A (k,ℓ) Span in Three Dimensional Projective Space PG(3,p) Over Galois Field where p=4

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Abstract

The purpose of this work is to study the three dimensional projective space PG(3,P) ,where p=4 ,By using the algebraic equations, we found the points ,lines and planes .In this space we construct (k, ℓ)-span which is a set of k lines no two of which intersect . we prove that the maximum complete (k,ℓ)-span in PG(3,4) is (17,ℓ)-span, which is the equal to all the points of the space that is called a spread.

Introduction

A projective 3-space PG(3,k) over a field K is a 3-dimensional projective space which consists of points, lines and planes with the incidence relation between them.[5]

PG(3,k) satisfying the following axioms:

1. Any two distinct points are contained in a unique line.
2. Any three distinct non-collinear points, also any line and point not on the line are contained in a unique plane.
3. Any two distinct coplanar lines intersect in a unique point.
4. Any line not on a given plane intersects the plane in a unique point.
5. Any two distinct planes intersect in a unique line.

A projective space PG(3,p) over Galois field GF(p) , where $p = q^m$ for some prime number q and some integer m, is a 3-dimensional projective space.

Any point in PG(3,p) has the form of a quadrable (x_1,x_2,x_3,x_4), where x_1,x_2,x_3,x_4 are elements in GF(p) with the exception of the quadrable consisting of four zero elements.

Two quadrables (x_1,x_2,x_3,x_4) and (y_1,y_2,y_3,y_4) represent the same point if there exists λ in $GF(p)\setminus\{0\}$ such that $(x_1,x_2,x_3,x_4) = \lambda\ (y_1,y_2,y_3,y_4)$. Similarly, any plane in PG(3,p) has the form of a quadrable $[x_1,x_2,x_3,x_4]$, where x_1,x_2,x_3,x_4 are elements in GF(p) with the exception of the quadrable consisting of four zero elements.

Two quadrables $[x_1,x_2,x_3,x_4]$ and $[y_1,y_2,y_3,y_4]$ represent the same plane if there exists λ in $GF(p)\setminus\{0\}$ such that $[x_1,x_2,x_3,x_4] = \lambda\ [y_1,y_2,y_3,y_4]$.

Finally, a point $p(x_1,x_2,x_3,x_4)$ is incident with the plane π $[a_1,a_2,a_3,a_4]$ iff $a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 = 0$.

Definition 1.1: "Plane π", [4]

A plane π in PG(3,p) is the set of all points $p(x_1,x_2,x_3,x_4)$ satisfying a linear equation $u_1x_1 + u_2x_2 + u_3x_3 + u_4x_4 = 0$. This plane is denoted by π $[u_1,u_2,u_3,u_4]$.

Theorem 1.2: [1]

The points of PG(3,p) have aunique forms which are (1,0,0,0), (x,1,0,0), (x,y,1,0), (x,y,z,1) for all x, y, z in GF(p).

There exists one point of the form (1,0,0,0).

There exists p points of the form (x,1,0,0).

There exists p^2 points of the form (x,y,1,0).

There exists p^3 points of the form (x,y,z,1).

Theorem 1.3: [1]

The planes of PG(3,p) have a unique forms which are [1,0,0,0], [x,1,0,0], [x,y,1,0], [x,y,z,1] for all x, y, z in GF(p).

There exists one plane of the form [1,0,0,0].

There exists p planes of the form [x,1,0,0].

There exists p^2 planes of the form [x,y,1,0].

There exists p^3 planes of the form [x,y,z,1].

Theorem 1.4: [6]

In PG(3,p) satisfies the following:

A) Every line contains exactly $p + 1$ points and every point is on exactly $p + 1$ lines.

B) Every plane contains exactly $p^2 + p + 1$ points (lines) and every point is on exactly $p^2 + p + 1$ planes.

C) There exist $p^3 + p^2 + p + 1$ of points and there exists $p^3 + p^2 + p + 1$ of planes.

D) Any two planes intersect in exactly $p + 1$ points and any line is on exactly $P+1$ planes. So any two points are on exactly $p + 1$ planes.

Theorem 1.5: [4]

There exists $(p^2+1)(p^2+p+1)$ of lines in PG(3,p).

Definition 1.6: [3]

A (k,ℓ)-span, $\ell \geq 1$ is a set of k spaces π_ℓ no two of which intersect.

Definition 1.7: [7]

A maximum (k,ℓ)-span is a set of k spaces π_ℓ which are every points of PG(3,p) lies in exactly one line of π_ℓ, and every two lines of π_ℓ are disjoint.

Definition 1.8: [1]

Every maximum (k,ℓ)-span is a spread.

2- The Additions and Multiplications Operation of GF(4): [2]

To find the addition and multiplication tables in GF(4), we have the order pairs (x_1,x_2) such that x_1, x_2 in GF(2), as follows:

$0 \equiv (0,0)$, $1 \equiv (1,0)$, $2 \equiv (0,1)$, $3 \equiv (1,1)$

Put these points in one orbit, (1,0) at the first point and by the principle of (1,0) A^i, i = 0,1,2,3 and $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, $(1,0)A \equiv (0,1)$ and $(1,0)A^2 \equiv (1,1)$, so

Now, in the left of the following table, m is the operation of multiplication and in the right n is the operation of addition in multiplication side we write the numeration of points as last, and the addition side takes the normal sequence.

Table (1) : The additions and multiplications operations of GF(4)

m(*)		(+)n = f(m)
1	(1,0)	0
2	(0,1)	1
3	(1,1)	2
mod 3		

In addition table, we have the following relation:

$(x_1,x_2) + (y_1,y_2) = (z_1,z_2)$ where $z_i = (y_i + x_i) \bmod (2)$, for i = 1, 2.

In multiplication table, we have the following relation

$$((1,0)\,A^{f(m_1)})\,A^{f(m_2)} \Leftrightarrow m_1 * m_2 = m_3$$
$$= (1,0)\,A^{(f(m_1)+f(m_2)) \pmod 3}$$
$$= (x_1,x_2)$$

For example: $2*3=1 \Leftrightarrow ((1,0)A^1)A^2 = (1,0)A^3$
$$= (1,0)A^0$$
$$= (1,0)$$

where (1,0) equal to 1 in multiplication side.

Now we have addition and multiplication tables:

Table (2)

The addition operation of GF(4)

+	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Table(3)

The multiplication operation of GF(4)

*	1	2	3
1	1	2	3
2	2	3	1
3	3	1	2

3- The Projective Space and The (k,ℓ)-span in PG(3,4):

3.1 The Projective Space in PG(3,4):

PG(3,4) contains 85 points and 85 planes such that each point is on 21 planes and every plane contains 21 points, any line contains 5 points and it is the intersection of 5 planes, all the points, planes and lines of PG(3,p) are given in tables 4 and 5.

3.2 The (k,ℓ)-span in PG(3,4):

In table (5) , Any two non-intersecting lines can be taken in PG(3,4), say ℓ_1={1,2,3,4,5} and ℓ_2={6,22,26,30,34}, then A={ℓ_1,ℓ_2} is a (2,ℓ)-span.

One can add another line ℓ_3={7,38,43,48,53} then B={ℓ_1,ℓ_2,ℓ_3} is a (3,ℓ)-span, since ℓ_3 cannot intersect ℓ_1 or ℓ_2.

The line ℓ_4={8,71,77,80,82} this line cannot intersect any line of B, then C=B∪{ℓ_4}={ℓ_1,ℓ_2,ℓ_3,ℓ_4} is a (4,ℓ)-span.

The line ℓ_5={9,55,60,62,69} this line cannot intersect any line of C, then D=C∪{ℓ_5}={ℓ_1,ℓ_2,ℓ_3,ℓ_4,ℓ_5} is a (5,ℓ)-span.

The line ℓ_6={10,32,52,56,76} this line cannot intersect any line of D, then E=D∪{ℓ_6}={ℓ_1,ℓ_2,ℓ_3,ℓ_4,ℓ_5,ℓ_6} is a (6,ℓ)-span.

The line ℓ_7={11,35,46,61,72} this line cannot intersect any line of E, then F=E∪{ℓ_7}={ℓ_1,ℓ_2,ℓ_3,ℓ_4,ℓ_5,ℓ_6,ℓ_7} is a (7,ℓ)-span.

Also, the line ℓ_8={12,27,41,68,78} cannot intersect any line of F, then G=F∪{ℓ_8}={ℓ_1,…,ℓ_8} is a (8,ℓ)-span.

The line ℓ_9={13,25,42,64,83} cannot intersect any line of G, then H=G∪{ℓ_9}={ℓ_1,…,ℓ_9} is a (9,ℓ)-span.

The line ℓ_{10}={14,29,49,57,85} cannot intersect any line of H, then I=H∪{ℓ_{10}}={ℓ_1,…,ℓ_{10}} is a (10,ℓ)-span.

The line ℓ_{11}={15,31,44,66,73} cannot intersect any line of I, then J=I∪{ℓ_{11}}={ℓ_1,…,ℓ_{11}} is a (11,ℓ)-span.

The line ℓ_{12}={16,37,40,63,74} cannot intersect any line of J, then M=J∪{ℓ_{12}}={ℓ_1,…,ℓ_{12}} is a (12,ℓ)-span.

The line ℓ_{13}={17,24,50,59,81} cannot intersect any line of M, then N=M∪{ℓ_{13}}={ℓ_1,…,ℓ_{13}} is a (13,ℓ)-span.

The line ℓ_{14}={18,23,47,67,75} cannot intersect any line of N, then O=N∪{ℓ_{14}}={ℓ_1,…,ℓ_{14}} is a (14,ℓ)-span.

The line ℓ_{15}={19,33,39,58,84} cannot intersect any line of O, then P=O∪{ℓ_{15}}={ℓ_1,ℓ_2,…,ℓ_{15}} is a (15,ℓ)-span.

The line ℓ_{16}={20,28,51,65,70} cannot intersect any line of P, then Q=P∪{ℓ_{16}} is a (16,ℓ)-span.

Finally, one can add the line ℓ_{17}={21,36,45,54,79} to Q this line cannot intersect any line of Q, then R=Q∪{ℓ_{17}}={ℓ_1,ℓ_2,…,ℓ_{17}} is a (17,ℓ)-span, which is the maximum (k,ℓ)-span of PG(3,4) can be obtained. Thus R is called a spread of seventeen lines of PG(3,4) which partitions PG(3,4); that is every point of PG(3,4) lies in exactly one line of R, and every two lines of R are disjoint, i.e. {ℓ_1,ℓ_2,…,ℓ_{17}} = PG(3,4).

Results and conclusions

From the above results ,the number of the planes in the projective space PG(3,4) are 85 planes ,and each plane contains 21 lines ,therefore the total number of the lines in PG(3,4) are 1785 .we found that the number of the lines do not intersect with some of them are seventeen lines ,these lines contains the whole points of the projective space PG(3,4) ,and called him a (17,ℓ)-span ,i.e.

(17, ℓ)-span}={$\ell_1,\ell_2,\ldots,\ell_{17}$}=PG(3,4)={1,2,3,…,85}

Moreover ,we found that a (17,ℓ)-span is a maximum complete (k,ℓ)-span in PG(3,4) .

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Table (4) Points and Plans of PG (3 , 4)

i	P_i	Π_i																				
1	(1,0,0,0)	2	6	10	14	18	22	26	30	34	38	42	46	50	54	58	62	66	70	74	78	82
2	(0,1,0,0)	1	6	7	8	9	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37
3	(1,1,0,0)	3	6	11	16	21	22	26	30	34	39	43	47	51	56	60	64	68	73	77	81	85
4	(2,1,0,0)	5	6	13	15	20	22	26	30	34	41	45	49	53	55	59	63	67	72	76	80	84
5	(3,1,0,0)	4	6	12	17	19	22	26	30	34	40	44	48	52	57	61	65	69	71	75	79	83
6	(0,0,1,0)	1	2	3	4	5	22	23	24	25	38	39	40	41	54	55	56	57	70	71	72	73
7	(1,0,1,0)	2	7	11	15	19	22	27	32	37	38	43	48	53	54	59	64	69	70	75	80	85
8	(2,0,1,0)	2	9	13	17	21	22	29	31	36	38	45	47	52	54	61	63	68	70	77	79	84
9	(3,0,1,0)	2	8	12	16	20	22	28	33	35	38	44	49	51	54	60	65	67	70	76	81	83
10	(0,1,1,0)	1	10	11	12	13	22	23	24	25	42	43	44	45	62	63	64	65	82	83	84	85
11	(1,1,1,0)	3	7	10	17	20	22	27	32	37	39	42	49	52	56	61	62	67	73	76	79	82
12	(2,1,1,0)	5	9	10	16	19	22	29	31	36	41	42	48	51	55	60	62	69	72	75	81	82
13	(3,1,1,0)	4	8	10	15	21	22	28	33	35	40	42	47	53	57	59	62	68	71	77	80	82
14	(0,2,1,0)	1	18	19	20	21	22	23	24	25	46	47	48	49	66	67	68	69	74	75	76	77
15	(1,2,1,0)	4	7	13	16	18	22	27	32	37	40	45	46	51	57	60	63	66	71	74	81	84
16	(2,2,1,0)	3	9	12	15	18	22	29	31	36	39	44	46	53	56	59	65	66	73	74	80	83
17	(3,2,1,0)	5	8	11	17	18	22	28	33	35	41	43	46	52	55	61	64	66	72	74	79	85
18	(0,3,1,0)	1	14	15	16	17	22	23	24	25	50	51	52	53	58	59	60	61	78	79	80	81
19	(1,3,1,0)	5	7	12	14	21	22	27	32	37	41	44	47	50	55	58	65	68	72	77	78	83
20	(2,3,1,0)	4	9	11	14	20	22	29	31	36	40	43	49	50	57	58	64	67	71	76	78	85
21	(3,3,1,0)	3	8	13	14	19	22	28	33	35	39	45	48	50	56	58	63	69	73	75	78	84
22	(0,0,0,1)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
23	(1,0,0,1)	2	6	10	14	18	23	27	31	35	39	43	47	51	55	59	63	67	71	75	79	83
24	(2,0,0,1)	2	6	10	14	18	25	29	33	37	41	45	49	53	57	61	65	69	73	77	81	85

25	(3,0,0,1)	2	6	10	14	18	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84
26	(0,0,1,1)	1	2	3	4	5	26	27	28	29	42	43	44	45	58	59	60	61	74	75	76	77
27	(1,0,1,1)	2	7	11	15	19	23	26	33	36	39	42	49	52	55	58	65	68	71	74	81	84
28	(2,0,1,1)	2	9	13	17	21	25	26	32	35	41	42	48	51	57	58	64	67	73	74	80	83
29	(3,0,1,1)	2	8	12	16	20	24	26	31	37	40	42	47	53	56	58	63	69	72	74	79	85
30	(0,0,2,1)	1	2	3	4	5	34	35	36	37	50	51	52	53	66	67	68	69	82	83	84	85
31	(1,0,2,1)	2	8	12	16	20	23	29	32	34	39	45	48	50	55	61	64	66	71	77	80	82
32	(2,0,2,1)	2	7	11	15	19	25	28	31	34	41	44	47	50	57	60	63	66	73	76	79	82
i	P_i	Π_i																				
33	(3,0,2,1)	2	9	13	17	21	24	27	33	34	40	43	49	50	56	59	65	66	72	75	81	82
34	(0,0,3,1)	1	2	3	4	5	30	31	32	33	46	47	48	49	62	63	64	65	78	79	80	81
35	(1,0,3,1)	2	9	13	17	21	23	28	30	37	39	44	46	53	55	60	62	69	71	76	78	85
36	(2,0,3,1)	2	8	12	16	20	25	27	30	36	41	43	46	52	57	59	62	68	73	75	78	84
37	(3,0,3,1)	2	7	11	15	19	24	29	30	35	40	45	46	51	56	61	62	67	72	77	78	83
38	(0,1,0,1)	1	6	7	8	9	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53
39	(1,1,0,1)	3	6	11	16	21	23	27	31	35	38	42	46	50	57	61	65	69	72	76	80	84
40	(2,1,0,1)	5	6	13	15	20	25	29	33	37	38	42	46	50	56	60	64	68	71	75	79	83
41	(3,1,0,1)	4	6	12	17	19	24	28	32	36	38	42	46	50	55	59	63	67	73	77	81	85
42	(0,1,1,1)	1	10	11	12	13	26	27	28	29	38	39	40	41	66	67	68	69	78	79	80	81
43	(1,1,1,1)	3	7	10	17	20	23	26	33	36	38	43	48	53	57	60	63	66	72	77	78	83
44	(2,1,1,1)	5	9	10	16	19	25	26	32	35	38	45	47	52	56	59	65	66	71	76	78	85
45	(3,1,1,1)	4	8	10	15	21	24	26	31	37	38	44	49	51	55	61	64	66	73	75	78	84
46	(0,1,2,1)	1	14	15	16	17	34	35	36	37	38	39	40	41	62	63	64	65	74	75	76	77
47	(1,1,2,1)	3	8	13	14	19	23	29	32	34	38	44	49	51	57	59	62	68	72	74	79	85
48	(2,1,2,1)	5	7	12	14	21	25	28	31	34	38	43	48	53	56	61	62	67	71	74	81	84
49	(3,1,2,1)	4	9	11	14	20	24	27	33	34	38	45	47	52	55	60	62	69	73	74	80	83
50	(0,1,3,1)	1	18	19	20	21	30	31	32	33	38	39	40	41	58	59	60	61	82	83	84	85
51	(1,1,3,1)	3	9	12	15	18	23	28	30	37	38	45	47	52	57	58	64	67	72	75	81	82
52	(2,1,3,1)	5	8	11	17	18	25	27	30	36	38	44	49	51	56	58	63	69	71	77	80	82
53	(3,1,3,1)	4	7	13	16	18	24	29	30	35	38	43	48	53	55	58	65	68	73	76	79	82
54	(0,2,0,1)	1	6	7	8	9	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85
55	(1,2,0,1)	4	6	12	17	19	23	27	31	35	41	45	49	53	56	60	64	68	70	74	78	82
56	(2,2,0,1)	3	6	11	16	21	25	29	33	37	40	44	48	52	55	59	63	67	70	74	78	82
57	(3,2,0,1)	5	6	13	15	20	24	28	32	36	39	43	47	51	57	61	65	69	70	74	78	82
58	(0,2,1,1)	1	18	19	20	21	26	27	28	29	50	51	52	53	62	63	64	65	70	71	72	73
59	(1,2,1,1)	4	7	13	16	18	23	26	33	36	41	44	47	50	56	61	62	67	70	75	80	85
60	(2,2,1,1)	3	9	12	15	18	25	26	32	35	40	43	49	50	55	60	62	69	70	77	79	84
61	(3,2,1,1)	5	8	11	17	18	24	26	31	37	39	45	48	50	57	59	62	68	70	76	81	83
62	(0,2,2,1)	1	10	11	12	13	34	35	36	37	46	47	48	49	58	59	60	61	70	71	72	73
63	(1,2,2,1)	4	8	10	15	21	23	29	32	34	41	43	46	52	56	58	63	69	70	76	81	83
64	(2,2,2,1)	3	7	10	17	20	25	28	31	34	40	45	46	51	55	58	65	68	70	75	80	85
65	(3,2,2,1)	5	9	10	16	19	24	27	33	34	39	44	46	53	57	58	64	67	70	77	79	84
66	(0,2,3,1)	1	14	15	16	17	30	31	32	33	42	43	44	45	66	67	68	69	70	71	72	73
i	P_i	Π_i																				
67	(1,2,3,1)	4	9	11	14	20	23	28	30	37	41	42	48	51	56	59	65	66	70	77	79	84
68	(2,2,3,1)	3	8	13	14	19	25	27	30	36	40	42	47	53	55	61	64	66	70	76	81	83
69	(3,2,3,1)	5	7	12	14	21	24	29	30	35	39	42	49	52	57	60	63	66	70	75	80	85
70	(0,3,0,1)	1	6	7	8	9	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69
71	(1,3,0,1)	5	6	13	15	20	23	27	31	35	40	44	48	52	54	58	62	66	73	77	81	85
72	(2,3,0,1)	4	6	12	17	19	25	29	33	37	39	43	47	51	54	58	62	66	72	76	80	84
73	(3,3,0,1)	3	6	11	16	21	24	28	32	36	41	45	49	53	54	58	62	66	71	75	79	83
74	(0,3,1,1)	1	14	15	16	17	26	28	29	45	46	47	48	49	54	55	56	57	82	83	84	85
75	(1,3,1,1)	5	7	12	14	21	23	26	33	36	40	45	46	51	54	59	64	69	73	76	79	82
76	(2,3,1,1)	4	9	11	14	20	25	26	32	35	39	44	46	53	54	61	63	68	72	75	81	82
77	(3,3,1,1)	3	8	13	14	19	24	26	31	37	41	43	46	52	54	60	65	67	71	77	80	82
78	(0,3,2,1)	1	18	19	20	21	34	35	36	37	42	43	44	45	54	55	56	57	78	79	80	81
79	(1,3,2,1)	5	8	11	17	18	23	29	32	34	40	42	47	53	54	60	65	67	73	75	78	84
80	(2,3,2,1)	4	7	13	16	18	25	28	31	34	39	42	49	52	54	59	64	69	72	77	78	83
81	(3,3,2,1)	3	9	12	15	18	24	27	33	34	41	42	48	51	54	61	63	68	71	76	78	85
82	(0,3,3,1)	1	10	11	12	13	30	31	32	33	50	51	52	53	54	55	56	57	74	75	76	77
83	(1,3,3,1)	5	9	10	16	19	23	28	30	37	40	43	49	50	54	61	63	68	73	74	80	83
84	(2,3,3,1)	4	8	10	15	21	25	27	30	36	39	45	48	50	54	60	65	67	72	74	79	85
85	(3,3,3,1)	3	7	10	17	20	24	29	30	35	41	44	47	50	54	59	64	69	71	74	81	84

Table (5) Plans and lines of PG (3 , 4)

	2	6	10	14	18	22	26	30	34	38	42	46	50	54	58	62	66	70	74	78	82
	6	22	22	22	22	2	2	2	2	6	14	10	18	6	18	14	10	6	10	18	14
1	10	26	42	50	46	38	42	46	50	42	30	34	26	58	30	34	26	74	30	34	26
	14	30	62	58	66	54	58	62	66	46	66	58	62	62	38	38	38	78	50	42	46
	18	34	82	78	74	70	74	78	82	50	70	70	70	66	82	74	78	82	54	54	54
	1	6	7	8	9	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37
	6	22	22	22	22	1	7	6	6	9	6	1	8	9	7	1	9	1	7	8	8
2	7	26	27	28	29	23	26	28	29	25	23	26	23	23	25	30	24	35	24	25	24
	8	30	32	33	31	24	33	32	33	32	31	27	32	28	28	31	27	36	29	27	26
	9	34	37	35	36	25	36	36	37	35	35	29	34	37	34	33	34	37	30	30	31
	3	6	11	16	21	22	26	30	34	39	43	47	51	56	60	64	68	73	77	81	85
	6	22	22	22	22	3	11	21	16	6	3	16	21	6	11	3	3	16	11	21	6
3	11	26	43	51	47	39	39	39	39	43	26	26	26	60	34	30	34	30	30	34	73
	16	30	64	60	68	56	68	60	64	47	60	56	64	64	47	47	51	43	51	43	77
	21	34	85	81	77	73	81	85	77	51	77	85	73	68	73	81	85	68	56	56	81
	5	6	13	15	20	22	26	30	34	41	45	49	53	55	59	63	67	72	76	80	84
	6	22	22	22	22	5	13	13	20	6	15	13	20	6	20	15	5	6	5	5	15
4	13	26	45	53	49	41	41	53	45	45	30	34	26	59	30	34	34	76	26	30	26
	15	30	63	59	67	55	67	55	55	49	67	59	63	63	41	41	53	80	45	49	49
	20	34	84	80	76	72	80	76	80	53	72	72	72	67	84	76	84	84	59	63	55
	4	6	12	17	19	22	26	30	34	40	44	48	52	57	61	65	69	71	75	79	83
	6	22	22	22	22	4	4	4	4	6	17	17	12	6	12	17	12	19	6	19	19
5	12	26	44	52	48	40	44	48	52	44	30	26	30	61	34	34	26	26	71	34	30
	17	30	65	61	69	57	61	65	69	48	69	57	57	65	48	40	40	52	79	44	40
	19	34	83	79	75	71	75	79	83	52	71	83	75	69	71	75	79	65	83	57	61
	1	2	3	4	5	22	23	24	25	38	39	40	41	54	55	56	57	70	71	72	73
	2	22	22	22	22	1	2	2	2	1	5	3	4	5	1	5	3	1	3	4	4
6	3	38	39	40	41	23	39	40	41	39	24	25	23	23	54	25	23	71	24	25	24
	4	54	56	57	55	24	55	56	57	40	57	55	56	40	56	38	38	72	41	39	38
	5	70	73	71	72	25	71	72	73	41	70	70	70	73	57	71	72	73	54	54	55
	2	7	11	15	19	22	27	32	37	38	43	48	53	54	59	64	69	70	75	80	85
	7	22	22	22	22	2	2	2	2	7	15	11	19	15	7	15	11	7	11	19	19
7	11	27	43	53	48	38	43	48	53	43	32	37	27	27	54	37	27	75	32	37	32
	15	32	64	59	69	54	59	64	69	48	69	59	64	48	64	38	38	80	53	43	38
	19	37	85	80	75	70	75	80	85	53	70	70	70	85	69	75	48	85	54	54	59
	2	9	13	17	21	22	29	31	36	38	45	47	52	54	61	63	68	70	77	79	84
	9	22	22	22	22	2	2	2	2	21	9	13	21	13	9	17	13	17	9	21	17
8	13	29	45	52	47	38	45	47	52	31	38	36	29	31	54	36	29	31	70	36	29
	17	31	63	61	68	54	61	63	68	61	47	61	63	52	63	38	38	45	79	45	47
	21	36	84	79	77	70	77	79	84	84	52	70	70	77	68	77	79	68	84	54	54
	2	8	12	16	20	22	28	33	35	38	44	49	51	54	60	65	67	70	76	81	83
	8	22	22	22	22	2	2	2	2	8	16	16	12	8	12	16	12	20	8	20	20
9	12	28	44	51	49	38	44	49	51	44	33	28	33	60	35	35	28	28	70	35	33
	16	33	65	60	67	54	60	65	67	49	67	54	54	65	49	38	38	51	81	44	38
	20	35	83	81	76	70	76	81	83	51	70	83	76	67	70	76	81	65	83	54	60
	1	10	11	12	13	22	23	24	25	42	43	44	45	62	63	64	65	82	83	84	85
	10	22	22	22	22	1	10	10	10	1	12	11	12	13	1	13	13	1	11	11	12
10	11	42	43	44	45	23	43	44	45	43	25	25	23	23	62	25	24	83	24	23	24
	12	62	64	65	63	24	63	64	65	44	62	63	64	44	64	42	43	84	45	42	42
	13	82	85	83	84	25	83	84	85	45	84	82	82	85	65	83	82	85	62	65	63
	3	7	10	17	20	22	27	32	37	39	42	49	52	56	61	62	67	73	76	79	82
	7	22	22	22	22	3	3	10	10	10	7	17	20	20	20	7	17	7	17	3	3
11	10	27	42	52	49	39	42	52	49	27	39	27	27	37	32	56	32	76	37	32	37
	17	32	62	61	67	56	61	56	61	67	49	56	62	42	39	61	42	79	39	49	52
	20	37	82	79	76	73	76	76	73	79	52	82	73	79	82	67	73	82	62	62	67
12	5	9	10	16	19	22	29	31	36	41	42	48	51	55	60	62	69	72	75	81	82
	9	22	22	22	22	5	10	10	10	16	16	16	9	9	5	5	5	19	9	19	19
	10	29	42	51	48	41	41	51	48	36	31	29	41	60	29	31	36	29	72	36	31
	16	31	62	60	69	55	69	55	60	62	69	55	42	62	42	48	51	51	81	42	41

	19	36	82	81	75	72	81	75	72	75	72	82	48	69	75	81	82	62	82	55	60
13	4	8	10	15	21	22	28	33	35	40	42	47	53	57	59	62	68	71	77	80	82
	8	22	22	22	22	4	10	10	10	15	15	15	8	21	21	21	8	8	4	4	4
	10	28	42	53	47	40	40	53	47	35	33	28	40	35	33	28	57	77	28	33	35
	15	33	62	59	68	57	68	57	59	62	68	57	42	42	40	53	59	80	42	47	53
	21	35	82	80	77	71	80	77	71	77	71	82	47	80	82	71	62	82	59	62	68
14	1	18	19	20	21	22	23	24	25	46	47	48	49	66	67	68	69	74	75	76	77
	18	22	22	22	22	1	18	18	18	1	20	21	21	20	1	19	21	1	20	19	19
	19	46	48	49	47	23	47	48	49	47	24	25	24	23	66	23	23	75	25	25	24
	20	66	69	67	68	24	67	68	69	48	69	67	66	48	68	49	46	76	46	47	46
	21	74	75	76	77	25	75	76	77	49	74	74	75	77	69	74	76	77	68	66	67
15	4	7	13	16	18	22	27	32	37	40	45	46	51	57	60	63	66	71	74	81	84
	7	22	22	22	22	4	18	18	18	7	16	16	13	7	13	16	13	7	4	4	4
	13	27	45	51	46	40	51	40	45	45	32	27	32	60	37	37	27	74	27	32	37
	16	32	63	60	66	57	63	60	57	46	66	57	57	63	46	40	40	81	45	46	51
	18	37	84	81	74	71	71	84	81	51	71	84	74	66	71	74	81	84	60	63	66
16	3	9	12	15	18	22	29	31	36	39	44	46	53	56	59	65	66	73	74	80	83
	9	22	22	22	22	3	18	18	18	15	15	15	9	12	12	9	12	9	3	3	3
	12	29	44	53	46	39	53	39	44	36	31	29	39	31	36	56	29	74	29	31	36
	15	31	65	59	66	56	65	59	56	65	66	56	44	53	46	59	39	80	44	46	53
	18	36	83	80	74	73	73	83	80	74	73	83	46	74	73	66	80	83	59	65	66
17	5	8	11	17	18	22	28	33	35	41	43	46	52	55	61	64	66	72	74	79	85
	8	22	22	22	22	5	11	11	11	18	18	8	18	8	5	17	5	17	8	5	17
	11	28	43	52	46	41	41	52	46	33	35	41	28	61	28	35	35	33	72	33	28
	17	33	64	61	66	55	66	55	61	61	55	43	64	64	43	41	52	43	79	46	46
	18	35	85	79	74	72	79	74	72	85	79	52	72	66	74	74	85	66	85	64	55
18	1	14	15	16	17	22	23	24	25	50	51	52	53	58	59	60	61	78	79	80	81
	14	22	22	22	22	1	14	14	14	1	17	15	16	1	16	17	15	1	15	16	17
	15	50	53	51	52	23	51	52	53	51	25	23	24	59	25	23	24	79	25	23	24
	16	58	59	60	61	24	59	60	61	52	58	58	58	60	52	53	51	80	50	50	50
	17	78	80	81	79	25	79	80	81	53	80	81	79	61	78	78	78	81	60	61	59
19	5	7	12	14	21	22	27	32	37	41	44	47	50	55	58	65	68	72	77	78	83
	7	22	22	22	22	5	14	14	14	21	21	5	7	12	7	21	5	12	5	12	7
	12	27	44	50	47	41	47	44	41	32	37	32	41	32	55	27	37	37	27	27	72
	14	32	65	58	68	55	55	68	65	58	55	65	44	50	65	50	50	47	44	41	77
	21	37	83	78	77	72	83	72	77	83	78	78	47	77	68	72	83	58	58	68	78
20	4	9	11	14	20	22	29	31	36	40	43	49	50	57	58	64	67	71	76	78	85
	9	22	22	22	22	4	14	14	14	20	20	11	9	9	4	4	4	20	11	11	9
	11	29	43	50	49	40	49	43	40	31	36	36	40	58	29	31	36	29	31	29	71
	14	31	64	58	67	57	57	67	64	58	57	58	43	64	43	49	50	50	50	40	76
	20	36	85	78	76	71	85	71	76	85	78	71	49	67	76	78	85	64	57	67	78
21	3	8	13	14	19	22	28	33	35	39	45	48	50	56	58	63	69	73	75	78	84
	8	22	22	22	22	3	3	3	13	14	14	8	13	14	19	8	13	19	18	19	3
	13	28	45	50	48	39	45	48	48	28	33	39	33	28	33	56	28	28	73	35	35
	14	33	63	58	69	56	58	63	58	35	69	45	56	48	39	58	39	50	78	45	50
	19	35	84	78	75	73	75	78	73	63	73	50	75	84	84	69	78	63	84	56	69
22	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
	2	6	6	6	6	1	2	2	2	1	5	3	4	1	4	5	3	1	3	4	5
	3	10	11	12	13	7	11	12	13	11	8	9	7	15	8	9	7	19	8	9	7
	4	14	16	17	15	8	15	16	17	12	17	15	16	16	10	10	10	20	13	11	12
	5	18	21	19	20	9	19	20	21	13	18	18	18	17	21	19	20	21	14	14	14
23	2	6	10	14	18	23	27	31	35	39	43	47	51	55	59	63	67	71	75	79	83
	6	23	23	23	23	2	10	10	10	14	14	6	18	18	18	6	2	6	2	2	14
	10	27	43	51	47	39	39	51	47	35	31	39	27	35	31	55	35	75	27	31	27
	14	31	63	59	67	55	67	55	59	63	67	43	63	43	39	59	51	79	43	47	55
	18	35	83	79	75	71	79	75	71	75	71	51	71	79	83	67	83	83	59	63	47
24	2	6	10	14	18	25	29	33	37	41	45	49	53	57	61	65	69	73	77	81	85
	6	25	25	25	25	2	10	10	10	18	18	6	18	14	2	6	2	14	14	2	6
	10	29	45	53	49	41	41	53	49	33	37	41	29	29	29	57	37	33	37	33	73

	14	33	65	61	69	57	69	57	61	61	57	45	65	49	45	61	53	45	41	49	77
	18	37	85	81	77	73	81	77	73	85	81	53	73	85	77	69	85	69	65	65	81
	2	6	10	14	18	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84
	6	24	24	24	24	2	14	14	18	18	2	6	10	6	10	18	10	6	14	2	2
25	10	28	44	52	48	40	48	44	44	32	28	40	32	60	36	28	28	76	36	32	36
	14	32	64	60	68	56	56	68	56	60	60	44	56	64	48	52	40	80	40	48	52
	18	36	84	80	76	72	84	72	80	84	76	52	76	68	72	72	80	84	64	64	68
	1	2	3	4	5	26	27	28	29	42	43	44	45	58	59	60	61	74	75	76	77
	2	26	26	26	26	1	2	2	2	1	5	3	4	1	4	5	3	1	3	4	5
26	3	42	43	44	45	27	43	44	45	43	28	29	27	59	28	29	27	75	28	29	27
	4	58	60	61	59	28	59	60	61	44	61	59	60	60	42	42	42	76	45	43	44
	5	74	77	75	76	29	75	76	77	45	74	74	74	61	77	75	76	77	58	58	58
	2	7	11	15	19	23	26	33	36	39	42	49	52	55	58	65	68	71	74	81	84
	7	23	23	23	23	2	15	15	15	11	19	11	7	11	19	7	2	19	2	2	7
27	11	26	42	52	49	39	49	42	39	26	36	36	39	33	33	55	36	26	26	33	71
	15	33	65	58	68	55	55	68	65	68	55	58	42	52	39	58	52	52	42	49	74
	19	36	84	81	74	71	84	71	74	81	81	71	49	74	84	68	84	65	58	65	81
	2	9	13	17	21	25	26	32	35	41	42	48	51	57	58	64	67	73	74	80	83
	9	25	25	25	25	2	2	2	13	9	21	17	2	13	21	21	9	17	17	13	9
28	13	26	42	51	48	41	42	48	48	42	35	26	35	32	32	26	57	32	35	26	73
	17	32	64	58	67	57	58	64	58	48	57	57	67	51	41	51	58	42	41	41	74
	21	35	83	80	74	73	74	80	73	51	80	83	83	74	83	73	64	67	64	67	80
	2	8	12	16	20	24	26	31	37	40	42	47	53	56	58	63	69	72	74	79	85
	8	24	24	24	24	2	20	2	2	8	20	16	12	8	20	16	16	12	2	12	8
29	12	26	42	53	47	40	53	47	53	42	37	26	31	58	31	37	31	37	26	26	72
	16	31	63	58	69	56	63	63	69	47	56	56	56	63	40	40	42	47	42	40	74
	20	37	85	79	74	72	72	79	85	53	79	85	74	69	85	74	72	58	58	69	79
	1	2	3	4	5	34	35	36	37	50	51	52	53	66	67	68	69	82	83	84	85
	2	34	34	34	34	1	2	2	2	3	1	5	3	1	3	5	5	4	1	4	4
30	3	50	51	52	53	35	51	52	53	35	50	35	36	67	37	37	36	35	82	37	36
	4	66	68	69	67	36	67	68	69	69	52	66	66	68	52	50	51	53	84	51	50
	5	82	85	83	84	37	83	84	85	84	53	85	83	69	82	83	82	68	85	66	67
	2	8	12	16	20	23	29	32	34	39	45	48	50	55	61	64	66	71	77	80	82
	8	23	23	23	23	2	2	2	2	8	16	16	12	8	12	20	12	8	16	20	20
31	12	29	45	50	48	39	45	48	50	45	32	29	32	61	34	29	29	77	34	34	32
	16	32	64	61	66	55	61	64	66	48	66	55	55	64	48	50	39	80	39	45	39
	20	34	82	80	77	71	77	80	82	50	71	82	77	66	71	71	80	82	64	55	61
	2	7	11	15	19	25	28	31	34	41	44	47	50	57	60	63	66	73	76	79	82
	7	25	25	25	25	2	2	2	2	7	15	15	11	7	11	19	11	7	15	19	19
32	11	28	44	50	47	41	44	47	50	44	31	28	31	60	34	28	28	76	34	34	31
	15	31	63	60	66	57	60	63	66	47	66	57	57	63	47	50	41	79	41	44	41
	19	34	82	79	76	73	76	79	82	50	73	82	76	66	73	73	79	82	63	57	60
	2	9	13	17	21	24	27	33	34	40	43	49	50	56	59	65	66	72	75	81	82
	9	24	24	24	24	2	2	2	2	9	17	17	13	9	13	17	13	21	9	21	21
33	13	27	43	50	49	40	43	49	50	43	33	27	33	59	34	34	27	27	72	34	33
	17	33	65	59	66	56	59	65	66	49	66	56	56	65	49	40	40	50	81	43	40
	21	34	82	81	75	72	75	81	82	50	72	82	75	66	72	75	81	65	82	56	59
	1	2	3	4	5	30	31	32	33	46	47	48	49	62	63	64	65	78	79	80	81
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	6	23	23	23	23	5	5	5	5	6	15	15	13	6	13	15	13	20	6	20	20
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	15	31	62	58	66	54	58	62	66	48	66	54	54	62	48	40	40	52	81	44	40

	20	35	85	81	77	73	77	81	85	52	73	85	77	66	73	77	81	62	85	54	58
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	6	25	25	25	25	4	4	4	4	6	17	17	12	6	12	17	12	19	6	19	19
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	3	6	11	16	21	24	28	32	36	41	45	49	53	54	58	62	66	71	75	79	83
	6	24	24	24	24	3	3	3	3	6	16	16	11	6	11	16	11	21	6	21	21
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	1	14	15	16	17	26	28	29	46	47	47	48	49	54	55	56	57	82	83	84	85
	14	26	26	26	26	1	14	14	14	1	15	15	16	1	16	15	16	17	1	17	17
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	7	23	23	23	23	5	5	5	5	7	14	14	12	7	12	14	12	21	7	21	21
75	12	26	45	51	46	40	45	46	51	45	33	26	33	59	36	36	26	26	73	36	33
	14	33	64	59	69	54	59	64	69	46	69	54	54	64	46	40	40	51	76	45	40
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	9	25	25	25	25	4	4	4	4	9	14	14	11	9	11	14	11	20	9	20	20
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	5	8	11	17	18	23	29	32	34	40	42	47	53	54	60	65	67	73	75	78	84
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	9	24	24	24	24	3	3	3	3	9	15	15	12	9	12	15	12	18	9	18	18
81	12	27	42	51	48	41	42	48	51	42	33	27	33	61	34	34	27	27	71	34	33
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	10	30	30	30	30	1	10	10	10	1	12	11	11	1	12	12	11	13	13	1	13
82	11	50	51	52	53	31	51	52	53	51	33	33	32	55	32	31	31	32	33	74	31
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	13	74	77	75	76	33	75	76	77	53	76	74	75	57	77	74	76	57	56	77	54
83	5	9	10	16	19	23	28	30	37	40	43	49	50	54	61	63	68	73	74	80	83
	9	23	23	23	23	5	5	5	5	9	16	16	10	9	10	16	10	19	9	19	19
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	16	30	63	61	68	54	61	63	68	49	68	54	54	63	49	40	40	50	80	43	40
	19	37	83	80	74	73	74	80	83	50	73	83	74	68	73	74	80	63	83	54	61
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	8	25	25	25	25	4	4	4	4	8	15	15	10	8	10	15	10	21	8	21	21
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	3	7	10	17	20	24	29	30	35	41	44	47	50	54	59	64	69	71	74	81	84
	7	24	24	24	24	3	3	3	3	7	17	17	10	7	10	17	10	20	7	20	20
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	17	30	64	59	69	54	59	64	69	47	69	54	54	64	47	41	41	50	81	44	41
	20	35	84	81	74	71	74	81	84	50	71	84	74	69	71	74	81	64	84	54	59

الامتداد - (k,ℓ) في الفضاء الاسقاطي ثلاثي الأبعاد PG (3,p) حول حقل كالوا ، عندما p = 4

فاطمة فيصل كريم
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المستخلص

الغرض من هذا البحث هو دراسة الفضاء الاسقاطي ثلاثي الابعاد PG(3,p) ،عندما p=4 ،وباستخدام المعادلات الجبرية وجدنا النقاط والمستقيمات والمستويات ،وفي هذا الفضاء قمنا بإنشاء الامتداد - (k,ℓ) الذي هومجموعة k من المستقيمات بحيث لا يتقاطع أي اثنين منهما . وقد برهنا ان اعظم امتداد كامل (k,ℓ) في الفضاء الاسقاطي PG(3,4) هو الامتداد - (17,ℓ) ،وهو يساوي جميع نقاط الفضاء ويسمى ناشر.