closed set in (X, τ). But $f^{-1}(A^c)=X- f^{-1}(A)=(f^{-1}(A))^c$.
Hence, $f^{-1}(A)$ is a rgα-open set in (X, τ).

(ii) ⇒(i)Let A is a regular open set in (Y,σ). Then, A^c is regular closed set in(Y,σ). By assumption we get $f^{-1}(A^c)$ is a rgα-open set in (X, τ). But $f^{-1}(A^c) = X- f^{-1}(A) = (f^{-1}(A))^c$. Hence, $f^{-1}(A)$ is a rgα-closed set in (X, τ). Therefore, f is almost contra-rgα- continuous function.

(ii) ⇒(iii) Let A be any regular closed set in(Y,σ) Containing f(x). Then, $f^{-1}(A)$ is a rgα-open set in (X, τ),
and $x \in f^{-1}(A)$ by (ii). Take $B= f^{-1}(A)$. Then, $f(B) \subseteq A$.

(iii) ⇒(ii) Let A is a regular closed set in(Y,σ) and $x \in f^{-1}(A)$. From ***(iii)*** there exists a rgα-open set B in (X, τ) containing x , such that $B_x \subseteq f^{-1}(A)$. So we obtain $f^{-1}(A)=\cup\{B_x : x\in f^{-1}(A)\}$.
Hence,$f^{-1}(A)$is a rgα-open set in (X, τ).

(iii) ⇒(iv): Let G be a regular open set in Y non-containing f(x) .Then, G^c is a regular closed set
containing f(x). By ***(iii)*** **,**there exists a rgα-open set B in X containing x such that $f(B) \subseteq G^c$. Thus ,$B\subseteq f^{-1}(G^c) = X- f^{-1}(G)$. Then , $f^{-1}(G) \subseteq X- B$. Take H=X-B. we obtain that H is a rgα-closed set in X non-containing x such that $f^{-1}(G) \subseteq H$.

(iv)→(iii) Let A be any regular closed set in Y
containing f(x). Then ,A^c is a regular open set in Y non-containing f(x) .By ***(iv),*** there exists a rgα-closed set H in X not containing x, such that $f^{-1}(A^c) \subseteq H$. But
$f^{-1}(A^c) = X- f^{-1}(A)$. Hence , $X- f^{-1}(A) \subseteq H$ .Therefore, $H^c \subseteq f^{-1}(A)$.
Then, $f^{-1}(H^c) \subseteq A$. Take $B= H^c$= X-H. Hence,
B is a rgα-open set in X containing x such that $f(B) \subseteq A$.

Proposition(٤- ٣):
Every contra- rgα- continuous function is almost contra- rgα- continuous.
Proof:
Let f: (X, τ) →(Y, σ) be a contra- rgα- continuous function and let *A* be a regular open set in Y.Since every regular open set is an open set. Thus, *A* is an open set in Y and also since f is a contra- rgα- continuous. Then , $f^{-1}(A)$ is a rgα-closed set in X. Hence ,a function f is almost contra- rgα- continuous.
Corollary(٤- ٤):
Every contra- continuous (resp. contra- α- continuous) function is almost contra- rgα- continuous.
Proof:
It follows from the proposition(٣-٣) and proposition(٤-٣).
Corollary(٤- ٥):
Every contra- $rg\alpha^{*}$- continuous (resp. contra- $rg\alpha^{**}$ - continuous) function is almost contra- rgα- continuous.
Proof:
It follows from the corollary (٣-١٦) –step-ii- (resp. from proposition (٣-٢٤)) and proposition(٤-٣).
Remark(٤- ٦):
The converse of proposition(٤-٣) and corollaries (٤-٤), (٤-٥) respectively, need not be true , as seen from the following example
Example(٤- ١) :
Let X= Y ={a, b, c, d } with topologiesτ= {X ,ø ,{a}, {b } , {a, b} , {a, b, c}}. And σ = {Y, ,ø ,{a},{b},{a, b}, {b, c} , {a, b, c} , {a, b, d}} Define f: (X ,τ) →(Y ,σ) by f(a)= f(d)=c , f(b)=b and f(c)=d. It easy seen that f is almost contra- rgα- continuous function, but is not contra-continuous(contra- α- continuous, contra-rgα- continuous, contra- $rg\alpha^{*}$-continuous and contra- $rg\alpha^{**}$- continuous) function respectively. Since the set {b } is an open set and rgα-open set in (Y,σ) but f^{-1}({b })= {b } is not closed (resp. α-closed and rgα-closed) set in (X, τ).
Proposition(٤- ٧):
If f: (X, τ) →(Y, σ) is a rgα-irresolute and g: (Y,σ)→(Z,μ) is almost contra-rgα-continuous function. Then ,their composition gof: (X ,τ) →(Z ,μ) is almost contra- rgα-continuous function.
Proof:
Let *A* be a regular open set in Z.Thus, $g^{-1}(A)$is a rgα-closed set in Y . Since f is rgα-irresolute function. Then $f^{-1}(g^{-1}(A))$ is rgα-closed set in X.
But $f^{-1}(g^{-1}(A))= (gof)^{-1}(A)$. Hence , gof: (X ,τ) →(Z ,μ) is almost contra- rgα-continuous function.
Simillary , we prove the following corollary:

Corollary(٤-٨):

If f: (X ,τ) →(Y, σ) is a strongly rgα-continuous and g: (Y,σ)→(Z,μ) is almost contra- rgα-continuous . Then ,their composition gof: (X ,τ) →(Z ,μ) is almost contra- rgα-continuous function.

Proposition(٤-٩):

If f: (X, τ) →(Y ,σ) is a rgα-continuous function , g: (Y,σ)→(Z,μ) is almost contra- rgα-continuous function. and Y is a $T^*½$ space . Then their composition gof: (X, τ) →(Z, μ) is almost contra- rgα-continuous function.

Proof:

Let *A* be a regular open set in Z.Thus, $g^{-١}(A)$is a rgα-closed set in Y .Since Y is *a $T^*½$ space* and by using

Proposition(٣-٥) we get $g^{-١}(A)$is a closed set in Y and also since f is a rgα-continuous. Then , $f^{-١}(g^{-١}(A))$ is rgα-closed set in X .

But $f^{-١}(g^{-١}(A))= (gof)^{-١}(A)$. Hence , gof: (X ,τ) →(Z ,μ) is almost contra- rgα-continuous function.

Proposition(٤-١٠):

If f: (X,τ) →(Y,σ) and g: (Y,σ)→(Z,μ) be any two function .Let X , Y are both *$T^*½$ space* . Then gof: (X ,τ) →(Z, μ) is almost continuous if g is almost contra- rgα-continuous function and f is contra- rgα-continuous .

Proof: Let *A* be a regular open set in Z.Thus, $g^{-١}(A)$is a rgα-closed set in Y .Since Y is a *$T^*½$ space* and by using Proposition(٣-٥) we obtain

$g^{-١}(A)$is a closed set in Y. since f is a contra- rgα-continuous function . Then , $f^{-١}(g^{-١}(A))$ is rgα-open set in X and also since X is *$T^*½$ space* and by using corollary (٣-٦) we get $f^{-١}(g^{-١}(A))$ is an open set in X.

But $f^{-١}(g^{-١}(A))= (gof)^{-١}(A)$. Hence , gof: (X ,τ) →(Z ,μ) is almost continuous function

Simillary , we prove the following corollary.

Corollary(٤-١١):

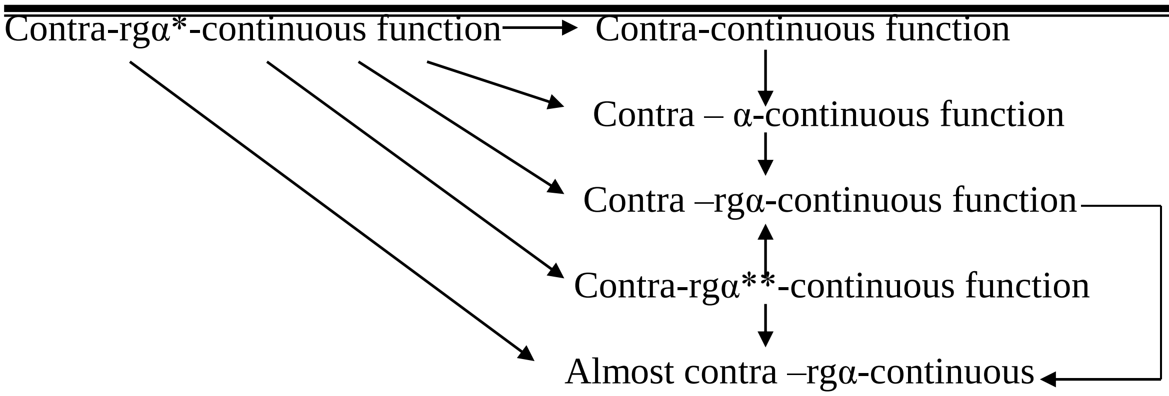
Let f: (X ,τ) →(Y ,σ) and g: (Y, σ)→(Z,μ) be any two functions, Let X, Y are both *$T^*½$ space* . Then ,their composition gof: (X, τ) →(Z ,μ) is almost continuous function if g is almost contra- rgα-continuous and

(i) f is contra- $rgα^{**}$-continuous .

(ii) f is contra- $rgα^{*}$-continuous .

Remark(٤-١٢) :

From the above discussion and know results we have the following implications :



Digram(١) Summarized The Relationships Between Contra-rgα -Continuous Function Types

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تقريبا rgα والدوال ضد المستمرة – rgαحول أنواع الدوال –ضد المستمرة -

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المستخلص :

*الهدف الرئيسي من هذا البحث هو دراسة بعض الصفوف الجديدة من الدوال ضد المستمرة وهي (الدالة ضد المستمرة – rgα , الدالة ضد المستمرة -*rgα والدالة ضد المستمرة -**rgα)في الفضاء التبولوجي وقدمنا بعض خواصها والعلاقات فيما بينها .وكذلك سنعرف وندرس نوع اخرمن الدوال ضد المستمرة - rgα*-تدعى الدوال-rgα - ضد المستمرة تقريباً.