Mathematical Morphology Operations on Grayscale Image

Dr.Amel H.Abbas
Al-Mustansiriyah University
College of science

Abstract

Mathematical Morphology is a tool for extracting image components useful in the representation and description of region shape, such as boundaries, skeletons and convex hulls. The mathematical morphology depends on set theory and it can apply to binary images and gray scale images, and the usual set operators can be applied to them.

The basic operation in mathematical morphology operate on two sets: the first one is the image, and the second one is the structuring element. The structuring element in practice is generally much smaller than the image, often 3x3 matrices.

This paper introduces the basic operations of mathematical morphology (dilation, erosion, open, close) and applying the morphology gradient, morphology Laplacian and morphology smoothing on images.

Introduction

The word morphology means "the form and structure of an object" or the arrangements and interrelation ships between the parts of an object. Digital morphology is a way to describe or analyze the shape of a digital object.

Morphology is an image processing operations that process images based on shapes. Morphological operations apply a structuring element to an input image, creating an output image of the same size. In a morphological operation, the value of each pixel in the output image is based on comparisons of the corresponding pixel in the input image with its neighbors. By choosing the size and shape of the neighborhood, you can construct a morphological operation that is sensitive to specific shapes in the input image.

The elements of mathematical morphology [1]

1 – Uses the set theory image analyses
2 – Can be used to find the boundaries, skeleton, etc.
3- Can be used for many pre and post image processing techniques.
4- Relies on two basic operations used to shrink and expand image features.
5- Originally used on binary images.

Fundamental definitions

The idea underlying digital morphology is that image consists of a set of picture elements (pixels) that collect into groups having a two- dimentional structure (shape). Certain mathematical operation on the set of pixels can be used to enhance specifics aspect of the shapes so that they might be counted or recognized. Mathematical morphology is a tool for processing shapes in image, including boundaries, skeleton, conves, hulls, etc.

For two sets A and B figure (1a) the fundamental operations associated with an object are the standard set operations {2]

- Complement set
  - » If $A \subset \Omega$, then its complement set $Ac = \{\omega |\ \omega \in \Omega$, and $\notin \omega A\}$
- Union ($\cup$)
  - » $A \cup B = \{\omega |\ \omega \in A$ **or** $\omega \in B\}$
- Intersection ($\cap$)
  - » $A \cap B = \{\omega |\ \omega \in A$ **and** $\omega \in B\}$

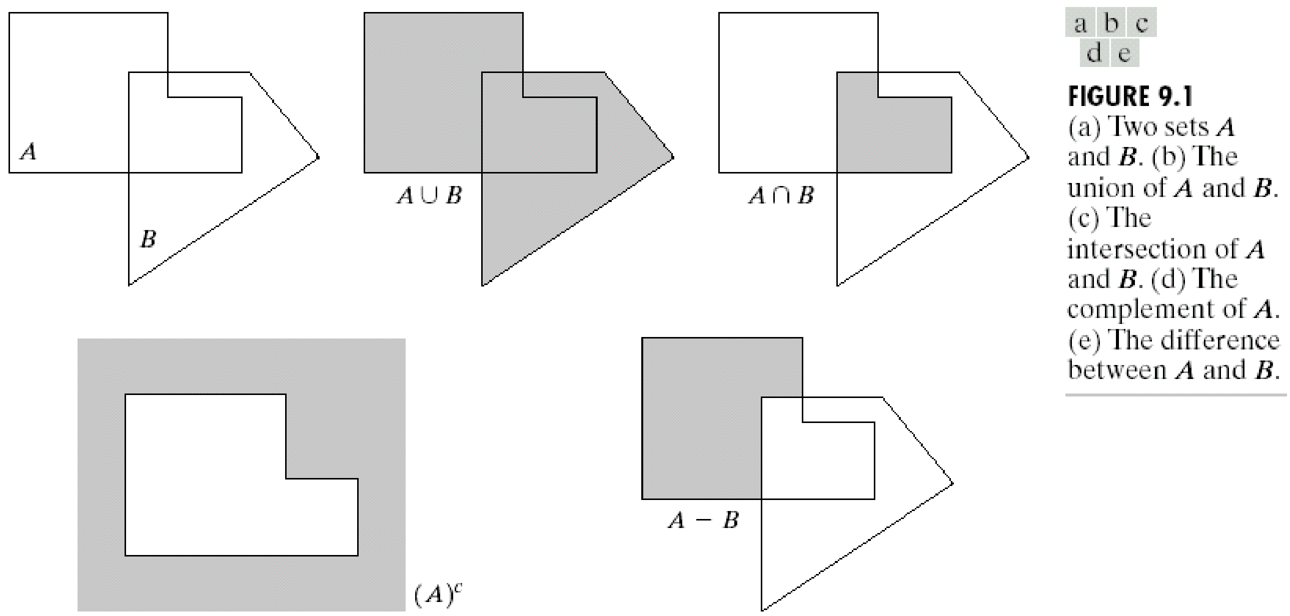


Figure 1 a) two sets A and B b) the union of A and B c) the intersection of A and B d) the complement of A

- The Translation of A by z figure (2) is defined

  Translation (A)z = $\{ c |\ c = a + z$, for $a \in A \}$

- The reflection of B figure (2)

:

$$\hat{B} = \{ w \mid w = -b, \text{for } b \in B \}$$

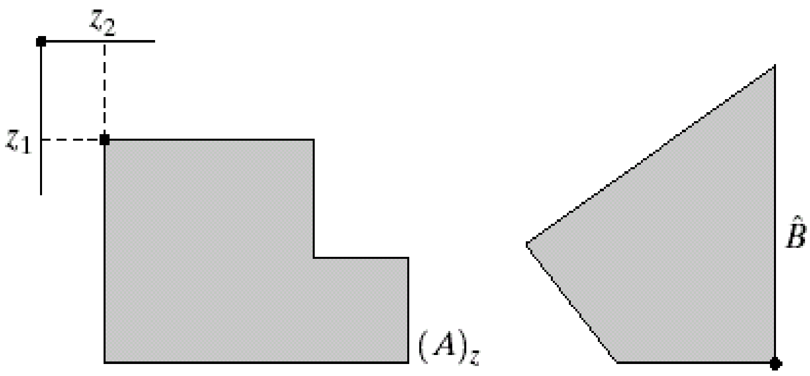


Figure 2 Translation of A by Z and Reflection of B

The Mathematical Morphology

The mathematical morphology operation includes [1]:

1 - Two basic operations
- Dilation
- Erosion

2 – Composite Relations
- Opening
- Closing

3 – Operations
- Edge Detection
- Thinning
- Thickening

The mathematical morphology operations covered by applying structuring element. The structure element is a binary image with size 3x3, 5x5, 7x7, etc there are various types of structuring element figure 3 show the typical shape of structure element:-

1	1	1
1	1	1
1	1	1

Square (3x3)

	1	1	1	
1				1
1				1
1				1
	1	1	1	

Ring (5x5)

	1	1	1	
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
	1	1	1	

Disk (5x5)

Figure 3 the types of structure element

Dilation and Erosion

The basic morphological operations, dilation and erosion, produce contrasting results when applied to either grayscale or binary images. Erosion shrinks image objects while dilation expands them. The specific actions of each operation are covered in the following sections.

The dilation

Dilation of the set *A* by set *B*, denoted by $A \oplus B$, is obtained by first reflecting *B* about its origin and then translating the result by *x*. All *x* such that *A* and reflected *B* translated by *x* that have at least one point in common form the dilated set [3].

$$A \oplus B = \{x \mid (\hat{B})_x \cap A \neq \phi\},$$

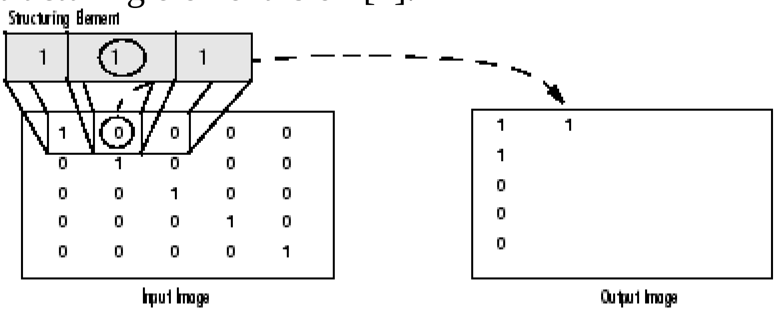
Where, $\hat{B}$ denotes the reflection of *B* $\hat{B} = \{x \mid x = -b, \text{for } b \in B\}$ i.e., $(B)_x$ and denotes the translation of B $x = (x_1, x_2)$ by $(B)_x = \{c \mid c = b + x, \text{for } b \in B\}$ i.e. Thus, dilation of *A* by *B* expands the boundary of *A*.

It is easier to implement for gray-scales than the above description suggests.

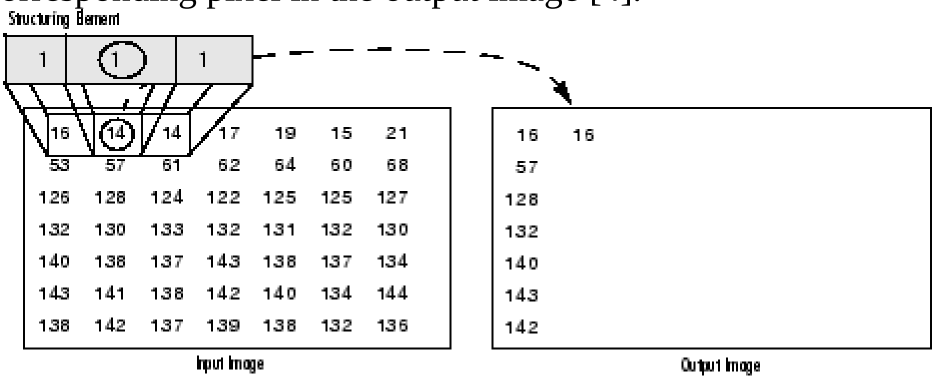
$$(f \oplus b)(s,t) = \max\{f(s-x,t-y)+b(x,y) \mid (s-x),(t-y) \in D_f; (x,y) \in \qquad \text{(A.1)}$$

Here, f and b denote images f(x,y) and b(x,y). f is being dilated and b is called the structuring element and D_f and D_b are the domains of f and b respectively. Thus, in dilation we choose the maximum value of f+b in a neighborhood defined by b. If all elements of *b* are positive, the dilated image is brighter than the original and the dark details are either reduced or eliminated.

The following figure illustrates the dilation of a binary image. Note how the structuring element defines the neighborhood of the pixel of interest, which is circled. The dilation function applies the appropriate rule to the pixels in the neighborhood and assigns a value to the corresponding pixel in the output image. In the figure, the morphological dilation function sets the value of the output pixel to 1 because one of the elements in the neighborhood defined by the structuring element is on [4].



The following figure illustrates this processing for a grayscale image. The figure shows the processing of a particular pixel in the input image. Note how the function applies the rule to the input pixel's neighborhood and uses the highest value of all the pixels in the neighborhood as the value of the corresponding pixel in the output image [4].



Characteristics of Dilation

- Dilation generally increases the sizes of objects, filling in holes and broken areas, and connecting areas that are separated by spaces smaller than the size of the structuring element.
- With grayscale images, dilation increases the brightness of objects by taking the neighborhood maximum when passing the structuring element over the image.
- With binary images, dilation connects areas that are separated by spaces smaller than the structuring element and adds pixels to the perimeter of each image object [5].

The erosion

Erosion of *A* by *B*, denoted by $A \ominus B$, is the set of all *x* such that *B* translated by *x* is completely contained in *A*, i.e.,

$$A \ominus B = \{x \mid (B)_x, \subseteq A\}.$$

For gray-scale images we have,

$$(f \ominus b)(s,t) = \min\{f(s+x, t+y) - b(x,y) \mid (s+x),(t+y) \in D_f; (x,y) \in D_b\}.$$

Erosion is thus based on choosing the minimum value of (f-b) in a neighborhood defined by the shape of *b*. If all elements of *b* are positive, the output image is darker than the original and the effect of bright details in the input image are reduced if they cover a region smaller than b [3]

Characteristics of Erosion

- Erosion generally decreases the sizes of objects and removes small anomalies by subtracting objects with a radius smaller than the structuring element.
- With grayscale images, erosion reduces the brightness (and therefore the size) of bright objects on a dark background by taking the neighborhood minimum when passing the structuring element over the image.
- With binary images, erosion completely removes objects smaller than the structuring element and removes perimeter pixels from larger image objects[5].

Rules for Dilation and Erosion

As a result the rules f dilation and erosion in binary and gray scale image are[4]:

Operation	**Rule**
Dilation	The value of the output pixel is the *maximum* value of all the pixels in the input pixel's neighborhood. In a binary image, if any of the pixels is set to the value 1, the output pixel is set to 1.
Erosion	The value of the output pixel is the *minimum* value of all the pixels in the input pixel's neighborhood. In a binary image, if any of the pixels is set to 0, the output pixel is set to 0.

Opening and closing

The subsequent morphological operators are defined in terms of erosion and dilation. Their expressions for gray-scale and binary images are the same and a distinction will not be henceforth made.

A is said to be opened by B if the erosion of A by B is followed by a dilation of the result by B.[3] $A \circ B = (A \ominus B) \oplus B$ (A.3)

Opening results in removal of narrow peaks. The initial erosion removes the small details and darkens the image. The following dilation increases the brightness but does not reintroduce the details removed by erosion.

Similarly, *A* is said to be closed by B if A is first dilated by B and the result is then eroded by B. Thus,

$$A \bullet B = (A \oplus B) \ominus B$$

Closing is used to remove dark details from an image. The initial dilation removes dark details and makes the image brighter. The erosion that follows darkens the image but does not reintroduce the details removed by dilation [3].

Morphological Gradient

Let f: E $\rightarrow$ R is a grayscale image, mapping points from a Euclidean space or discrete grid E (such as R^2 or Z^2) into the real line. Let b(x) be a grayscale structuring element. Then, the morphological gradient of f is given by

$$G(f) = f \oplus b - f \ominus b$$

Where $\oplus$ and $\ominus$ denote the dilation and the erosion, respectively.

An **internal gradient** is given by:

$$G_I(f) = f - f \ominus b$$

And an **external gradient** is given by:

$$G_e(f) = f \oplus b - f$$

The internal and external gradients are "thinner" than the gradient, but the gradient peaks are located on the edges, whereas the internal and external ones are located at each side of the edges. Notice that $G_i + G_e = G$. If, b (0) > = 0 then all the three gradients have non-negative values at all pixels [6].

The morphological gradient highlights sharp transitions in the input image. It depends less on edge directionality than the Sobel operator and is useful for locating faint but large scale structures. The morphological gradient is defined by [3]

$$g = (f \oplus b) - (f \ominus b). \qquad (A.5)$$

It is thus the difference between a dilated image and an eroded image. Dilation removes small scale dark features and erosion removes small scale bright features. Dilation brightens the image and erosion darkens it. When the difference is obtained, all small scale features are gone and the contrast between the large scale features improved.

For linear filters the gradient filter yields a vector representation with a magnitude and direction .The version presented here generates a morphological estimate of the gradi*ent* magnitude [7]:

$$\begin{aligned} Gradient(A,B) &= \frac{1}{2}\left(D_G(A,B) - E_G(A,B)\right) \\ &= \frac{1}{2}\left(\max(A) - \min(A)\right) \end{aligned}$$

Morphological Laplacian

Laplace operator is a second - order partial derivative in the orthogonal direction of a continues space, Laplace filter used in digital image processing in 3x3 mask with coefficients as show

0	1	0
1	-4	1
0	1	0

Laplace filter is no directional filter, so there is no any control over the orientation of the linear features therefore the Laplace operator will enhance details in all directions equally, it is important to note that the gradient gives both magnitude and direction information about the change in pixel values at a point, where the Laplace filter is a scalar giving only magnitude. Because the Laplacian is rotation invariant, this has the advantage that high spatial frequencies in all directions are equally enhanced at the same time. This may be a disadvantage because useful directional information will not be available.
The morphologically-based Laplacian filter is defined by [7]:

$$\begin{aligned} Laplacian(A,B) &= \frac{1}{2}\left(\left(D_G(A,B)-A\right)-\left(A-E_G(A,B)\right)\right) \\ &= \frac{1}{2}\left(D_G(A,B)+E_G(A,B)-2A\right) \\ &= \frac{1}{2}\left(\max(A)+\min(A)-2A\right) \end{aligned}$$

Morphological Smoothing

The morphological smoothing algorithm is based on the observation that a gray-level opening smoothes a gray-value image from above the brightness surface given by the function a[m,n] and the gray-level closing smoothes from below. by using a structuring element **B** the morphological soothing equation is[7]:

$$\begin{aligned} MorphSmooth(A,B) &= C_G(O_G(A,B),B) \\ &= \min(\max(\max(\min(A)))) \end{aligned}$$

Experimental Results

In this section the results of applying the mathematical morphology operations on images are illustrate. Figure 4 show the original grayscale images (camera and Lena) and the result of dilation (increases the brightness of objects) and erosion (reduces the brightness and the size of the bright objects).

Figure4

Original image dilation image erosion image

Figure 5 show the original grayscale images and the results of applying sobel edge detection operator (with manual threshold), morphological gradient, laplacian edge detector and morphological laplacian. The results show that the detected edge more smooth in morphological gradient method than sobel operator and the noise is removed. In laplacian operator the sharp edges have been detected with some thickness and the noise is very clear while in morphological laplacian the edges detected with suitable thickness.

Figure 5

Sobel operator morphological gradient Laplacian operator morphological Laplacian

Figure 6 show the original grayscale image and the result of applying morphological (open, close and smooth) operations. The results show that in open operation the narrow peaks and the bright details are removed, the close operation removes the dark details from the image and the smooth operation remove the bright and dark details from the image.

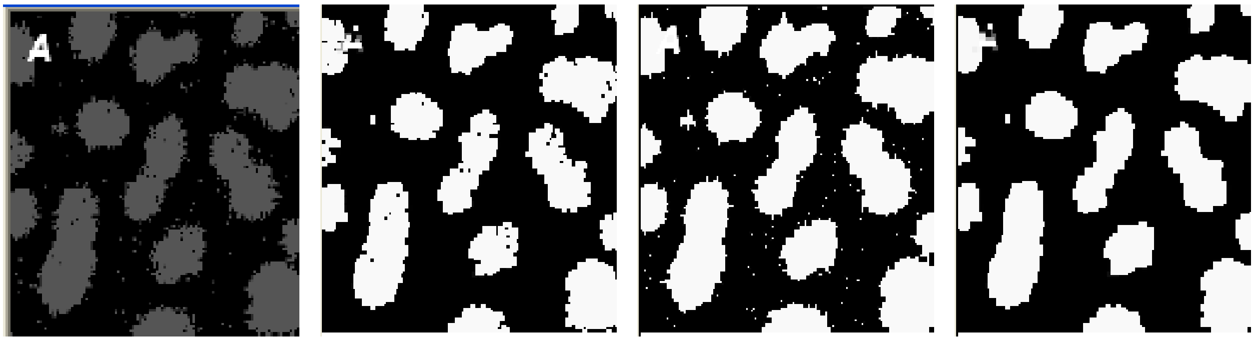


Figure 6

Original image morphological open morphological close morphological smooth

Conclusions

From the results of applying the basic morphological operations on selected images some conclusions can derive:

1- In sobel edge detection operator the manual threshold are used while in morphological gradient the results appears without using manual threshold.
2- In laplacian edge detector the sharp edges have been detected with some thickness and the noise is very clear while in morphological laplacian the edges detected with suitable thickness.
3- The morphological opening and closing can used as smoothing methods for removing noise and smoothing the contours of the objects

Future works

1- Applying the basic morphological operations (dilation, erosion, opening and closing) on color images.
2- Using the dilation and erosion operations in finger print processing.
3- Sing the basic morphological operations in boundary extracting.

References

1- Scott N.,"Edge Detection using Mathematical Morphology", Research Center for Integrated Microsystems University of Windsor, 2007.
2 –Dougherty E.,Lotufo R., "Morphological Image Processing" SPIE, 2003, http://www.dai.ed.ac.uk/cvoline/transf.htm
3 – Wilson F.,"Morphological Processing", 2009, http://www.astro.caltech.edu/aam
4 – The math works Accelerating the poce of engineering and science," Understanding Dilation and Erosion", 2009, http://www.mathwork.com/access/helpdesk...
5 – IDL Online Help, "Image Processing, Extracting and Analyzing Shapes, Eroding and Dilation Image Objects",2007,http//www.idlastro.gsfc.nasagov/idl-html..
6 - Wales J.,"Morphological Gradient", from wikipedia founder, 2009.
7- Young I.T. van vliet L.J. ,"Image Processing Fundamentals", by Netherlands Organization for Scientific Research (NWO),2007.

الخلاصة

م. د. امل حسين عباس
الجامعة المستنصرية
كلية العلوم
قسم الفيزياء

Mathematical Morphology هي وسيلة لاستخلاص مكونات الصور المفيدة في تمثيل ووصف اشكال المناطق في الصور مثل الحواف والهياكل والتحدب.
Mathematical Morphology يعتمد على نظرية المجموعات ويمكن ان يطبق على الصور الثنائية والصور الاعتيادية العمليات الاساسية تعمل على الصورة وعنصر البنية . Structuring Element هو في الغالب اصغر من الصورة ويكون غالبا مصفوفة ذات قياس ٣ X٣
في هذا البحث يتم استعراض العمليات الاساسية وهي dilation, erosion, open, close وتطبيق Morphological gradient, morphological Llaplacian and morphological smoothing على الصور.