Artin Exponent for the Special linear group SL(2,p^k) where p^k = 9, 25 and 27

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Abstract

In this paper we calculate the Artin exponent for the Special linear group SL(2,p^k) where p is 9, 25 and 27, from the cyclic subgroups of this groups which is equal to the order of the group.

Introduction

Let G be a finite group, all characters of G induced from a principal character of cyclic subgroups of G are called Artin characters of G. Artin induction theorem [1] states that any rational valued character of G is a rational linear combination of the induced principal character of its cyclic subgroups. Lam [7] proved a sharp form of Artin's theorem, he determined the least positive integer A(G) such that A(G)χ is an integral linear combination of Artin character, for any rational valued character χ of G, and he called A(G) the Artin exponent of G and studied it extensively for many groups.

The group of invertible $n \times n$ matrices over a field F denoted by GL(n,F). The determinant of these matrices is a homomorphism from GL(n,F) into F-{0} and we denote the kernel of this homomorphism by SL(n,F), the special linear group. Thus SL(n,F) is the subgroup of GL(n,F) which contains all matrices of determinant one over the field F

In this work we take n = 2 and choose F = p^k = 9, 25 and 27 and we count all cyclic subgroup of SL(2,p^k), then we found Artin characters (induced characters) tables from these cyclic subgroups of all cases of p^k, and written every rational valued character of SL(2,p^k) as an integral linear combination of Artin characters.

Also our main results is found Artin exponent of SL(2,p^k) for all cases of p^k which is equal to order SL(2,p^k) and denoted by A(SL(2,p^k)).

§.1 Preliminaries

In this section, we recall some definitions and theorems which we need.

Definition 1.1 : [5]

A rational valued character χ of G is a character whose valued are in Z, that is $\chi(x)\in Z$, for all $x\in G$.

Definition 1.2 **: [5]**

Let H be a subgroup of a group G, and ϕ be a class function of H. Then $\phi\uparrow^G$, the induced class function on G, is given by

$$\phi\uparrow(g)=\frac{1}{|H|}\sum_{x\in G}\phi^{\circ}(xgx^{-1})\ ,$$

where ϕ° is defined by $\phi^{\circ}(h)=\phi(h)$ if $h\in H$ and $\phi^{\circ}(y)=0$ if $y\notin H$.

Observe that $\phi\uparrow^G$ is a class function on G and $\phi\uparrow^G(1)=[G:H]\ \phi(1)$.

Another useful formula for computing $\phi\uparrow^G(g)$ explicitly is to choose representatives $x_1, x_2, \ldots, x_m$ for the m classes of H contained in the conjugacy class c_g in G which is given by

$$\phi\uparrow^G(g)=|C_G(g)|\sum_{i=1}^{m}\frac{\phi(x_i)}{|C_H(x_i)|}$$

where it is understood that $\varphi\uparrow^G(C_\alpha)=0$ if $H\cap Cl(g)=\varnothing$. This formula is immediate from the definition of $\phi\uparrow^G$ since as x runs over G, $xgx^{-1}=x_i$ for exactly $|C_G(g)|$ values of x. If H is a cyclic subgroup then

$$\phi\uparrow^G(g)=\frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\phi(x_i) \qquad \ldots(1.1)$$

Definition 1.3**: [8]**

The character induced from the principal character of a cyclic subgroups of G is called Artin character.

Definition 1.4 **: [8]**

Let G be a finite group and let χ be any rational valued character on G. The smallest positive number n such that,

$$n\chi=\sum_{c}a_c\phi_c$$

where $a_c\in Z$ and ϕ_c is Artin character, is called the Artin exponent of G and denoted by A(G).

Theorem 1.5 **: [8]**

Let 1 denote the principal character of G and $d\in Z$, then d is an Artin exponent of G if there exists (uniquely) integers $a_k\in Z$ such that

$$d\cdot 1=\sum_{k=1}^{q}a_k\mu_k$$

where $\mu_1, \ldots, \mu_k$ are the Artin characters.

Theorem 1.6 **: [8]**

For a subgroup H in G, A(H) divides A(G).

§.2 The Special Linear Group SL(2,p^k)

In this section, we introduce some concepts about the Special linear group.

Definition 2.1 **: [8]**

The general linear group is the group of invertible n×n matrices over a field F denoted by GL(n,F). The determinant of these matrices is a homomorphism from GL(n,F) into F^* and we denote the kernel of this homomorphism by SL(n,F), the special linear group. Thus SL(n,F) is the subgroup of GL(n,F) which contains all matrices of determinant one.

In this work we are interested in finite special linear group, and we choose F to be finite, we consider the case when n=2 and F=p^k, where p^k=9,25 and 27.

Theorem 2.2 **: [3]**

The order of SL(2,p^k) is $p^k\,(p^{2k} - 1)$.

Theorem 2.3 **: [3]**

G = SL(2,p^k) has exactly $p^k + 4$ conjugacy classes :

$$1, z, c, d, zc, zd, a, a^2, \ldots, a^{\frac{p^k-3}{2}}, b, b^2, \ldots, b^{\frac{p^k-1}{2}}$$

Let v be the generator of the cyclic multiplicative group F^*, $1 \le \ell \le (p^k - 3)/2$, $1 \le m \le (p^k - 1)/2$.Thus this conjugacy classes is satisfied.

So table (2.1) represented the conjugacy classes of SL(2,p^k)

g ∈ G	Notation	C_g	$\|C_g\|$	$\|C_G(g)\|$
$\begin{pmatrix}1&0\\0&1\end{pmatrix}$	1	C_1	1	$p^k\,(p^{2k} - 1)$
$\begin{pmatrix}-1&0\\0&-1\end{pmatrix}$	z	C_z	1	$p^k\,(p^{2k}-1)$
$\begin{pmatrix}1&0\\1&1\end{pmatrix}$	c	C_c	$(p^{2k} - 1)/2$	$2p^k$
$\begin{pmatrix}1&0\\v&1\end{pmatrix}$	d	C_d	$(p^{2k} - 1)/2$	$2p^k$
$\begin{pmatrix}-1&0\\-1&-1\end{pmatrix}$	zc	C_{zc}	$(p^{2k} - 1)/2$	$2p^k$
$\begin{pmatrix}-1&0\\-v&-1\end{pmatrix}$	zd	C_{zd}	$(p^{2k} - 1)/2$	$2p^k$
$\begin{pmatrix}v^\ell&0\\0&v^{-\ell}\end{pmatrix}$	a^ℓ	$C_{a\,\ell}$	$p^k\,(p^k + 1)$	$p^k - 1$
Element of order(p^k+1)m	b^m	$C_{b\,m}$	$p^k\,(p^k - 1)$	$p^k + 1$

Table (2.1)

Now, in the following notation we refer to how the rational valued character of SL(2,p^k) is obtain.

Notation 2.4 **: [2]**

Let G = SL(2,p^k) for some prime $p \neq 2$, e and e' denote divisors of $p^k - 1$ such that $e < \frac{p^k-1}{2}$ and $e' < \frac{p^k-1}{2}$, f and f' denote divisors of $p^k + 1$ such that $f < \frac{p^k+1}{2}$ and $f' < \frac{p^k+1}{2}$, ρ_e is a primitive $(\frac{p^k-1}{e})$-th root of unity , σ_f is a primitive $(\frac{p^k+1}{f})$-th root of unity, 1, z, c, d, a, b are as in theorem (2.3), $\varepsilon = (-1)^{(p^k-1)/2}$, $\rho \in$ C be a $(p^k - 1)$-th root of unity and $\sigma \in$ C be a $(p^k + 1)$-th root of unity.

$$B(k)=\begin{cases}1 & \text{if } k \text{ is even}\\ 2 & \text{otherwise}\end{cases},\ E(p^k)=\begin{cases}1 & \text{if } p^k \equiv 3 \bmod 4\\ 2 & \text{otherwise}\end{cases},\ A(e)=\frac{1}{2}\Phi(\frac{p^k-1}{e}),\ C(f)=\frac{1}{2}\Phi(\frac{p^k+1}{f})$$

$\tau_1(e,e') = \sum_{\alpha\in\Gamma}(\rho_e^{e'}+\rho_e^{-e'})^{\alpha} = \frac{\Phi(\frac{p^k-1}{e})}{\Phi(\frac{p^k-1}{ee'})}\mu(\frac{p^k-1}{ee'})$, where $\Gamma = \Gamma(Q(\chi_e):Q)$.

[Note that $\Gamma = \Gamma(Q(\rho_e + \rho_e^{-1}):Q)$].

$\tau_2(f,f') = \sum_{\alpha\in\Gamma_1}(\sigma_f^{f'}+\sigma_f^{-f'})^{\alpha} = \frac{\Phi(\frac{p^k+1}{f})}{\Phi(\frac{p^k+1}{ff'})}\mu(\frac{p^k+1}{ff'})$,where $\Gamma_1 = \Gamma(Q(\theta_f):Q)$.

[Note that $\Gamma = \Gamma(Q(\sigma_f + \sigma_f^{-1}):Q)$]. $\chi_e = B(e)\sum_{\alpha\in\Gamma}\chi_i^{\alpha}$ where $e = (i,p^k - 1)$. $\theta_j = B(f)\sum_{\alpha\in\Gamma_1}\theta_j^{\alpha}$ where $f = (i,p^k + 1)$. ξ' and η' denote the irreducible characters of the rational representations of G arising from ξ_1 (or ξ_2) and η_1 (or η_2) respectively where k is odd. Also we know that the column for the class zc is obtained from the relation $\chi(zc) = \frac{\chi(z)}{\chi(1)}\chi(c)$ where χ is an irreducible character of G.

The character table of rational representations of SL(2,p^k), p an odd prime, k odd is described in table (2.2).

Table (2.2)

§.3 Artin Character and Artin Exponent of SL(2,p^k)

Cg	1	z	c	zc	$a^{e'}$	$b^{f'}$
| Cg |	1	1	$(p^{2k}-1)/2$	$(p^{2k}-1)/2$	$p^k(p^k+1)$	$p^k(p^k-1)$
| $C_G(g)$ |	$p^k(p^{2k}-1)$	$p^k(p^{2k}-1)$	$2p^k$	$2p^k$	p^k-1	p^k+1
1_G	1	1	1	1	1	1
ψ	p^k	p^k	0	0	1	-1
χ_e	$(p^k+1)A(e)B(e)$	$(-1)^e(p^k+1)A(e)B(e)$	$A(e)B(e)$	$(-1)^e A(e)B(e)$	$B(e)\tau_1(e,e')$	0
θ_f	$(p^k-1)C(f)B(f)$	$(-1)^f(p^k-1)C(f)B(f)$	$-C(f)B(f)$	$-(-1)^f C(f)B(f)$	0	$-B(e)\tau_2(f,f')$
ζ'	(p^k+1)	$\varepsilon(p^k+1)$	1	ε	$(-1)^{e'}2$	0
η'	$(p^k-1)E(p^k)$	$-\varepsilon(p^k-1)E(p^k)$	-1	ε	0	$(-1)^{f'+1}2E(p^k)$

In this section, we found the induced character (Artin character) of SL(2,p^k), for $p^k = 9, 25$ and 27 from the cyclic subgroups of SL(2,p^k) by using the formula (1.1) in definition (1.2), and we written the rational valued character of SL(2,p^k) as a linear combination of Artin character, and count the least positive integer A(SL(2,p^k)) such that A(SL(2,p^k))χ is an integral linear combination of Artin character for all rational valued character χ of SL(2,p^k).

(3.1) Artin Character and Artin Exponent of SL(2,9):

This group has 8 cyclic subgroups which are:

H_1 , H_z , H_c , H_{zc} , H_a , H_{a^2} , H_b and H_{b^2} .

The rational valued character table of SL(2,9) is given in the table (3.1.1), [6].

Table (3.1.1)

C_g	1	z	c	zc	a	a^2	b	b^2
$\|C_g\|$	1	1	40	40	90	90	72	72
$\|C_G(g)\|$	720	720	18	18	8	8	10	10
1_G	1	1	1	1	1	1	1	1
ψ	9	9	0	0	1	1	-1	-1
χ_1	20	-20	2	-2	0	0	0	0
χ_2	10	10	1	1	0	-2	0	0
θ_1	16	-16	-2	2	0	0	-1	1
θ_2	16	16	-2	-2	0	0	1	1
ξ'	10	10	1	1	-2	2	0	0
η'	8	-8	-1	1	0	0	2	-2

We calculate the Artin character (induced character) of the cyclic subgroups by using the formula

$$\phi_H \uparrow^{SL(2,9)}(\alpha) = \frac{|C_{SL(2,9)}(X)|}{|C_H(\alpha)|} \sum_{\substack{\alpha \to X \\ H \to SL(2,9)}} \phi(\alpha)$$

and given it in the table (3.1.2)

Table (3.1.2)

C_g	1	z	c	zc	a	a^2	b	b^2
Φ_1	720	0	0	0	0	0	0	0
Φ_2	360	360	0	0	0	0	0	0
Φ_3	80	0	2	0	0	0	0	0
Φ_4	42	120	3	3	0	0	0	0
Φ_5	90	180	0	0	2	0	0	0
Φ_6	180	360	0	0	0	4	0	0
Φ_7	72	144	0	0	0	0	2	0
Φ_8	144	0	0	0	0	0	0	2

From tables (3.1.1) and (3.1.2) we can written the rational valued character of SL(2,9) as a linear combination of induced characters, and count the least positive integer A(SL(2,9)) (Artin exponent) such that $A(SL(2,9))\chi_i$ is an integral linear combination of Φ_i, i=1,…,8 where χ_i is any rational valued character.

$$1=\frac{1}{2}\Phi_8+\frac{1}{2}\Phi_7+\frac{1}{4}\Phi_6+\frac{1}{2}\Phi_5+\frac{1}{3}\Phi_4-\frac{291}{360}\Phi_2+\frac{80}{720}\Phi_1$$

$$\psi=-\frac{1}{2}\Phi_8-\frac{1}{2}\Phi_7+\frac{1}{4}\Phi_6+\frac{1}{2}\Phi_5-\frac{99}{360}\Phi_2+\frac{126}{720}\Phi_1$$

$$\chi_1=-\frac{2}{3}\Phi_4+2\Phi_3+\frac{60}{360}\Phi_2-\frac{332}{720}\Phi_1$$

$$\chi_2=-\frac{1}{2}\Phi_6+\frac{1}{3}\Phi_4+\frac{150}{360}\Phi_2-\frac{64}{720}\Phi_1$$

$$\theta_1=\frac{1}{2}\Phi_8-\frac{1}{2}\Phi_7+\frac{2}{3}\Phi_4-2\Phi_3-\frac{24}{360}\Phi_2+\frac{136}{720}\Phi_1$$

$$\theta_2=\frac{1}{2}\Phi_8+\frac{1}{2}\Phi_7-\frac{2}{3}\Phi_4+\frac{24}{360}\Phi_2-\frac{88}{720}\Phi_1$$

$$\zeta'=\frac{1}{2}\Phi_6-\Phi_5+\frac{1}{3}\Phi_4-\frac{30}{360}\Phi_2+\frac{26}{720}\Phi_1$$

$$\eta'=-\Phi_8+\Phi_7+\frac{1}{3}\Phi_4-\Phi_3-\frac{192}{360}\Phi_2+\frac{338}{720}\Phi_1$$

Note that $720\cdot\chi_i = (Z)\,\Phi_i$, i=1,…,8. So A(SL(2,9)) = 720.

***(3.2) Artin Character and Artin Exponent of SL(2,25)*:**

This group has 10 cyclic subgroups which are:

H_1 , H_Z, H_c , H_{zc} , H_a , H_{a^2} , H_{a^3} , H_{a^4} , H_{a^6} , H_{a^8} , H_b , and H_{b^2} .

The rational valued character table of SL(2,25) is given in the table (3.2.1), [6]

Table (3.2.1)

C_g	1	z	c	zc	a	a^2	a^3	a^4	a^6	a^8	b	b^2
$\lvert C_g \rvert$	1	1	312	312	650	650	650	650	650	650	600	600
$\lvert C_G(g) \rvert$	15600	15600	50	50	24	24	24	24	24	24	26	26
1_G	1	1	1	1	1	1	1	1	1	1	1	1
ψ	25	25	0	0	1	1	1	1	1	1	-1	-1
χ_1	104	-104	4	-4	0	0	0	4	0	-4	0	0
χ_2	52	52	2	2	0	2	0	-2	-4	-2	0	0
χ_3	52	-52	2	-2	0	0	0	-4	0	4	0	0
χ_4	26	26	1	1	1	-1	-2	-1	2	-1	0	0
χ_6	26	26	1	1	0	-2	0	2	-2	2	0	0
χ_8	26	26	1	1	-1	-1	2	-1	2	-1	0	0
θ_1	144	-144	-6	6	0	0	0	0	0	0	-1	1
θ_2	144	144	-6	-6	0	0	0	0	0	0	1	1
ξ'	26	26	1	1	-2	2	-2	2	2	2	0	0

η´	24	-24	-1	1	0	0	0	0	0	0	2	-2

We calculate the Artin character (induced character) of the cyclic subgroups by using the formula

$$\phi_H \uparrow^{SL(2,25)}(\alpha) = \frac{|C_{SL(2,25)}(x)|}{|C_H(\alpha)|} \sum_{\substack{\alpha \to x \\ H \to SL(2,25)}} \phi(\alpha)$$

and given it in the table (3.2.2).

Table (3.2.2)

C_g	1	z	c	zc	a	a^2	a^3	a^4	a^6	a^8	b	b^2
Φ_1	15600	0	0	0	0	0	0	0	0	0	0	0
Φ_2	7800	7800	0	0	0	0	0	0	0	0	0	0
Φ_3	624	0	2	0	0	0	0	0	0	0	0	0
Φ_4	312	936	3	3	0	0	0	0	0	0	0	0
Φ_5	650	1300	0	0	2	0	0	0	0	0	0	0
Φ_6	1300	2600	0	0	0	4	0	0	0	0	0	0
Φ_7	1950	3900	0	0	0	0	6	0	0	0	0	0
Φ_8	2600	0	0	0	0	0	0	4	0	0	0	0
Φ_9	3900	660	0	0	0	0	0	0	6	0	0	0
Φ_{10}	5200	0	0	0	0	0	0	0	0	4	0	0
Φ_{11}	600	1200	0	0	0	0	0	0	0	0	2	0
Φ_{12}	1200	0	0	0	0	0	0	0	0	0	0	2

From tables (3.2.1) and (3.2.2) we can written the rational valued character of SL(2,25) as a linear combination of induced characters, and count the least positive integer A(SL(2,25)) (Artin exponent) such that A(SL(2,25))χ_i is an integral linear combination(Z) of Φ_i, i = 1,…,12 whereχ_i is any rational valued character.

$$1 = \frac{1}{2}\Phi_{12} + \frac{1}{2}\Phi_{11} + \frac{1}{4}\Phi_{10} + \frac{1}{6}\Phi_9 + \frac{1}{4}\Phi_8 + \frac{1}{6}\Phi_7 + \frac{1}{4}\Phi_6 + \frac{1}{2}\Phi_5 + \frac{1}{3}\Phi_4 - \frac{2971}{7800}\Phi_2 - \frac{1612}{15600}\Phi_1$$

$$\psi = -\frac{1}{2}\Phi_{12} - \frac{1}{2}\Phi_{11} + \frac{1}{4}\Phi_{10} + \frac{1}{6}\Phi_9 + \frac{1}{4}\Phi_8 + \frac{1}{6}\Phi_7 + \frac{1}{4}\Phi_6 + \frac{1}{2}\Phi_5 - \frac{1435}{7800}\Phi_2 - \frac{1220}{15600}\Phi_1$$

$$\chi_1 = -\Phi_{10} + \Phi_8 - \frac{4}{3}\Phi_4 - \frac{1}{2}\Phi_3 + \frac{1144}{7800}\Phi_2 - \frac{1712}{15600}\Phi_1$$

$$\chi_2 = -\frac{1}{2}\Phi_{10} - \frac{2}{3}\Phi_9 - \frac{1}{2}\Phi_8 + \frac{1}{2}\Phi_6 + \frac{2}{3}\Phi_4 - \frac{1432}{7800}\Phi_2 - \frac{6158}{15600}\Phi_1$$

$$\chi_3 = \Phi_{10} - \Phi_8 - \frac{2}{3}\Phi_4 - \frac{1}{2}\Phi_3 + \frac{572}{7800}\Phi_2 - \frac{2600}{15600}\Phi_1$$

$$\chi_4 = -\frac{1}{4}\Phi_{10} + \frac{1}{3}\Phi_9 - \frac{1}{4}\Phi_8 - \frac{1}{3}\Phi_7 - \frac{1}{4}\Phi_6 + \frac{1}{2}\Phi_5 + \frac{1}{3}\Phi_4 + \frac{742}{7800}\Phi_2 + \frac{1964}{15600}\Phi_1$$

$$\chi_6 = \frac{1}{2}\Phi_{10} - \frac{1}{3}\Phi_9 + \frac{1}{2}\Phi_8 - \frac{1}{2}\Phi_6 + \frac{1}{3}\Phi_4 + \frac{1234}{7800}\Phi_2 - \frac{3262}{15600}\Phi_1$$

$$\chi_8 = -\frac{1}{4}\Phi_{10} + \frac{1}{3}\Phi_9 - \frac{1}{4}\Phi_8 + \frac{1}{3}\Phi_7 - \frac{1}{4}\Phi_6 - \frac{1}{2}\Phi_5 + \frac{1}{3}\Phi_4 - \frac{506}{7800}\Phi_2 + \frac{1078}{15600}\Phi_1$$

$$\theta_1 = \frac{1}{2}\Phi_{12} - \frac{1}{2}\Phi_{11} + 2\Phi_4 - 6\Phi_3 - \frac{1416}{7800}\Phi_2 + \frac{4380}{15600}\Phi_1$$

$$\theta_2 = \frac{1}{2}\Phi_{12} + \frac{1}{2}\Phi_{11} - 2\Phi_4 + \frac{1416}{7800}\Phi_2 - \frac{1548}{15600}\Phi_1$$

$$\zeta' = \frac{1}{2}\Phi_{10} + \frac{1}{3}\Phi_9 + \frac{1}{2}\Phi_8 - \frac{1}{3}\Phi_7 + \frac{1}{2}\Phi_6 - \Phi_5 + \frac{1}{3}\Phi_4 + \frac{794}{7800}\Phi_2 - \frac{1078}{15600}\Phi_1$$

$$\eta' = -\Phi_{12} + \Phi_{11} + \frac{1}{3}\Phi_4 - \Phi_3 - \frac{1536}{7800}\Phi_2 + \frac{2680}{15600}\Phi_1$$

Note that $15600\cdot\chi_i = (Z)\,\Phi_i$, i=1,…,12. So A(SL(2,25)) = 15600.

***3.3 Artin Character and Artin Exponent of SL(2,27)*:**

This group has 10 cyclic subgroups which are:

$H_1, H_z, H_c, H_{zc}, H_a, H_{a^2}, H_b\ H_{b^2}, H_{b^4}$ a nd H_{b^7}

The rational valued character table of SL(2,27) is given in the table (3.3.1), [6]

Table (3.3.1)

C_g	1	z	c	zc	a	a^2	b	b^2	b^4	b^7
$\lvert C_g \rvert$	1	1	364	364	756	756	702	702	702	702
$\lvert C_G(g) \rvert$	19656	19656	54	54	26	26	28	28	28	28
1_G	1	1	1	1	1	1	1	1	1	1
ψ	27	27	0	0	1	1	-1	-1	-1	-1
χ_1	168	-168	6	-6	1	-1	0	0	0	0
χ_2	168	168	6	6	-1	-1	0	0	0	0
θ_1	156	-156	-6	6	0	0	0	-2	2	0
θ_2	78	78	-3	-3	0	0	-1	1	1	-1
θ_4	78	78	-3	-3	0	0	1	1	1	1
θ_7	26	-26	-1	1	0	0	0	2	0	0
ξ'	28	-28	1	-1	-2	2	0	0	0	0
η'	26	26	-1	-1	0	0	2	-2	2	-2

We calculate the Artin character (induced character) of the cyclic subgroups by using the formula

$$\phi_H \uparrow^{SL(2,27)}(\alpha) = \frac{\left|C_{SL(2,27)}(x)\right|}{\left|C_H(\alpha)\right|} \sum_{\substack{\alpha\to x \\ H\to SL(2,27)}} \phi(\alpha)$$

and given it in the table (3.3.2).

Table (3.3.2)

C_g	1	z	c	zc	a	a^2	b	b^2	b^4	b^7
Φ_1	19656	0	0	0	0	0	0	0	0	0
Φ_2	9828	9828	0	0	0	0	0	0	0	0
Φ_3	728	0	2	0	0	0	0	0	0	0
Φ_4	366	1092	3	3	0	0	0	0	0	0
Φ_5	756	1512	0	0	2	0	0	0	0	0
Φ_6	1512	728	0	0	0	2	0	0	0	0
Φ_7	702	1404	0	0	0	0	2	0	0	0
Φ_8	1404	2808	0	0	0	0	0	4	0	0
Φ_9	2808	0	0	0	0	0	0	0	4	0
Φ_{10}	702	1404	0	0	0	0	0	0	0	2

From tables (3.3.1) and (3.3.2) we can written the rational valued character of SL(2,27) as a linear combination of induced characters, and count the least positive integer A(SL(2,27)) (Artin exponent) such that A(SL(2,27))χ_i is an integral linear combination(Z) of Φ_i, i = 1,…,10 whereχ_i is any rational valued character.

$$1 = \frac{1}{2}\Phi_{10} + \frac{1}{4}\Phi_9 + \frac{1}{4}\Phi_8 + \frac{1}{2}\Phi_7 + \frac{1}{2}\Phi_6 + \frac{1}{2}\Phi_5 + \frac{1}{3}\Phi_4 - \frac{3589}{9828}\Phi_2 + \frac{579}{19656}\Phi_1$$

$$\psi = -\frac{1}{2}\Phi_{10} - \frac{1}{4}\Phi_9 - \frac{1}{4}\Phi_8 - \frac{1}{2}\Phi_7 + \frac{1}{2}\Phi_6 + \frac{1}{2}\Phi_5 + \frac{1013}{9828}\Phi_2 - \frac{365}{19656}\Phi_1$$

$$\chi_1 = -\frac{1}{2}\Phi_6 + \frac{1}{2}\Phi_5 - 2\Phi_4 + 6\Phi_3 + \frac{1624}{9828}\Phi_2 - \frac{4714}{19656}\Phi_1$$

$$\chi_2 = -\frac{1}{2}\Phi_6 - \frac{1}{2}\Phi_5 + 2\Phi_4 - \frac{896}{9828}\Phi_2 + \frac{1466}{19656}\Phi_1$$

$$\theta_1 = \frac{1}{2}\Phi_9 - \frac{1}{4}\Phi_8 + 2\Phi_4 - 6\Phi_3 - \frac{936}{9828}\Phi_2 + \frac{4026}{19656}\Phi_1$$

$$\theta_2 = -\frac{1}{2}\Phi_{10} + \frac{1}{4}\Phi_9 + \frac{1}{4}\Phi_8 - \frac{1}{2}\Phi_7 - \Phi_4 + \frac{1872}{9828}\Phi_2 - \frac{1779}{19656}\Phi_1$$

$$\theta_4 = \frac{1}{2}\Phi_{10} + \frac{1}{4}\Phi_9 + \frac{1}{4}\Phi_8 + \frac{1}{2}\Phi_7 - \Phi_4 - \frac{936}{9828}\Phi_2 - \frac{375}{19656}\Phi_1$$

$$\theta_7 = \frac{1}{2}\Phi_8 + \frac{1}{3}\Phi_4 - \Phi_3 - \frac{1794}{9828}\Phi_2 + \frac{1724}{19656}\Phi_1$$

$$\zeta' = \Phi_6 - \Phi_5 - \frac{1}{3}\Phi_4 + \Phi_3 + \frac{1120}{9828}\Phi_2 - \frac{2454}{19656}\Phi_1$$

$$\eta' = -\Phi_{10} + \frac{1}{2}\Phi_9 - \frac{1}{2}\Phi_8 + \Phi_7 - \frac{1}{3}\Phi_4 + \frac{1794}{9828}\Phi_2 - \frac{2348}{19656}\Phi_1$$

Note that 19656·χ_i = (Z) Φ_i, i=1,…,10. So A(SL(2,27)) = 19656.

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أس آرتن للزمرة الخطية الخاصة
SL(2,p^k) عندما $p^k = 9, 25 , 27$

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المستخلص

في هذا البحث قمنا بحساب الشواخص المحتثة للزمرة الخطية الخاصة SL(2,p^k) حيث ان p^k يساوي 9 و 25 و 27 من الزمر الجزئية لتلك الزمر والذي يكون مساوي لرتبة الزمرة.