Some Types of (1,2)*- M -πgb Closed Mappings

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Abstract:
In this paper, we introduce and study new types of (1,2)*- M -πgb closed mappings which are ((1,2)*- M*-πgb closed mapping and (1,2)*- M**-πgb closed mapping) in bitopological spaces. Also some characterizations and basic properties will be study.

Keywords:
bitopological spaces, Closed mappings, M-(1,2)*-πgb closed mapping.

1- Introduction:
Ravi and LellisThivagar [4] have introduced the concepts of (1,2)*-closed sets, (1,2)*-semi-open sets, (1,2)*-regular open sets and (1,2)*-generalized closed sets in bitopological spaces. Further, LellisTivagar and Ravi [3] unleashed the study of (1,2)*-g-closed maps. While the concepts ((1,2)*-πgb-closed sets, (1,2)*-πgb-open sets, (1,2)*-πgb-continuous maps, (1,2)*-πgb-irresoules maps, (1,2)*-πgb-closed maps and (1,2)*-πgb-M-closed maps) were discussed and introduced by (Sreeja and Janaki, 2012, in [9], [10]).

In this work, we introduce a new type of (1,2)*-M-πgb-closed mapping which are ((1,2)*-M*-closed and (1,2)*-M**-closed)) maps in bitopological spaces and study their basic properties. We also investigate its relationship with other types of (1,2)*-closed maps.

Throughout this paper, X, Y and Z are spaces, and ((X, τ₁, τ₂), (Y,σ₁,σ₂) and (Z,μ₁,μ₂)) are denote bitopological spaces respectively.

2- Preliminaries:
In this section, we recall some definitions and results which are used in this paper.

Definition (2.1), [4]:- A triplet (X, τ₁, τ₂), where X is a nonempty set and τ₁ and τ₂ are topologies on X is called bitopological space.

Definition (2.2), [4]:- A subset S of bitopological space (X, τ₁, τ₂), is said to be τ₁τ₂ - open if S = A ∪ B, where A ∈τ₁ and B ∈τ₂. The complement of τ₁τ₂ - open set is called τ₁τ₂ - closed. The family of all τ₁τ₂ - open (resp. τ₁τ₂ -closed) sets of (X, τ₁, τ₂), is denoted by (1,2)*-o(X) (resp. (1,2)*-c(X)).
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Example (2.1): Let X = {a, b, c}, τ₁ = {X, φ, {a}} and τ₂ = {X, φ, {b}}. Then the sets in {X, φ, {a}, {b}, {a, b}} are called τ₁ τ₂ - open and the sets in {X, φ, {c}, {a, c}, {b, c}} are called τ₁ τ₂ -closed.

Remark (2.3), [4]: τ₁ τ₂ -open subsets of (X, τ₁, τ₂), need not necessarily form a topology.

Definition (2.4), [4]: Let S be a subset of a bitopological space (X, τ₁, τ₂). Then:
(i) τ₁ τ₂ - closure of S denoted by τ₁ τ₂ -cl(S) is defined by ∩ {F: S ⊆ F and F is τ₁ τ₂ -closed}
(ii) τ₁ τ₂ - interior of S denoted by ∪ {U: U ⊆ S and U is τ₁ τ₂ -open}.

Definition (2.5): A subset S of (X, τ₁, τ₂), is said to be a:
(i) (1,2)*-semi-open [4] if S ⊆ τ₁ τ₂ -cl (τ₁ τ₂ -int(S)).
(ii) (1,2)*-regular open [4] if S = τ₁ τ₂ -int(τ₁ τ₂ -cl(S)).
(iii) (1,2)*-π-open [6] if S is the finite union of (1,2)*-regular open sets.
(iv) (1,2)*-b-open [1] if S ⊆ τ₁ τ₂ -cl (τ₁ τ₂ -int(S)) ∪ τ₁ τ₂ -int (τ₁ τ₂ -cl(S)).

The complements of all the above mentioned open sets are called their respective closed sets and we denoted the family of all (1,2)*-regular open (resp. (1,2)*-regular closed sets) by (1,2)*RO (resp. (1,2)*RC).

The (1,2)*-b-closure of a subset S of (X, τ₁, τ₂), is denoted by (1,2)*-bcl(S) and defined as the intersection of all (1,2)*-b-closed sets containing S.

Definition (2.6): A subset A of bitopological space (X, τ₁, τ₂), is said to be a:
(i) (1,2)*-generalized closed set [4] (briefly, (1,2)*-g-closed set) if τ₁ τ₂-cl(A) ⊆ U whenever A ⊆ U and U ∈ (1,2)*-open set in (X, τ₁, τ₂).
(ii) (1,2)*-semi-generalized-star-closed set [7] (briefly, (1,2)*-sg*-closed set) if τ₁ τ₂-cl(A) ⊆ U whenever A ⊆ U and U is (1,2)*- semi open set in (X, τ₁, τ₂).
(iii) (1,2)*-π-generalized b-closed set [9] (briefly, (1,2)*-πgb-closed set) if τ₁ τ₂-bcl(A) ⊆ U whenever A ⊆ U and U is τ₁ τ₂-π-open set in (X, τ₁, τ₂).

The family of all (1,2)*-g-closed sets (resp. (1,2)*-sg*-closed and (1,2)*-πgb-closed) of (X, τ₁, τ₂) will be denoted by (1,2)*-GC(X) (resp. (1,2)*-SG*C(X) and (1,2)*-πGBC(X))

Remark (2.7): In [7], [9] it is proved that in any bitopological spaces (X, τ₁, τ₂):
(1) Every τ₁ τ₂-closed set (resp. (1,2)*-g-closed set) is (1,2)*-πgb-closed set in (X, τ₁, τ₂).
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(2) Every \(\tau_1 \tau_2\)-closed set (resp. (1,2)*-\(sg\)*-closed set) is (1,2)*-\(g\)-closed set in \((X, \tau_1, \tau_2)\) is (1,2)*-\(\pi\)gb-closed set in \((X, \tau_1, \tau_2)\).

The converse of the above Remark need not be true, as shown in the following example:

**Examples (2.2):**

(i) Let \(X=\{a,b,c\}, \tau_1=\{X,\phi,\{a\}\}\) and \(\tau_2=\{X,\phi,\{b\},\{a,b\}\}\). Then (1,2)*-\(o(X) = \{X,\phi,\{a\},\{b\},\{a,b\}\}\), (1,2)*-\(c(X) = \{X,\phi,\{c\},\{a,c\},\{b,c\}\}\) = (1,2)*-\(GC(X)\) and (1,2)*-\(\pi\)GB(C(X)) = \(\{X,\phi,\{a\},\{b\},\{c\},\{a,c\},\{b,c\}\}\). Then, clearly the set \(A = \{a\}\) in \((X, \tau_1, \tau_2)\), is (1,2)*-\(\pi\)gb-closed sets, but is not \(\tau_1\tau_2\)-closed sets and (1,2)*-\(g\)-closed in \((X, \tau_1, \tau_2)\).

(ii) Let \(X=\{a,b,c\}, \tau_1=\{X,\phi\}\) and \(\tau_2=\{X,\phi,\{a\}\}\). Then (1,2)*-\(O(X) = \{X,\phi,\{a\}\}\), (1,2)*-\(C(X) = \{X,\phi,\{b,c\}\}\), (1,2)*-\(SG\)*-\(C(X) = \{X,\phi,\{b,c\}\}\) and \(GC(X) = \{X,\phi,\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\}\}\). Then clearly the set \(A = \{b\}\) is (1,2)*-\(g\)-closed sets in \((X, \tau_1, \tau_2)\), but is not \(\tau_1\tau_2\)-closed set and (1,2)*-\(sg\)*-closed set in \((X, \tau_1, \tau_2)\).

**Definition (2.8):** A map \(f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)\) is said to be:

1. **(1,2)*-closed** or \(\tau_1 \tau_2\)-closed [8] if \(f(A)\) is \(\sigma_1\sigma_2\)-closed set in \((Y, \sigma_1, \sigma_2)\) for every \(\tau_1\tau_2\)-closed set \(A\) in \((X, \tau_1, \tau_2)\).
2. **(1,2)*-\(g\)-closed** [3] if \(f(A)\) is (1,2)*-\(g\)-closed set in \((Y, \sigma_1, \sigma_2)\) for every \(\tau_1\tau_2\)-closed set \(A\) in \((X, \tau_1, \tau_2)\).
3. **(1,2)*-\(sg\)*-closed** [7] if \(f(A)\) is (1,2)*-\(sg\)*-closed set in \((Y, \sigma_1, \sigma_2)\) for every \(\tau_1\tau_2\)-closed set \(A\) in \((X, \tau_1, \tau_2)\).
4. **Pre-(1,2)*-\(sg\)*-closed** [7] if \(f(A)\) is (1,2)*-\(sg\)*-closed set in \((Y, \sigma_1, \sigma_2)\) for every (1,2)*-\(sg\)*-closed set \(A\) in \((X, \tau_1, \tau_2)\).
5. **(1,2)*-\(\pi\)gb-closed** [10] if \(f(A)\) is (1,2)*-\(\pi\)gb-closed set in \((Y, \sigma_1, \sigma_2)\) for every \(\tau_1\tau_2\)-closed set \(A\) in \((X, \tau_1, \tau_2)\).
6. **M-(1,2)*-\(\pi\)gb-closed** [10] if \(f(A)\) is (1,2)*-\(\pi\)gb-closed set in \((Y, \sigma_1, \sigma_2)\) for every (1,2)*-\(\pi\)gb-closed set \(A\) in \((X, \tau_1, \tau_2)\).

**Definition (2.9):** A map \(f: ((X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)\) is said to be a:

1. **(1,2)*-continuous** [5] if \(f^1(A)\) is \(\tau_1\tau_2\)-closed set in \((X, \tau_1, \tau_2)\) for every \(\sigma_1\sigma_2\)-closed set \(A\) in \((Y, \sigma_1, \sigma_2)\).
2. **(1,2)*-\(g\)-continuous** [3] if \(f^1(A)\) is (1,2)*-\(g\)-closed set in \((X, \tau_1, \tau_2)\) for every \(\tau_1\tau_2\)-closed set \(A\) in \((Y, \sigma_1, \sigma_2)\).
3. **(1,2)*-\(\pi\)gb-continuous** [9] if \(f^1(A)\) is (1,2)*-\(\pi\)gb-closed set in \((X, \tau_1, \tau_2)\) for every \(\tau_1\tau_2\)-closed set \(A\) in \((Y, \sigma_1, \sigma_2)\).
4. **(1,2)*-\(\pi\)gb-irresolute** [9] if \(f^1(A)\) is (1,2)*-\(\pi\)gb-closed set in \((X, \tau_1, \tau_2)\) for every (1,2)*-\(\pi\)gb-closed set \(A\) in \((Y, \sigma_1, \sigma_2)\).

**Definition (2.10):** A bitopological space \((X, \tau_1, \tau_2)\) is called a:

1. **(1,2)*-\(T_{1/2}\)-space** [2] if every (1,2)*-\(g\)-closed set is \(\tau_1\tau_2\)-closed.
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(2) (1,2)*-\(gb\)-space [9] if every (1,2)*-\(gb\)-closed set is \(\tau_1\tau_2\)-closed.

(3) RM-space [7] if every subset in \((X, \tau_1, \tau_2)\) is either \(\tau_1\tau_2\)-open or \(\tau_1\tau_2\)-closed.

**Theorem (2.11)**, [7]:- In RM-space \((X, \tau_1, \tau_2)\) every (1,2)*-\(sg\)*-closed set is \(\tau_1\tau_2\)-closed.

**Theorem (2.12)**, [9]:- Let \(B \subseteq A \subseteq X\) where \(A\) is (1,2)*-\(gb\)-closed and \(\tau_1\tau_2\)-\(\sigma\)-open set. Then \(B\) is (1,2)*-\(gb\)-closed in \(X\).

3- Some Types of (1,2)*-M-\(gb\)-closed Maps:

We now introduce the following a new definition of (1,2)*-M-\(gb\)-closed mapping.

**Definition (3.1):** A map \(f : (X, \tau_1, \tau_2) \longrightarrow (Y,\sigma_1,\sigma_2)\) is said to be (1,2)*-M*-\(gb\)-closed if

\(f\) (\(A\)) is \(\sigma_1\sigma_2\)-closed set in \((Y,\sigma_1,\sigma_2)\) for every (1,2)*-\(gb\)-closed set \(A\) in \((X, \tau_1, \tau_2)\).

**Example (3.1):** Let \(X=\{a,b,c\}\), \(\tau_1=\{X,\phi,\{a\}\}\) and \(\tau_2=\{X,\phi,\{b\}\}\). Then the sets \(\{X,\phi,\{a\}\}\) are \(\tau_1\tau_2\)-open in \((X,\tau_1,\tau_2)\), the sets \(\{X,\phi,\{a\}\}\) are \(\tau_1\tau_2\)-closed in \((X,\tau_1,\tau_2)\) and \(GBC(X,\tau_1,\tau_2)=\{X,\phi,\{a\}\}\). Define \(f : (X, \tau_1, \tau_2) \longrightarrow (Y,\sigma_1,\sigma_2)\) by \(f(a)=f(b)=b\) and \(f(c)=c\). Then clearly \(f\) is (1,2)*-M*-\(gb\)-closed map.

**Proposition (3.2):** If \(f : (X, \tau_1, \tau_2) \longrightarrow (Y,\sigma_1,\sigma_2)\) is (1,2)*-M*-\(gb\)-closed map. Then \(f\) is a:

(i) (1,2)*-closed map.
(ii) (1,2)*-\(g\)-closed map.
(iii)(1,2)*-\(sg\)-closed map.
(iv)(1,2)*-\(gb\)-closed map.
(v) (1,2)*-M-\(gb\)-closed map.
(vi)Pre (1,2)*-\(sg\)-closed map.

**Proof:**

(i) Let \(A\) be a \(\tau_1\tau_2\)-closed set in \((X, \tau_1, \tau_2)\) . By Remark (2.7) Part (1) every \(\tau_1\tau_2\)-closed set is (1,2)*-\(gb\)-closed. Then \(A\) is (1,2)*-\(gb\)-closed set in \((X, \tau_1, \tau_2)\) . Since \(f\) is (1,2)*-M*-\(gb\)-closed map. Thus \(f\) (\(A\)) is \(\sigma_1\sigma_2\)-closed set in \((Y,\sigma_1,\sigma_2)\) . Hence, \(f\) is (1,2)*-closed map.

(ii) Let \(A\) be a \(\tau_1\tau_2\)-closed set in \((X, \tau_1, \tau_2)\) . By Remark (2.7) Part (1) we get \(A\) is (1,2)*-\(gb\)-closed set in \((X, \tau_1, \tau_2)\) .Since \(f\) is (1,2)*-M*-\(gb\)-closed map. Thus \(f\) (\(A\)) is \(\sigma_1\sigma_2\)-closed set in \((Y,\sigma_1,\sigma_2)\) . Also, since
Since every $\sigma_1\sigma_2$-closed set is $(1,2)^*\pi g$-closed, [4]). Therefore $f(A)$ is $(1,2)^*\pi g$-closed set in $(Y,\sigma_1,\sigma_2)$ . Hence, $f$ is $(1,2)^*\pi g$-closed map.

(iii) Let $A$ be a $\tau_1\tau_2$-closed set in $(X, \tau_1, \tau_2)$ . By Remark (2.7) we get $A$ is $(1,2)^*\pi gb$-closed set in $(X, \tau_1, \tau_2)$ . Since $f$ is $(1,2)^*M^*\pi gb$-closed map. Thus $f (A)$ is $\sigma_1\sigma_2$-closed set in $(Y,\sigma_1,\sigma_2)$ . Also, since every $\sigma_1\sigma_2$-closed set is $(1,2)^*\pi g$-closed, [4] and every $(1,2)^*\pi g$-closed set is $(1,2)^*sg^*$-closed set, [7]). Therefore, $f (A)$ is $(1,2)^*sg^*$-closed set in $(Y,\sigma_1,\sigma_2)$ . Hence, $f$ is $(1,2)^*sg^*$-closed map.

(iv) Let $A$ be a $\tau_1\tau_2$-closed set in $(X, \tau_1, \tau_2)$ . Since (every $\tau_1\tau_2$-closed set is $(1,2)^*\pi gb$-closed). Then $A$ is $(1,2)^*\pi gb$-closed set in $(X, \tau_1, \tau_2)$ . Thus, $f (A)$ is $\sigma_1\sigma_2$-closed set in $(Y,\sigma_1,\sigma_2)$ . Also, by using Remark (2.7) Part (1) we get $f (A)$ is $(1,2)^*\pi gb$-closed set in $(Y,\sigma_1,\sigma_2)$ . Therefore, $f$ is $(1,2)^*\pi gb$-closed map.

The proof of Parts (v) and (vi) are similar to above.

Remark (3.3): The converse of Proposition (3.2) may not be true in general. Consider the following example:

Example (3.2):- Let $X=Y=\{a,b,c\}$, $\tau_1=\{X,\phi\}$ and $\tau_2=\{X,\phi,\{a\}\}$, $\sigma_1=\{Y,\phi,\{a\}\}$, $\sigma_2=\{Y,\phi,\{b\},\{a,b\}\}$. Then the sets $\{X,\phi,\{a\}\}$ are $\tau_1\tau_2$-open sets in $(X, \tau_1, \tau_2)$ . the sets $\{X,\phi,\{b,c\}\}$ are $\tau_1\tau_2$-closed sets in $(X, \tau_1, \tau_2)$ , and the sets $\{Y,\phi,\{a\},\{b\},\{a,b\}\}$ are $\sigma_1\sigma_2$-open set in $(Y,\sigma_1,\sigma_2)$ and the sets $\{Y,\phi,\{c\},\{a,c\},\{b,c\}\}$ are $\sigma_1\sigma_2$-closed set in $(Y,\sigma_1,\sigma_2)$ . Define $f : (X, \tau_1, \tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ by $f (a) = b, f (b) = a$ and $f (c) = c$. Then, $f$ is $(1,2)^*\pi g$-closed map, but $f$ is not $(1,2)^*M^*\pi gb$-closed map. Since for $(1,2)^*\pi gb$-closed set $A = \{a\}$ in $(X, \tau_1, \tau_2)$ , $f (A) = f (\{a\}) = \{b\}$ is not $\sigma_1\sigma_2$-closed set in $(Y,\sigma_1,\sigma_2)$.

Example (3.3):- Let $X=\{a,b,c\}$, $\tau_1=\{X,\phi,\{a\}\}$ and $\tau_2=\{X,\phi,\{b\},\{a,b\}\}$. Then the sets in $\{X,\phi,\{a\},\{b\},\{a,b\}\}$ are $\tau_1\tau_2$-open sets $(X,\tau_1,\tau_2)$ , the sets $\{X,\phi,\{c\},\{a,c\},\{b,c\}\}$ are $\tau_1\tau_2$-closed sets $(X,\tau_1,\tau_2)$ and $(1,2)^*\pi g$-closed set $(X,\tau_1,\tau_2)$ . Let $f : (X, \tau_1, \tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ be the identity map. Then $f : (X, \tau_1, \tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ is $(1,2)^*\pi g$-closed, $(1,2)^*sg^*$-closed and $(1,2)^*sg^*$-closed map but $f$ is not $(1,2)^*M^*\pi gb$-closed map. Since for $(1,2)^*\pi gb$-closed set $A = \{a\}$ in $(X, \tau_1, \tau_2)$ , $f (A) = f (\{a\}) = \{A\}$ is not $\tau_1\tau_2$-closed set in $(X, \tau_1, \tau_2)$.

Example (3.4):- Let $X=\{a,b,c\}$, $\tau_1=\{X,\phi\}$ and $\tau_2=\{X,\phi,\{a\}\}$. Then the sets $\{X,\phi,\{a\}\}$ are $\tau_1\tau_2$-open sets $(X,\tau_1,\tau_2)$ , the sets $\{X,\phi,\{b,c\}\}$ are $\tau_1\tau_2$-closed sets.
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In (X, τ₁, τ₂) and πGBC(X, τ₁, τ₂) = \{X, φ, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}. Define \(f : (X, τ₁, τ₂) \rightarrow (X, τ₁, τ₂)\) by \(f(a) = c, f(b) = a\) and \(f(c) = b\). Then clearly \(f\) is (1,2)*-πgb-closed map and (1,2)*-M-πgb-closed map, but \(f\) is not (1,2)*-M*-πgb-closed. Since for a \(τ₁τ₂\)-closed set \(A = \{a\}\) in 
\((X, τ₁, τ₂), f(A) = f(\{a\}) = \{c\}\) is not \(τ₁τ₂\)-closed set in \((X, τ₁, τ₂)\).

The following Propositions give the condition to make the Proposition (3.2) true.

**Proposition (3.4):** Let \(f : (X, τ₁, τ₂) \rightarrow (Y, σ₁, σ₂)\) be any map and X, Y be two (1,2)*-πgb-space. Then \(f\) is (1,2)*-M*-πgb-closed map, if \(f\) is a:

(i) (1,2)*-closed map.

(ii) (1,2)*-πgb-closed map.

(iii)(1,2)*-M*-πgb-closed map.

**Proof:**

(i) Let \(A\) be a πgb-closed set in \((X, τ₁, τ₂)\). Since \((X, τ₁, τ₂)\) is a (1,2)*-πgb-space and by using Definition (2.10) Part (ii) we get, \(A\) is a \(τ₁τ₂\)-closed set in \((X, τ₁, τ₂)\). Also, since \(f\) is a (1,2)*-closed map. Thus \(f(A)\) is a \(σ₁σ₂\)-closed set in \((Y, σ₁, σ₂)\). Hence \(f\) is (1,2)*-M*-πgb-closed map.

(ii) Let \(A\) be a πgb-closed set in \((X, τ₁, τ₂)\). Since \((X, τ₁, τ₂)\) is a (1,2)*-πgb-space and by Definition (2.10) we have \(A\) is a \(τ₁τ₂\)-closed set in \((X, τ₁, τ₂)\). By hypothesis \(f\) is (1,2)*-πgb-closed map. Thus \(f(A)\) is a (1,2)*-πgb-closed set in \((Y, σ₁, σ₂)\). Also, since \(Y\) is a (1,2)*-πgb-space and by Definition (2.10) Part (ii) we obtain \(f(A)\) is \(σ₁σ₂\)-closed set in \((Y, σ₁, σ₂)\). Therefore \(f\) is (1,2)*-M*-πgb-closed map.

The proof of Part (iii) is similar to Parts (i) and (ii).

**Proposition (3.5):**

Let \(X\) be a (1,2)*-πgb-space and \(Y\) be a (1,2)*-\(T_{1/2}\)-space. If \(f : (X, τ₁, τ₂) \rightarrow (Y, σ₁, σ₂)\) is a (1,2)*-\(g\)-closed map. Then \(f\) is (1,2)*-M*-πgb-map.

**Proof:-** Let \(A\) be a (1,2)*-πgb-closed set in \((X, τ₁, τ₂)\). Since \(X\) is a (1,2)*-πgb-space and by Definition (2.10) Part (ii) we get, \(A\) is a \(τ₁τ₂\)-closed set in \((X, τ₁, τ₂)\). Also, since \(f\) is a (1,2)*-\(g\)-closed map. Thus, \(f(A)\) is a (1,2)*-\(g\)-closed set in \((Y, σ₁, σ₂)\). By hypothesis \(Y\) is a (1,2)*-\(T_{1/2}\)-space and by Definition (2.10) Part (i) we have \(f(A)\) is a \(σ₁σ₂\)-closed set in \((Y, σ₁, σ₂)\). Hence, \(f\) is a (1,2)*-M*-πgb-closed map.

**Proposition (3.6):** Let \(X\) be a (1,2)*-πgb-space and \(Y\) be a RM-space. Then a map \(f : (X, τ₁, τ₂) \rightarrow (Y, σ₁, σ₂)\) is a(1,2)*-M*-πgb- closed. If \(f\) is a:

(i) (1,2)*-\(sg\*)-closed map.

(ii) Pre (1,2)*-\(sg\*)-closed map.
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Proof:
(i) Let A be a (1,2)*-πgb-closed set in (X, τ1, τ2). Since X is (1,2)*-πgb-space. Then A is a τ1τ2-closed set in (X, τ1, τ2). Also, since f is a (1,2)*-sg*-closed map. Thus f(A) is a (1,2)*-sg*-closed set in (Y, σ1, σ2). By hypothesis Y is a RM-space and by Definition (2.10) Part (iii) we get f(A) is a σ1σ2-closed set in (Y, σ1, σ2). Therefore, f is a (1,2)*-M*-πgb-closed map.

The proof of Part (ii) is similar to Part (i).

Proposition (3.7):- If f: (X, τ1, τ2) → (Y, σ1, σ2) is a (1,2)*-M*-πgb-closed map and A is (1,2)*-πgb-closed and τ1τ2-π-open set in (X, τ1, τ2). Then f|A:(A, τ1, τ2) → (Y, σ1, σ2) is (1,2)*-M*-πgb-closed map.

Proof:
Let B be a (1,2)*-πgb-closed subset of A. Then by Theorem (2.12) we get B is a (1,2)*-πgb-closed in (X, τ1, τ2). Since f is a (1,2)*-M*-πgb-closed map. Thus f(B) is a σ1σ2-closed set in (Y, σ1, σ2). But f(B) = (f|A)(B). Therefore (f|A)(B) is a τ1τ2-closed set in (Y, σ1, σ2). Hence, f|A is a (1,2)*-M*-πgb-closed map.

Corollary (3.8):- If f: (X, τ1, τ2) → (Y, σ1, σ2) is a (1,2)*-M*-πgb-closed map and A is (1,2)*-πgb-closed and τ1τ2-open set in (X, τ1, τ2). Then f|A:(A, τ1, τ2) → (Y, σ1, σ2) is a:
(i) (1,2)*-closed map
(ii) (1,2)*-πgb-closed map.
(iii) (1,2)*-M-πgb-closed map.

Proof:
(i) This follows from Propositions (3.7) and (3.2) Part (i).
(ii) This follows from Propositions (3.7) and (3.2) Part (ii).
(iii) This follows from Propositions (3.7) and (3.2) Part (iii).

Proposition (3.9): Let f: (X, τ1, τ2) → (Y, σ1, σ2)) and g: (Y, σ1, σ2) → (Z, μ1, μ2) be any two maps. Then g ∘ f : (X, τ1, τ2) → (Z, μ1, μ2) is (1,2)*-M*-πgb-closed map if:
(i) f and g are two (1,2)*-M*-πgb-closed maps.
(ii) f is a (1,2)*-M*-πgb-closed map and g is a (1,2)*-closed.
(iii) f is (1,2)*-M-πgb-closed map and g is (1,2)*-M*-πgb-closed.

Proof:
(i) Let A be a (1,2)*-πgb-closed set in (X, τ1, τ2). Since f is a (1,2)*-M*-πgb-closed map. Thus, f(A) is τ1τ2-closed set in (Y, σ1, σ2). By Remark (2.7) Part (i) we get f(A) is a (1,2)*-πgb-closed set in (Y, σ1, σ2). Also, since g is a (1,2)*-M*-πgb-closed map. Hence, g(f
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\(A\) is a \(\mu_1\mu_2\)-closed set in \((Z,\mu_1,\mu_2)\). But \(g(f(A)) = g \circ f(A)\). Then \(g \circ f\)
\(A\) is a \(\mu_1\mu_2\)-closed set in \((Z,\mu_1,\mu_2)\). Therefore, \(g \circ f\) is a \((1,2)^*\)-\(M^*\)-\(\pi gb\)-closed map.

The prove of Part -ii- and -iii- are similar to Part -i-.

**Proposition (3.10):** Let \(f: (X, \tau_1, \tau_2) \longrightarrow (Y,\sigma_1,\sigma_2)\) and \(g: (Y,\sigma_1,\sigma_2) \longrightarrow (Z,\mu_1,\mu_2)\) be any two maps. Then \(g \circ f: (X, \tau_1, \tau_2) \longrightarrow (Z,\mu_1,\mu_2)\) is \((1,2)^*\)-\(\pi gb\)-closed map iff \(g\) is a \((1,2)^*\)-\(M^*\)-\(\pi gb\)-closed map and \(f\) is a

(i) \((1,2)^*\)-\(\pi gb\)-closed map.

(ii) \((1,2)^*\)-\(M^{**}\)-\(\pi gb\)-closed map.

**Proof:**

(i) Let \(A\) be a \(\tau_1\tau_2\)-closed set in \((X, \tau_1, \tau_2)\). By Remark (2.7) Part (i) we get \(A\) is a \((1,2)^*\)-\(\pi gb\)-closed set in \((X, \tau_1, \tau_2)\). Since \(f\) is a \((1,2)^*\)-\(M^*\)-\(\pi gb\)-closed map. Thus, \(f(A)\) is a \(\sigma_1\sigma_2\)-closed set in \((Y,\sigma_1,\sigma_2)\). Also, since \(g\) is a \((1,2)^*\)-\(\pi gb\)-closed map. Hence, \(g(f(A))\) is a \((1,2)^*\)-\(\pi gb\)-closed set in \((Z,\mu_1,\mu_2)\). But, \(g(f(A)) = g(f(A))\). Then, \(g \circ f(A)\) is \((1,2)^*\)-\(\pi gb\)-closed set in \((Z,\mu_1,\mu_2)\). Therefore, \(g \circ f\) is a \((1,2)^*\)-\(\pi gb\)-closed map.

The proof of Part (ii) is similar to Part (i).

However the following proposition holds. The proof is easy and hence omitted.

**Proposition (3.11):** Let \(f: (X, \tau_1, \tau_2) \longrightarrow (Y,\sigma_1,\sigma_2)\) and \(g: (Y,\sigma_1,\sigma_2) \longrightarrow (Z,\mu_1,\mu_2)\) be any two maps. Then \(g \circ f: (X, \tau_1, \tau_2) \longrightarrow (Z,\mu_1,\mu_2)\) is \((1,2)^*\)-\(\pi gb\)-closed map if \(g\) is a \((1,2)^*\)-\(M^*\)-\(\pi gb\)-closed map and \(f\) is a

(i) \((1,2)^*\)-\(\pi gb\)-closed maps.

(ii) \((1,2)^*\)-\(\pi gb\)-closed map.

**Proposition (3.12):** Let \(f: (X, \tau_1, \tau_2) \longrightarrow (Y,\sigma_1,\sigma_2)\) and \(g: (Y,\sigma_1,\sigma_2) \longrightarrow (Z,\mu_1,\mu_2)\) be any two maps, such that \(g \circ f: (X, \tau_1, \tau_2) \longrightarrow (Z,\mu_1,\mu_2)\) is \((1,2)^*\)-\(\pi gb\)-closed map if \(f\) is a \((1,2)^*\)-\(\pi gb\)-irresolute map and surjective, then \(g\) is \((1,2)^*\)-\(M^{**}\)-\(\pi gb\)-closed map.

**Proof:** Let \(A\) be a \((1,2)^*\)-\(\pi gb\)-closed set in \((Y,\sigma_1,\sigma_2)\). Since \(f\) is \((1,2)^*\)-\(\pi gb\)-irresolute. Thus, \(f^{-1}(A)\) is a \(\pi gb\)-closed set in \((X, \tau_1, \tau_2)\). Also, since \(g \circ f\) is a \((1,2)^*\)-\(M^{**}\)-\(\pi gb\)-closed map. Then, \((g \circ f)(f^{-1}(A))\) is a \(\mu_1\mu_2\)-closed set in \((Z,\mu_1,\mu_2)\). By hypothesis \(g \circ f\) is surjective. Hence, \((g \circ f)(f^{-1}(A)) = g(A)\). Therefore \(g(A)\) is a \(\mu_1\mu_2\)-closed set in \((Z,\mu_1,\mu_2)\). Hence, \(g\) is \((1,2)^*\)-\(M^{**}\)-\(\pi gb\)-closed map.
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**Proposition (3.13):** Let \(f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)\) and \(g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2)\) be two mappings, such that \(g \circ f: (X, \tau_1, \tau_2) \rightarrow (Z, \mu_1, \mu_2)\) be \((1,2)^*\)-\(M^*\)-\(\pi gb\)-closed mapping. Then \(g\) is \((1,2)^*\)-\(M^*\)-\(\pi gb\)-closed map. If a surjective map \(f\) is a:

(i) \((1,2)^*\)-continuous map and \((Y, \sigma_1, \sigma_2)\) is \((1,2)^*\)-\(\pi gb\)-space.

(ii) \((1,2)^*\)-\(g\)-continuous map, \((X, \tau_1, \tau_2)\) is \((1,2)^*\)-\(T_{1/2}\) space and \((Y, \sigma_1, \sigma_2)\) is \((1,2)^*\)-\(\pi gb\)-space.

(iii) \((1,2)^*\)-\(\pi gb\)-continuous map, \((X, \tau_1, \tau_2)\) is \((1,2)^*\)-\(\pi gb\)-space.

**Proof:**

(i) Let \(A\) be a \((1,2)^*\)-\(\pi gb\)-closed set in \((Y, \sigma_1, \sigma_2)\). Since \((Y, \sigma_1, \sigma_2)\) is a \((1,2)^*\)-\(\pi gb\)-space and by using Definition (2.10), Part (ii) we get \(A\) is a \(\sigma_1 \sigma_2\)-closed set in \((Y, \sigma_1, \sigma_2)\). Also, since \(f\) is a \((1,2)^*\)-continuous map. Thus \(f^{-1}(A)\) is \(\tau_1 \tau_2\)-closed set in \((X, \tau_1, \tau_2)\). By Remark (2.7), Part (i) (every \(\tau_1 \tau_2\)-closed set is \((1,2)^*\)-\(\pi gb\)-closed). So, we have \(f^{-1}(A)\) is a \((1,2)^*\)-\(\pi gb\)-closed set in \((X, \tau_1, \tau_2)\) and by hypothesis \(g \circ f\) is \((1,2)^*\)-\(M^*\)-\(\pi gb\)-closed map. Then, \((g \circ f)(f^{-1}(A))\) is a \(\mu_1 \mu_2\)-closed set in \((Z, \mu_1, \mu_2)\). Since \(f\) is a surjective and \((g \circ f)(f^{-1}(A)) = g(f \circ f^{-1}(A)) = g(A)\). That is \(g(A)\) is a \(\mu_1 \mu_2\)-closed set in \((Z, \mu_1, \mu_2)\). Therefore, \(g\) is a \((1,2)^*\)-\(M^*\)-\(\pi gb\)-closed map.

(ii) Let \(A\) be a \((1,2)^*\)-\(\pi gb\)-closed set in \((Y, \sigma_1, \sigma_2)\). By hypothesis \((Y, \sigma_1, \sigma_2)\) is a \((1,2)^*\)-\(\pi gb\)-space and by using Definition (2.10), Part (ii) we get \(A\) is a \(\sigma_1 \sigma_2\)-closed set in \((Y, \sigma_1, \sigma_2)\). Also, since \(f\) is a \((1,2)^*\)-\(g\)-continuous map. Thus \(f^{-1}(A)\) is a \((1,2)^*\)-\(g\)-closed set in \((X, \tau_1, \tau_2)\), since \((X, \tau_1, \tau_2)\) is a \((1,2)^*\)-\(T_{1/2}\)-space and by using Definition (2.10), Part (i) we have \(f^{-1}(A)\) is a \(\tau_1 \tau_2\)-closed set in \((X, \tau_1, \tau_2)\). By Remark (2.7), Part (i) we get \(f^{-1}(A)\) is a \((1,2)^*\)-\(\pi gb\)-closed set in \((X, \tau_1, \tau_2)\) and since \(g \circ f\) is \((1,2)^*\)-\(M^*\)-\(\pi gb\)-closed map. Then, \((g \circ f)(f^{-1}(A))\) is a \(\mu_1 \mu_2\)-closed set in \((Z, \mu_1, \mu_2)\). But \(f\) is a surjective and \((g \circ f)(f^{-1}(A)) = g(f \circ f^{-1}(A)) = g(A)\). Hence, \(g(A)\) is a \(\mu_1 \mu_2\)-closed set in \((Z, \mu_1, \mu_2)\). Therefore, \(g\) is a \((1,2)^*\)-\(M^{**}\)-\(\pi gb\)-closed map.

The proof of Part (iii) is similar to Part (i).

**Proposition (3.14):** Let \(f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)\) and \(g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2)\) be two maps, such that \(g \circ f: (X, \tau_1, \tau_2) \rightarrow (Z, \mu_1, \mu_2)\) be \((1,2)^*\)-\(M^*\)-\(\pi gb\)-closed map. Then \(f\) is a \((1,2)^*\)-\(M^{**}\)-\(\pi gb\)-closed map. If:

(i) \(g\) is \((1,2)^*\)-continuous and injective.
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(ii) $g$ is $(1,2)^*\text{-}\pi gb$-continuous, injective and $(Y,\sigma_1,\sigma_2)$ is a $(1,2)^*\text{-}\pi gb$-space.

(iii) $g$ is $(1,2)^*\text{-}M\text{-}\pi gb$-continuous, injective and $(Y,\sigma_1,\sigma_2)$ is a $(1,2)^*\text{-}\pi gb$-space.

(iv) $g$ is $(1,2)^*\text{-}g$-continuous, injective and $(Y,\sigma_1,\sigma_2)$ is a $(1,2)^*\text{-}T_{1/2}$-space.

Proof:

(i) Let $A$ be a $(1,2)^*\text{-}\pi gb$-closed set in $(X,\tau_1,\tau_2)$. Since $g \circ f$ is $(1,2)^*\text{-}M^*\text{-}\pi gb$-closed map. Thus, $(g \circ f)(A)$ is a $\mu_1\mu_2$-closed set in $(Z,\mu_1,\mu_2)$. Also, since $g$ is a $(1,2)^*\text{-}\pi gb$-closed set in $(Y,\sigma_1,\sigma_2)$. Since $g$ is injective and $g^{-1}(g \circ f)))(A) = (g^{-1} \circ g) f(A) = f(A)$. Hence, $f(A)$ is a $\sigma_1\sigma_2$-closed set in $(Y,\sigma_1,\sigma_2)$. Therefore $f$ is a $(1,2)^*\text{-}M^*\text{-}\pi gb$-closed map.

(ii) Let $A$ be a $(1,2)^*\text{-}\pi gb$-closed set in $(X,\tau_1,\tau_2)$. Since $g \circ f$ is $(1,2)^*\text{-}M^*\text{-}\pi gb$-closed map. Thus $(g \circ f)(A)$ is a $\mu_1\mu_2$-closed set in $(Z,\mu_1,\mu_2)$. Since $g$ is a $(1,2)^*\text{-}\pi gb$-continuous map. Then $g^{-1}(g \circ f)))(A) = (g^{-1} \circ g) f(A) = f(A)$. Hence, $f(A)$ is a $\sigma_1\sigma_2$-closed set in $(Y,\sigma_1,\sigma_2)$. Also, since $g$ is injective and $(g^{-1}(g \circ f)))(A) = (g^{-1} \circ g) f(A) = f(A)$. Hence, $f(A)$ is a $\sigma_1\sigma_2$-closed set in $(Y,\sigma_1,\sigma_2)$. Therefore $f$ is a $(1,2)^*\text{-}M^*\text{-}\pi gb$-closed map.

The proof of Part (iii) and (iv) as similar to Part (ii).

Now, we given other type of $(1,2)^*\text{-}M$-closed mapping is called $(1,2)^*\text{-}M^**\text{-}\pi gb$-closed.

Definition (3.15): A mapping $f : (X,\tau_1,\tau_2) \longrightarrow (Y,\sigma_1,\sigma_2)$ is said to be $(1,2)^*\text{-}M^**\text{-}\pi gb$-closed if $f(A)$ is $(1,2)^*\text{-}\pi gb$-closed set in $(Y,\sigma_1,\sigma_2)$ for every $(1,2)^*\text{-}\pi gb$-closed set $A$ in $(X,\tau_1,\tau_2)$.

Example (3.5):

Let $X = \{a,b\}, Y = \{a,b,c\}, \tau_1 = \{X,\phi\}$ and $\tau_2 = \{X,\phi,\{a\}\}, \sigma_1 = \{Y,\phi,\{a\}\}$, $\sigma_2 = \{Y,\phi,\{b,c\}\}$. Then $\tau_1 \tau_2$-open sets in $(X,\tau_1,\tau_2)$ are $\{X,\phi,\{a\}\}$ and $\sigma_1 \sigma_2$-open set in $(Y,\sigma_1,\sigma_2)$ are $\{Y,\phi,\{a\}\}$, $(1,2)^*\pi GBC(X,\tau_1,\tau_2) = \{Y,\phi,\{a\}\}$ and $(1,2)^*\pi GC(Y,\sigma_1,\sigma_2) = \{Y,\phi,\{a\}\}$. Let $f : (X,\tau_1,\tau_2) \longrightarrow (Y,\sigma_1,\sigma_2)$ defined by $f(a) = f(b) = a$. Then, clearly $f$ is $(1,2)^*\text{-}M^**\text{-}\pi gb$-closed mapping.

Proposition (3.16): Let $f : (X,\tau_1,\tau_2) \longrightarrow (Y,\sigma_1,\sigma_2)$ be a $(1,2)^*\text{-}M^**\text{-}\pi gb$-closed mapping, then $f$ is $(1,2)^*\text{-}M^*\text{-}\pi gb$-closed.
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Proof:-

Let $A$ be $(1,2)^*\text{-}\pi gb$-closed set in $(X, \tau_1, \tau_2)$. Since $f$ is $(1,2)^*\text{-M}^*\text{-}\pi gb$-closed map, thus $f(A)$ is $(1,2)^*$-regular closed set in $(Y, \sigma_1, \sigma_2)$, (since every $(1,2)^*$-regular closed set is $(1,2)^*$-closed set, [4]). Therefore, $f(A)$ is $\sigma_1 \sigma_2$-closed set in $(Y, \sigma_1, \sigma_2)$. Hence, $f$ is $(1,2)^*\text{-M}^*\text{-}\pi gb$-closed map.

The converse of Proposition (3.16) may not be true in general as shown in the following.

Example (3.6):-

Let $X=\{a,b,c\}$, $\tau_1=\{X,\phi,\{a\},\{b\},\{a,b\}\}$ and $\tau_2=\{X,\phi,\{b,c\}\}$. Then $\tau_1 \tau_2$-open sets in $(X, \tau_1, \tau_2) = \{X,\phi,\{a\},\{b\},\{a,b\},\{b,c\}\}$, $\tau_1 \tau_2$-closed set in $(X, \tau_1, \tau_2) = \{X,\phi,\{a\},\{c\},\{a,c\},\{b,c\}\}$.

By Proposition (3.16) we have $f: (X, \tau_1, \tau_2) \longrightarrow (X, \tau_1, \tau_2)$ by $f(a) = a$, $f(b) = b$ and $f(c) = c$. It is clear that $f$ is $(1,2)^*\text{-M}^*\text{-}\pi gb$-closed map, but is not $(1,2)^*\text{-M}^*\text{-}\pi gb$-closed map, since for a subset $A = \{a,c\}$ is $(1,2)^*\text{-}\pi gb$-closed set in $(X, \tau_1, \tau_2)$, but $f(A) = f(\{a,c\}) = \{a,c\}$ is not $(1,2)^*$-regular closed set in $(X, \tau_1, \tau_2)$.

Corollary (3.17): Let $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ be a $(1,2)^*\text{-M}^*\text{-}\pi gb$-closed mapping, then $f$ is

(i) $\tau_1 \tau_2$-closed map.
(ii) $(1,2)^*\text{-g}$-closed map.
(iii) $(1,2)^*\text{-sg}$-closed map.
(iv) $(1,2)^*\text{-}\pi gb$-closed map.
(v) $(1,2)^*\text{-M}$-closed map.

Proof:-

Since $f$ is $(1,2)^*\text{-M}^*\text{-}\pi gb$-closed map. Then by using Proposition (3.16) we get $f$ is $(1,2)^*\text{-M}$-closed map and by Proposition (3.2) we have $f$ is $\tau_1 \tau_2$-closed map (resp. $(1,2)^*\text{-g}$-closed, $(1,2)^*\text{-sg}$-closed, $(1,2)^*\text{-}\pi gb$-closed and $M(1,2)^*\text{-}\pi gb$-closed map.

The converse of Corollary (3.17) need not be true in general.

Example (3.7):-

Let $X=\{a,b,c\}$, $\tau_1=\{X,\phi\}$ and $\tau_2=\{X,\phi,\{a\}\}$. Then $\tau_1 \tau_2$-open sets in $(X, \tau_1, \tau_2)$ are $\{X,\phi,\{a\},\{b\},\{a,b\},\{b,c\}\}$, $\tau_1 \tau_2$-closed set in $(X, \tau_1, \tau_2)$ are $\{X,\phi,\{b,c\}\}$ and $(1,2)^*\text{-RC}(X, \tau_1, \tau_2)=\{X,\phi\}$. Define $f : (X, \tau_1, \tau_2) \longrightarrow (X, \tau_1, \tau_2)$ by $f(a) = a$, $f(b) = b$ and $f(c) = c$. Then, $f$ is $\tau_1 \tau_2$-closed (resp. $(1,2)^*\text{-g}$-closed, $(1,2)^*\text{-sg}$-closed, $(1,2)^*\text{-}\pi gb$-closed and $M(1,2)^*\text{-}\pi gb$-closed) map, but $f$ is not $(1,2)^*\text{-M}$-closed set, since for $(1,2)^*\text{-}\pi gb$-closed set $A = \{a,c\}$ in $(X, \tau_1, \tau_2)$. $f(A) = f(\{a,c\}) = \{a,c\}$ is not $(1,2)^*$-regular closed set in $(X, \tau_1, \tau_2)$.
Proposition (3.18): Let \( f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) and \( g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2) \) be both \((1,2)^*\)-M**-\(\pi\)gb-closed mappings. Then \( g \circ f: (X, \tau_1, \tau_2) \rightarrow (Z, \mu_1, \mu_2) \) is also \((1,2)^*\)-M**-\(\pi\)gb-closed mapping

Proof:-

Let \( A \) be \((1,2)^*\)-\(\pi\)gb-closed set in \((X, \tau_1, \tau_2)\), thus \( f(A) \) is \((1,2)^*\)-regular closed set in \((Y, \sigma_1, \sigma_2)\), (since every \((1,2)^*\)-regular closed set is \((1,2)^*\)-closed set, [4]). Hence, \( f(A) \) is \( \sigma_1 \sigma_2 \)-closed set in \((Y, \sigma_1, \sigma_2)\) and by Remark (2.7) Part (i) we get \( f(A) \) is \((1,2)^*\)-\(\pi\)gb-closed set in \((Y, \sigma_1, \sigma_2)\). Also, since \( g \) is \((1,2)^*\)-M**-\(\pi\)gb-closed map. Thus \( g(f(A)) \) is \((1,2)^*\)-regular closed set in \((Z, \mu_1, \mu_2)\). But \( g(f(A)) = g \circ f(A) \). Thus, \( g \circ f(A) \) is \((1,2)^*\)-regular closed set in \((Z, \mu_1, \mu_2)\). Therefore, \( g \circ f: (X, \tau_1, \tau_2) \rightarrow (Z, \mu_1, \mu_2) \), is \((1,2)^*\)-M**-\(\pi\)gb-closed mapping

The proof of the following Propositions are clear. Thus we omitted it.

Proposition (3.19): Let \( f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) be \((1,2)^*\)-M**-\(\pi\)gb-closed mapping and \( g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2) \) be any mapping. Then \( g \circ f: (X, \tau_1, \tau_2) \rightarrow (Z, \mu_1, \mu_2) \) is \((1,2)^*\)-\(\pi\)gb-closed (resp. M-\((1,2)^*\)-\(\pi\)gb-closed) map if \( g \) is

(i) \((1,2)^*\)-\(\pi\)gb-closed map.
(ii) M-\((1,2)^*\)-\(\pi\)gb-closed map.
(iii) \((1,2)^*\)-M*-\(\pi\)gb-closed map.

Proposition (3.20): Let \( f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) be any and \( g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2) \) be \((1,2)^*\)-M*-\(\pi\)gb-closed mapping. Then \( g \circ f: (X, \tau_1, \tau_2) \rightarrow (Z, \mu_1, \mu_2) \) is \((1,2)^*\)-M*-\(\pi\)gb-closed map iff is

(i) M-\((1,2)^*\)-\(\pi\)gb-closed map.
(ii) \((1,2)^*\)-M*-\(\pi\)gb-closed map.
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References

بعض أنواع من التطبيقات المغلقة $\mathcal{M} - \pi gb$ من النطاق

المستخلص:
في هذا البحث، نحن بدراسة أنواع جديدة من التطبيقات المغلقة $\mathcal{M} - \pi gb$ والتطبيقات المغلقة $\mathcal{M}^*$ من النطاق $(1,2)$ - $(\pi gb)$-

ندرس بعض تطبيقاته وصفاته.