Some Types of (1,2)*- M -πgb Closed Mappings

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Abstract:

In this paper, we introduce and study new types of (1,2)*- M -πgb closed mappings which are ((1,2)*- M*-πgb closed mapping and (1,2)*- M**-πgb closed mapping) in bitopological spaces. Also some characterizations and basic properties will be study.

Keywords:
bitopological spaces ,Closed mappings, M-(1,2)*-πgb closed mapping.

1- Introduction:

Ravi and LellisThivagar [4] have introduced the concepts of $\tau_1\tau_2$-closed sets, (1,2)*-semi-open sets, (1,2)*-regular open sets and (1,2)*-generalized closed sets in bitopological spaces. Further, LellisTivagar and Ravi [3] unleashed the study of (1,2)*-g-closed maps. While the concepts ((1,2)*-πgb-closed sets, (1,2)*-πgb-open sets, (1,2)*-πgb-continuous maps, (1,2)*-πgb-irresoultes maps, (1,2)*-πgb-closed maps and (1,2)*-πgb-M-closed maps) were discussed and introduced by (Sreeja and Janaki, 2012, in [9], [10]).

In this work, we introduce a new type of (1,2)*-M-πgb-closed mapping which are ((1,2)*-M*-closed and (1,2)*-M**-closed)) maps in bitopological spaces and study their basic properties. We also investigate its relationship with other types of (1,2)*-closed maps.

Throughout this paper, X, Y and Z are spaces, and ((X, τ_1, τ_2), (Y,σ_1,σ_2) and (Z,μ_1,μ_2)) are denote bitopological spaces respectively.

2- Preliminaries:

In this section, we recall some definitions and results which are used in this paper.

Definition (2.1), [4]:- A triplet (X, τ_1, τ_2),where X is a nonempty set and $\tau_1$and τ_2 are topologies on X is called bitopological space .

Definition (2.2), [4]:- A subset S of bitopological space (X, τ_1, τ_2),is said to be $\boldsymbol{\tau_1\tau_2}$ **- open** if S = A ∪ B, where A $\in\tau_1$ and B $\in\tau_2$. The complement of $\tau_1\tau_2$ - open set is called $\boldsymbol{\tau_1\tau_2}$ **- closed** .The family of all $\tau_1\tau_2$ - open (resp. $\tau_1\tau_2$ -closed) sets of (X, τ_1, τ_2),is denoted by (1,2)*-o(X) (resp. (1,2)*-c(X)).

Example (2.1): Let X ={a,b,c}, τ_1= {X,ϕ,{a}} and τ_2 = { X,ϕ,{b}}. Then the sets in { X,ϕ,{a},{b},{a ,b}} are called $\tau_1\tau_2$ - open and the sets in {X,ϕ,{c},{a,c},{b,c}} are called $\tau_1\tau_2$ -closed.

Remark (2.3), [4]: $\tau_1\tau_2$ -open subsets of (X, τ_1, τ_2), need not necessarily form a topology.

Definition (2.4), [4]:- Let S be a subset of a bitopological space (X, τ_1, τ_2) ,Then:

(i) $\tau_1\tau_2$- **closure** of S denoted by $\tau_1\tau_2$ -cl(S) is defined by ∩ {F:S ⊆ F and F is $\tau_1\tau_2$ -closed}

(ii) $\tau_1\tau_2$ - **interior** of S denoted by ∪ {U:U ⊆ S and U is $\tau_1\tau_2$ -open}.

Definition (2.5): A subset S of (X, τ_1, τ_2), is said to be a:

(i) **(1,2)*-semi-open** [4] if S ⊂$\tau_1\tau_2$ -cl ($\tau_1\tau_2$ int(S)).

(ii) **(1,2)*-regular open** [4] if S = $\tau_1\tau_2$ -int($\tau_1\tau_2$ -cl(S)).

(iii) **(1,2)*-π-open** [6] if S is the finite union of (1,2)*-regular open sets.

(iv) **(1,2)*-b-open** [1] if S ⊂$\tau_1\tau_2$ -cl ($\tau_1\tau_2$ -int(S)) ∪ $\tau_1\tau_2$ -int ($\tau_1\tau_2$ -cl(S)).

The complements of all the above mentioned open sets are called their respective closed sets and we denoted the family of all (1,2)*-regular open (resp. (1,2)*-regular closed sets) by (1,2)*-RO (resp. (1,2)*RC).

The (1,2)*-b-closure of a subset S of (X, τ_1, τ_2), is denoted by (1,2)*-bcl(S) and defined as the intersection of all (1,2)*-b-closed sets containing S.

Definition (2.6):- A subset A of bitopological space (X, τ_1, τ_2), is said to be a :

(i) **(1,2)*-generalized closed set** [4] (briefly, (1,2)*-g-closed set) if $\tau_1\tau_2$-cl(A) ⊂ U whenever A ⊂ U and U ∈ (1,2)*-open set in (X, τ_1, τ_2).

(ii) **(1,2)*-semi-generalized-star-closed set** [7] (briefly, (1,2)*-sg*-closed set) if $\tau_1\tau_2$-cl(A) ⊂ U whenever A ⊂ U and U is (1,2)*- semi open set in (X, τ_1, τ_2).

(iii) **(1,2)*-π-generalized b-closed set** [9] (briefly, (1,2)*-πgb-closed set) if $\tau_1\tau_2$-bcl(A) ⊂ U whenever A ⊂ U and U is $\tau_1\tau_2$-π-open set in (X, τ_1, τ_2).

The family of all (1,2)*-g-closed sets (resp. (1,2)*-sg*-closed and (1,2)*-πgb-closed) of (X,τ_1,τ_2) will be denoted by (1,2)*-GC(X) (resp. (1,2)*-SG*C(X) and (1,2)*-πGBC(X))

Remark (2.7): In [7], [9] it is proved that in any bitopological spaces (X, τ_1, τ_2):

(1) Every $\tau_1\tau_2$-closed set (resp. (1,2)*-g-closed set) is (1,2)*-πgb-closed set in (X, τ_1, τ_2).

(2) Every $\tau_1\tau_2$-closed set (resp. (1,2)*-sg*-closed set) is (1,2)*-g-closed set in (X, τ_1, τ_2) is (1,2)*-πgb-closed set in (X, τ_1, τ_2).

The converse of the above Remark need not be true, as shown in the following example:

Examples (2.2):-

(i) Let X={a,b,c}, τ_1={X,ϕ,{a}} and τ_2={X,ϕ,{b},{a,b}}. Then (1,2)*o(X) = {X,ϕ,{a},{b},{a,b}}, (1,2)*c(X) ={X,ϕ,{c},{a,c},{b,c}} = (1,2)*-GC(X) and (1,2)*-πGBC(X) = {X,ϕ,{a},{b},{c},{a,c},{b,c}}. Then, clearly the set A = {a} in (X, τ_1, τ_2),is (1,2)*-πgb-closed sets, but is not $\tau_1\tau_2$-closed sets and (1,2)*-g-closed in (X , τ_1, τ_2).

(ii) Let X={a,b,c}, τ_1={X,ϕ} and τ_2={X,ϕ,{a}}. Then (1,2)*-O(X) = {X,ϕ,{a}}, (1,2)*-C(X)={X,ϕ,{b,c}}, (1,2)*-SG*C(X))={X,ϕ,{b,c}} and GC(X)= {X,ϕ,{b},{c},{a,b},{a,c},{b,c}}. Then clearly the set A = {b} is (1,2)*-g-closed sets in (X, τ_1, τ_2),but is not $\tau_1\tau_2$-closed set and (1,2)*-sg*-closed set in (X, τ_1, τ_2).

Definition (2.8): A map f : (X, τ_1, τ_2) ⟶ (Y,σ_1,σ_2) is said to be:

(1) **(1,2)*-closed** or $\tau_1\tau_2$-closed [8] if f (A) is $\sigma_1\sigma_2$-closed set in (Y,σ_1,σ_2) for every $\tau_1\tau_2$-closed set A in (X, τ_1, τ_2).

(2) **(1,2)*-g-closed** [3] if f (A) is (1,2)*-g-closed set in (Y,σ_1,σ_2) for every $\tau_1\tau_2$-closed set A in (X, τ_1, τ_2).

(3) **(1,2)*-sg*-closed** [7] if f (A) is (1,2)*-sg*-closed set in (Y,σ_1,σ_2) for every $\tau_1\tau_2$-closed set A in (X, τ_1, τ_2).

(4) **Pre-(1,2)*-sg*-closed** [7] if f (A) is (1,2)*-sg*-closed set in (Y,σ_1,σ_2) for every (1,2)*-sg*-closed set A in (X, τ_1, τ_2).

(5) **(1,2)*-πgb-closed** [10] if f (A) is (1,2)*-πgb-closed set in (Y,σ_1,σ_2) for every $\tau_1\tau_2$-closed set A in (X, τ_1, τ_2).

(6) **M-(1,2)*-πgb-closed** [10] if f (A) is (1,2)*-πgb-closed set in (Y,σ_1,σ_2) for every (1,2)*-πgb-closed set A in (X, τ_1, τ_2).

Definition (2.9): A map f : ((X, τ_1, τ_2) ⟶ (Y,σ_1,σ_2) is said to be a :

(1) **(1,2)*-continuous** [5] if f^{-1}(A) is $\tau_1\tau_2$-closed set in (X, τ_1, τ_2) for every $\sigma_1\sigma_2$-closed set A in (Y,σ_1,σ_2) .

(2) **(1,2)*-g-continuous** [3] if f^{-1}(A) is (1,2)*-g-closed set in (X, τ_1, τ_2) for every $\tau_1\tau_2$-closed set in (Y,σ_1,σ_2) .

(3) **(1,2)*-πgb-continuous** [9] if f^{-1}(A) is (1,2)*-πgb-closed set in (X, τ_1, τ_2) for every $\tau_1\tau_2$ -closed set in (Y,σ_1,σ_2) .

(4) **(1,2)*-πgb-irresolute** [9] if f^{-1}(A) is (1,2)*-πgb-closed set in (X, τ_1, τ_2) for every (1,2)*-πgb-closed set in (Y,σ_1,σ_2) .

Definition (2.10): A bitopological space (X, τ_1, τ_2) is called a :

(1) **(1,2)*-$T_{1/2}$-space** [2] if every (1,2)*-g-closed set is $\tau_1\tau_2$-closed.

(2) **(1,2)*-πgb-space** [9] if every (1,2)*-πgb-closed set is $\tau_1\tau_2$-closed.
(3) **RM-space** [7] if every subset in (X, τ_1, τ_2) is either $\tau_1\tau_2$-open or $\tau_1\tau_2$-closed.

Theorem (2.11) ,[7]:- In RM-space (X, τ_1, τ_2) every (1,2)*-sg*-closed set is $\tau_1\tau_2$-closed.

Theorem (2.12) , [9]:- Let B $\subseteq$ A $\subseteq$ X where A is (1,2)*-πgb-closed and $\tau_1\tau_2$-π-open set. Then B is (1,2)*-πgb-closed in X.

3- Some Types of (1,2)*-M-πgb-closed Maps:

We now introduce the following a new definition of (1,2)*-M-πgb-closed mapping.

Definition (3.1): A map f : (X, τ_1, τ_2) $\longrightarrow$ (Y,σ_1,σ_2) is said to be **(1,2)*-M*-πgb-closed** if
f (A) is $\sigma_1\sigma_2$-closed set in (Y,σ_1,σ_2) for every (1,2)*-πgb-closed set A in (X, τ_1, τ_2).

Example(3.1):- Let X=Y={a,b,c}, τ_1={X,ϕ,{a}} and τ_2={X,ϕ,{b},{a,b}}, σ_1={Y,ϕ,{a},{a,c}}, σ_2 ={Y,ϕ,{a,b}}. Then the sets {X,ϕ,{a},{b},{a,b}} are $\tau_1\tau_2$-open in (X,τ_1,τ_2), the sets {X,ϕ,{c},{a,c},{b,c}} are $\tau_1\tau_2$-closed in (X, τ_1, τ_2) and GBC(X, τ_1, τ_2) = {X,ϕ,{a},{b},{c],{a,c},{b,c}}, the sets {Y,ϕ,{a},{a,c}, {a,b}} are $\sigma_1\sigma_2$-open in (Y,σ_1,σ_2) and the sets {Y,ϕ,{c},{b},{b,c}} are $\sigma_1\sigma_2$-closed in (Y,σ_1,σ_2) . Define f: (X, τ_1, τ_2) $\longrightarrow$ (Y,σ_1,σ_2) by $f(a) = f(b)$ = b and $f(c)$ = c. Then clearly f is (1,2)*-M*-πgb-closed map.

Proposition (3.2): If f: (X, τ_1, τ_2) $\longrightarrow$ (Y,σ_1,σ_2) is (1,2)*-M*-πgb-closed map. Then f is a:
(i) (1,2)*-closed map.
(ii) (1,2)*-g-closed map.
(iii)(1,2)*-sg*-closed map.
(iv)(1,2)*-πgb-closed map.
(v) (1,2)*-M-πgb-closed map.
(vi) Pre (1,2)*-sg*-closed map.

Proof:
(i) Let A be a $\tau_1\tau_2$-closed set in (X, τ_1, τ_2) . By Remark (2.7) Part (1) (every $\tau_1\tau_2$-closed set is (1,2)*-πgb-closed). Then A is (1,2)*-πgb-closed set in (X, τ_1, τ_2) . Since f is (1,2)*-M*-πgb-closed map. Thus f (A) is $\sigma_1\sigma_2$-closed set in (Y,σ_1,σ_2) . Hence, f is (1,2)*-closed map.
(ii) Let A be a $\tau_1\tau_2$-closed set in (X, τ_1, τ_2) . By Remark (2.7) Part (1) we get A is (1,2)*-πgb-closed set in (X, τ_1, τ_2) .Since f is (1,2)*-M*-πgb-closed map. Thus f (A) is $\sigma_1\sigma_2$-closed set in (Y,σ_1,σ_2) . Also, since

(every $\sigma_1\sigma_2$-closed set is (1,2)*-g-closed, [4]). Therefore *f*(A) is (1,2)*-g-closed set in (Y,σ_1,σ_2) . Hence, *f* is (1,2)*-g-closed map.

(iii) Let A be a $\tau_1\tau_2$-closed set in (X, τ_1, τ_2) .By Remark (2.7) we get A is (1,2)*-πgb-closed set in (X, τ_1, τ_2) .Since *f* is (1,2)*-M*-πgb-closed map. Thus *f* (A) is $\sigma_1\sigma_2$-closed set in (Y,σ_1,σ_2) . Also, since (every $\sigma_1\sigma_2$-closed set is (1,2)*-g-closed, [4] and every (1,2)*-g-closed set is (1,2)*-sg*-closed set, [7]). Therefore, *f* (A) is (1,2)*-sg*-closed set in (Y,σ_1,σ_2). Hence, *f* is (1,2)*-sg*-closed map.

(iv) Let A be a $\tau_1\tau_2$-closed set in (X, τ_1, τ_2) .Since (every $\tau_1\tau_2$-closed set is (1,2)*-πgb-closed). Then A is (1,2)*-πgb-closed set in (X, τ_1, τ_2) .Thus, *f* (A) is $\sigma_1\sigma_2$-closed set in (Y,σ_1,σ_2) .Also, by using Remark (2.7) Part (1) we get *f* (A) is (1,2)*-πgb-closed set in (Y,σ_1,σ_2) .Therefore , *f* is (1,2)*-πgb-closed map.

The proof of Parts (v) and (vi) are similar to above.

Remark (3.3): The converse of Proposition (3.2) may not be true in general. Consider the following example:

<u>Example (3.2):-</u>

Let X=Y={a,b,c}, τ_1={X,ϕ} and τ_2={X,ϕ,{a}}, σ_1={Y,ϕ,{a}}, σ_2={Y,ϕ,{b},{a,b}}. Then the sets {X,ϕ,{a}} are $\tau_1\tau_2$-open sets in (X, τ_1, τ_2) , the sets {X,ϕ,{b,c}}are $\tau_1\tau_2$-closed sets in (X, τ_1, τ_2) , πGBC(X, τ_1, τ_2)={X,ϕ,{a},{b},{c},{a,b},{a,c},{b,c}}, the sets {Y,ϕ,{a},{b},{a,b}} are $\sigma_1\sigma_2$-open set in (Y,σ_1,σ_2) and the sets {Y,ϕ,{c},{a,c},{b,c}} are $\sigma_1\sigma_2$ -closed set in (Y,σ_1,σ_2) .Define *f* :(X, τ_1, τ_2) $\longrightarrow$ (Y,σ_1,σ_2) by *f* (a) = b, *f* (b) = a and *f* (c) = c. Then, *f* is (1,2)*-closed map, but *f* is not (1,2)*-M*-πgb-closed map. Since for (1,2)*-πgb-closed set A = {a} in (X, τ_1, τ_2) , *f* (A) = *f* ({a}) = {b} is not $\sigma_1\sigma_2$-closed set in (Y,σ_1,σ_2).

<u>Example(3.3):-</u>Let X={a,b,c}, τ_1={X,ϕ,{a}} and τ_2={X,ϕ,{b},{a,b}}. Then the sets in {X,ϕ,{a},{b},{a,b}}are $\tau_1\tau_2$-open sets (X,τ_1,τ_2), the sets {X,ϕ,{c},{a,c},{b,c}} are $\tau_1\tau_2$-closed sets (X, τ_1, τ_2) and (1,2)*-GC(X, τ_1, τ_2) (1,2)SG*C((X,τ_1,τ_2))={X,ϕ,{c},{a,c},{b,c}}. The identity map**,** *f:* (X, τ_1, τ_2)$\longrightarrow$(X, τ_1, τ_2) is (1,2)*-g-closed, (1,2)*-sg*-closed and pre(1,2)*-sg*-closed) map but *f* is not (1,2)*-M*-πgb-closed map. Since for (1,2)*-πgb-closed set A = {a} in (X, τ_1, τ_2) , *f* (A) = *f* ({a}) = {A} is not $\tau_1\tau_2$-closed set in (X, τ_1, τ_2)**.**

<u>Example (3.4):-</u>

Let X={a,b,c}, τ_1={X,ϕ} and τ_2={X,ϕ,{a}}. Then the sets {X,ϕ,{a}} are $\tau_1\tau_2$-open sets in (X, τ_1, τ_2) , the sets {X,ϕ,{b,c}} are $\tau_1\tau_2$-closed sets in (X, τ_1, τ_2) and πGBC(X, τ_1, τ_2)={X, ϕ, {a}, {b}, {c}, {a,b}, {a,c}, {b,c}}. Define *f* : (X, τ_1, τ_2) ⟶(X, τ_1, τ_2) by *f* (a) = c, *f* (b) = a and *f* (c) = b. Then clearly *f* is (1,2)*-πgb-closed map and (1,2)*-M-πgb-closed map, but *f* is not (1,2)*-M*-πgb-closed. Since for a $\tau_1\tau_2$-closed set A = {a} in (X, τ_1, τ_2) ,*f* (A) = *f* ({a}) = {c} is not $\tau_1\tau_2$-closed set in (X, τ_1, τ_2).

The following Propositions give the condition to make the Proposition (3.2) true.

Proposition (3.4): Let *f*: (X, τ_1, τ_2) ⟶ (Y,σ_1,σ_2) be any map and X, Y be two (1,2)*-πgb-space. Then *f* is (1,2)*-M*-πgb-closed map, if *f* is a:

(i) (1,2)*-closed map.

(ii) (1,2)*-πgb-closed map.

(iii)(1,2)*-M-πgb-closed map.

Proof:

(i) Let A be a πgb-closed set in (X, τ_1, τ_2) .Since (X, τ_1, τ_2) is a (1,2)*-πgb-space and by using Definition (2.10) Part (ii) we get, A is a $\tau_1\tau_2$-closed set in (X, τ_1, τ_2) .Also, since *f* is a (1,2)*-closed map. Thus *f* (A) is a $\sigma_1\sigma_2$-closed set in (Y,σ_1,σ_2) .Hence *f* is (1,2)*-M*-πgb-closed map.

(ii) Let A be a πgb-closed set in (X, τ_1, τ_2) .Since (X, τ_1, τ_2) is a (1,2)*-πgb-space and by Definition (2.10) we have A is a $\tau_1\tau_2$-closed set in (X, τ_1, τ_2) .By hypothesis *f* is (1,2)*-πgb-closed map. Thus *f* (A) is a (1,2)*-πgb-closed set in (Y,σ_1,σ_2) .Also, since Y is a (1,2)*-πgb-space and by Definition (2.10) Part (ii) we obtain *f* (A) is $\sigma_1\sigma_2$-closed set in (Y,σ_1,σ_2). Therefore *f* is (1,2)*-M*-πgb-closed map.

The proof of Part (iii) is similar to Parts (i) and (ii).

Proposition (3.5):

Let X be a (1,2)*-πgb-space and Y be a (1,2)*-$T_{1/2}$-space. If *f* :(X, τ_1, τ_2) ⟶ (Y,σ_1,σ_2) is a (1,2)*-g-closed map. Then *f* is (1,2)*-M*-πgb-map.

Proof:- Let A be a (1,2)*-πgb-closed set in (X, τ_1, τ_2) .Since X is (1,2)*-πgb-space and by Definition (2.10) Part (ii) we get, A is a $\tau_1\tau_2$-closed set in (X, τ_1, τ_2) .Also, since *f* is a (1,2)*-g-closed map. Thus, *f* (A) is a (1,2)*-g-closed set in (Y,σ_1,σ_2) .By hypothesis Y is a (1,2)*-$T_{1/2}$-space and by Definition (2.10) Part (i) we have *f* (A) is a $\sigma_1\sigma_2$-closed set in (Y,σ_1,σ_2) .Hence , *f* is a (1,2)*-M*-πgb-closed map.

Proposition (3.6): Let X be a (1,2)*-πgb-space and Y be a RM-space. Then a map *f*: (X, τ_1, τ_2)⟶(Y,σ_1,σ_2) is a(1,2)*-M*-πgb- closed. If *f* is a:

(i) (1,2)*-sg*-closed map.

(ii) Pre (1,2)*-sg*-closed map.

Proof:

(i) Let A be a (1,2)*-πgb-closed set in (X, τ_1, τ_2) .Since X is (1,2)*-πgb-space. Then A is a $\tau_1\tau_2$-closed set in (X, τ_1, τ_2).Also, since *f* is a (1,2)*-sg*-closed map. Thus *f* (A) is a (1,2)*-sg*-closed set in (Y,σ_1,σ_2) .By hypothesis Y is a RM-space and by Definition (2.10) Part (iii) we get *f* (A) is a $\sigma_1\sigma_2$-closed set in (Y,σ_1,σ_2) .Therefore, *f* is a (1,2)*-M*-πgb-closed map.

The proof of Part (ii) is similar to Part (i).

Proposition (3.7):- If *f*: (X, τ_1, τ_2) ⟶ (Y,σ_1,σ_2) is a (1,2)*-M*-πgb-closed map and A is (1,2)*-πgb-closed and $\tau_1\tau_2$-π-open set in (X, τ_1, τ_2). Then $f|_A$:(A, τ_1, τ_2)⟶ (Y,σ_1,σ_2) is (1,2)*-M*-πgb-closed map.

Proof:

Let B be a (1,2)*-πgb-closed subset of A. Then by Theorem (2.12) we get B is a (1,2)*-πgb-closed in (X,τ_1,τ_2). Since *f* is a (1,2)*-M*-πgb-closed map. Thus *f* (B) is a $\sigma_1\sigma_2$-closed set in (Y,σ_1,σ_2). But *f* (B) = ($f|_A$)(B). Therefore ($f|_A$)(B) is a $\tau_1\tau_2$-closed set in (Y,σ_1,σ_2) .Hence, $f|_A$ is a (1,2)*-M*-πgb-closed map.

Corollary (3.8):- If *f* :(X, τ_1, τ_2)⟶ (Y,σ_1,σ_2) is a (1,2)*-M*-πgb-closed map and A is (1,2)*-πgb-closed and $\tau_1\tau_2$-open set in (X,τ_1,τ_2). Then $f|_A$:(A,,τ_1,τ_2)⟶ (Y,σ_1,σ_2) is a:

(i) (1,2)*-closed map

(ii) (1,2)*-πgb-closed map.

(iii) (1,2)*-M-πgb-closed map.

Proof:

(i) This follows from Propositions (3.7) and (3.2) Part (i).

(ii) This follows from Propositions (3.7) and (3.2) Part (ii).

(iii) This follows from Propositions (3.7) and (3.2) Part (iii).

Proposition (3.9): Let *f*: (X, τ_1, τ_2) ⟶ (Y,σ_1,σ_2)) and g: (Y,σ_1,σ_2) ⟶ (Z,μ_1,μ_2)be any two maps. Then g∘*f* : (X, τ_1, τ_2) ⟶ (Z,μ_1,μ_2)is (1,2)*-M*-πgb-closed map if :

(i) *f* and g are two (1,2)*-M*-πgb-closed maps.

(ii) *f* is a (1,2)*-M*-πgb-closed map and g is a (1,2)*-closed.

(iii)*f* is (1,2)*-M-πgb-closed map and g is (1,2)*-M**-πgb-closed.

Proof:

(i) Let A be a (1,2)*-πgb-closed set in (X, τ_1, τ_2) .Since *f* is a (1,2)*-M*-πgb-closed map. Thus, *f* (A) is $\tau_1\tau_2$-closed set in (Y,σ_1,σ_2) . By Remark (2.7) Part (i) we get *f* (A) is a (1,2)*-πgb-closed set in (Y,σ_1,σ_2) .Also, since g is a (1,2)*-M*-πgb-closed map. Hence, g(*f*

(A)) is a $\mu_1\mu_2$-closed set in (Z,μ_1,μ_2). But g(f (A)) = g∘f (A). Then g∘f (A) is a $\mu_1\mu_2$-closed set in (Z,μ_1,μ_2).Therefore, g∘f is a (1,2)*-M*-πgb-closed map.

The prove of Part -ii- and -iii- are similar to Part -i- **.**

Proposition (3.10): Let f: (X, τ_1, τ_2) ⟶ (Y,σ_1,σ_2) and g: (Y,σ_1,σ_2) ⟶ (Z,μ_1,μ_2) be any two maps. Then g∘f :(X, τ_1, τ_2) ⟶ (Z,μ_1,μ_2) is (1,2)*-πgb-closed map iff is a (1,2)*-M*-πgb-closed map and g is a

(i) (1,2)*-πgb-closed maps.

(ii) (1,2)*-M-πgb-closed map.

Proof:

(i) Let A be a $\tau_1\tau_2$-closed set in (X, τ_1, τ_2) .By Remark (2.7) Part (i) we get A is a (1,2)*-πgb-closed set in (X, τ_1, τ_2) .Since f is a (1,2)*-M*-πgb-closed map. Thus, f (A) is a $\sigma_1\sigma_2$-closed set in (Y,σ_1,σ_2) . Also, since g is a (1,2)*-πgb-closed map. Hence, g(f (A)) is a (1,2)*-πgb-closed set in (Z,μ_1,μ_2).But, g(f (A)) = g∘f (A). Then, g∘f (A) is a(1,2)*-πgb-closed set in (Z,μ_1,μ_2).Therefore, g∘f is a (1,2)*-πgb-closed map.

The proof of Part (ii) is similar to Part (i).

However the following proposition holds. The proof is easy and hence omitted.

Proposition (3.11): Let f :(X, τ_1, τ_2) ⟶ (Y,σ_1,σ_2) and g: (Y,σ_1,σ_2) ⟶ (Z,μ_1,μ_2) be any two maps. Then g∘f :(X, τ_1, τ_2) ⟶ (Z,μ_1,μ_2)is (1,2)*-πgb-closed map if g is a (1,2)*-M*-πgb-closed map and f is a

(i) (1,2)*-closed maps.

(ii) (1,2)*-πgb-closed map.

Proposition (3.12): Let f : (X, τ_1, τ_2) ⟶ (Y,σ_1,σ_2) and g: (Y,σ_1,σ_2) ⟶ (Z,μ_1,μ_2)be any two maps, such that g∘f: (X, τ_1, τ_2) ⟶ (Z,μ_1,μ_2)is (1,2)*-M*-πgb-closed map if f is a (1,2)*-πgb-irresolute map and surjective, then g is (1,2)*-M*-πgb-closed map.

Proof:- Let A be a (1,2)*-πgb-closed set in (Y,σ_1,σ_2) .Since f is (1,2)*-πgb-irresolute. Thus, f^{-1}(A) is a πgb-closed set in (X, τ_1, τ_2) .Also, since g∘f is a (1,2)*-M**-πgb-closed map. Then, (g∘f)(f^{-1}(A)) is a $\mu_1\mu_2$-closed set in (Z,μ_1,μ_2).By hypothesis g∘f is surjective. Hence, (g∘f)(f^{-1}(A))= g∘(f∘f^{-1})(A) = g(A). Therefore g(A) is a $\mu_1\mu_2$-closed set in (Z,μ_1,μ_2). Hence, g is (1,2)*-M**-πgb-closed map.

Proposition (3.13): Let f :(X, τ_1, τ_2) ⟶ (Y,σ_1,σ_2) and g: (Y,σ_1,σ_2) ⟶ (Z,μ_1,μ_2)be two mappings, such that g∘ f: (X, τ_1, τ_2) ⟶ (Z,μ_1,μ_2)be (1,2)*-M*-πgb-closed mapping. Then g is (1,2)*-M*-πgb-closed map. If a surjective map f is a:

(i) (1,2)*-continuous map and (Y,σ_1,σ_2) is (1,2)*-πgb-space.

(ii) (1,2)*-g-continuous map, (X, τ_1, τ_2) is (1,2)*-$T_{1/2}$ space and (Y,σ_1,σ_2) is (1,2)*-πgb-space.

(iii) (1,2)*-πgb -continuous map, (X, τ_1, τ_2) is (1,2)*-πgb-space.

Proof:

(i) Let A be a (1,2)*-πgb-closed set in (Y,σ_1,σ_2) .Since (Y,σ_1,σ_2) is a (1,2)*-πgb-space and by using Definition (2.10) Part (ii) we get A is a $\sigma_1\sigma_2$-closed set in (Y,σ_1,σ_2) .Also, since f is a (1,2)*-continuous map. Thus f^{-1}(A) is $\tau_1\tau_2$-closed set in (X, τ_1, τ_2) . By Remark (2.7) Part (i) (every $\tau_1\tau_2$-closed set is (1,2)*-πgb-closed). So, we have f^{-1}(A) is a (1,2)*-πgb-closed set in (X, τ_1, τ_2) and by hypothesis g∘f is (1,2)*-M*-πgb-closed map. Then,(g∘f)(f^{-1}(A) is a $\mu_1\mu_2$-closed set in (Z,μ_1,μ_2).Since f is a surjective and (g∘f)(f^{-1}(A))) = g(f∘f^{-1}(A)) = g(A). That is g(A) is a $\mu_1\mu_2$-closed set in (Z,μ_1,μ_2). Therefore, g is a (1,2)*-M**-πgb-closed map.

(ii) Let A be a (1,2)*-πgb-closed set in (Y,σ_1,σ_2) .By hypothesis (Y,σ_1,σ_2) is a (1,2)*-πgb-space and by using Definition (2.10) Part (ii) we get A is a $\sigma_1\sigma_2$-closed set in (Y,σ_1,σ_2) .Also, since f is a (1,2)*-g-continuous map. Thus, f^{-1}(A) is a (1,2)*-g-closed set in (X, τ_1, τ_2) ,since (X, τ_1, τ_2) is a (1,2)*-$T_{1/2}$-space and by using Definition (2.10) Part (i) we have f^{-1}(A) is a $\tau_1\tau_2$-closed set in (X, τ_1, τ_2) .By Remark (2.7) Part (i) we get f^{-1}(A) is a (1,2)*-πgb-closed set in (X, τ_1, τ_2) and since g∘f is (1,2)*-M*-πgb-closed map. Then, (g∘f)(f^{-1}(A)) is a $\mu_1\mu_2$-closed set in (Z,μ_1,μ_2).But f is a surjective and (g∘f)(f^{-1}(A)) = g(f∘f^{-1}(A)) = g(A). Hence, g(A) is a $\mu_1\mu_2$-closed set in (Z,μ_1,μ_2).Therefore, g is a (1,2)*-M**-πgb-closed map.

The proof of Part (iii) is similar to Part (i).

Proposition (3.14): Let f: (X, τ_1, τ_2) ⟶ (Y,σ_1,σ_2) and g: (Y,σ_1,σ_2) ⟶ (Z,μ_1,μ_2).be two maps, such that g∘f: (X, τ_1, τ_2) ⟶ (Z,μ_1,μ_2).be (1,2)*-M*-πgb-closed map. Then f is a (1,2)*-M**-πgb-closed map. If:

(i) g is (1,2)*-continuous and injective.

(ii) g is (1,2)*-πgb-continuous, injective and (Y,σ_1,σ_2) is a (1,2)*-πgb-space.

(iii) g is (1,2)*-M-πgb–continuous, injective and (Y,σ_1,σ_2) is a (1,2)*-πgb-space.

(iv) g is (1,2)*-g-continuous, injective and (Y,σ_1,σ_2) is a (1,2)*-$T_{1/2}$-space.

Proof:

(i) Let A be a (1,2)*-πgb-closed set in (X, τ_1, τ_2) .Since g∘*f* is (1,2)*-M*-πgb-closed map. Thus, (g∘*f*)(A) is a $\mu_1\mu_2$-closed set in (Z,μ_1,μ_2). Also, since g is a (1,2)*-continuous map. Then(g^{-1}(g∘*f*))(A) is a $\sigma_1\sigma_2$-closed set in (Y,σ_1,σ_2) .Since g is injective and (g^{-1}(g∘*f*))(A) = (g^{-1}∘g) *f* (A) = *f* (A). Hence, *f* (A) is a $\sigma_1\sigma_2$-closed set in (Y,σ_1,σ_2) .Therefore *f* is a (1,2)*-M*-πgb-closed map.

(ii) Let A be a (1,2)*-πgb-closed set in (X, τ_1, τ_2) .Since g∘*f* is (1,2)*-M*-πgb-closed map. Thus (g∘*f*)(A) is a $\mu_1\mu_2$-closed set in (Z,μ_1,μ_2).Since g is a (1,2)*-πgb-continuous map. Then(g^{-1}(g∘*f*))(A) is a (1,2)*-πgb-closed set in (Y,σ_1,σ_2) .By hypothesis (Y,σ_1,σ_2) is a (1,2)*-πgb-space and by Definition (2.10) Part (ii) we have (g^{-1}(g∘*f*))(A) is a $\sigma_1\sigma_2$-closed set in (Y,σ_1,σ_2) .Also, since g is injective and (g^{-1}(g∘*f*))(A) = (g^{-1}∘g) *f* (A) = *f* (A). Hence, *f* (A) is a $\sigma_1\sigma_2$-closed set in (Y,σ_1,σ_2) .Therefore *f* is a (1,2)*-M*-πgb-closed map.

The proof of Part (iii) and (iv) as similar to Part (ii).

Now, we given other type of (1,2)*- M-closed mapping is called (1,2)*-M**-πgb-closed.

Definition (3.15): A mapping *f* : (X, τ_1, τ_2) ⟶ (Y,σ_1,σ_2) is said to be **(1,2)*-M**-πgb-closed** if *f* (A) is (1,2)*-regular closed set in (Y,σ_1,σ_2) for every (1,2)*-πgb-closed set A in (X, τ_1, τ_2)**.**

<u>Example (3.5):-</u>

Let X= {a,b}, Y={a,b,c}, τ_1={X,ϕ} and τ_2={X,ϕ,{a}}, σ_1={Y,ϕ,{a}}, σ_2={Y,ϕ,{b,c}}. Then $\tau_1\tau_2$-open sets in (X, τ_1, τ_2) are {X,ϕ,{a}} and $\sigma_1\sigma_2$-open set in (Y,σ_1,σ_2) are {Y,ϕ,{a},{b,c}}.(1,2)*-πGBC(X, τ_1, τ_2)={X,ϕ,{a},{b}} and (1,2)*-RC(Y,σ_1,σ_2)={Y,ϕ,{a},{b,c}} , let *f* :(X, τ_1, τ_2) ⟶ (Y,σ_1,σ_2) defined by *f* (a) = *f* (b) = a. Then, clearly *f* is (1,2)*-M**-πgb-closed mapping.

Proposition (3.16): Let *f*: (X, τ_1, τ_2) ⟶ (Y,σ_1,σ_2) be a (1,2)*-M**-πgb-closed mapping, then *f* is (1,2)*-M*-πgb-closed.

Proof:-

Let A be (1,2)*-πgb-closed set in (X, τ_1, τ_2) .Since *f* is (1,2)*-M**-πgb-closed map, thus *f* (A) is (1,2)*-regular closed set in (Y,σ_1,σ_2) , (since every (1,2)*-regular closed set is (1,2)*-closed set, [4]). Therefore, *f* (A) is $\sigma_1\sigma_2$-closed set in (Y,σ_1,σ_2) . Hence, *f* is (1,2)*-M*-πgb-closed map.

The converse of Proposition (3.16) may not be true in general as shown in the following.

Example (3.6):-

Let X= {a,b,c}, τ_1={X,ϕ,{a},{b},{a,b}} and τ_2={X,ϕ,{b,c}}. Then $\tau_1\tau_2$-open sets in (X, τ_1, τ_2) = {X,ϕ,{a},{b},{a,b},{b,c}}, $\tau_1\tau_2$-closed set in (X, τ_1, τ_2) = {X,ϕ,{a},{c},{a,c},{b,c}}, (1,2)*-πGBC(X, τ_1, τ_2)={X,ϕ,{a},{b},{c},{a,b},{a,c}, {b,c}} and (1,2)*-RC(X , τ_1, τ_2)={X,ϕ,{a},{b,c}}.Define *f* : (X, τ_1, τ_2) ⟶ (X, τ_1, τ_2) by *f* (a) = a, *f* (b) = b and *f* (c) = c. It is clear that *f* is (1,2)*-M*-πgb-closed map, but is not (1,2)*-M**-πgb-closed map, since for a subset A = {a,c} is (1,2)*-πgb-closed set in (X, τ_1, τ_2) , but *f* (A) = *f* ({a,c}) = {a,c} is not (1,2)*-regular closed set in (X, τ_1, τ_2) .

Corollary (3.17): Let *f* :(X, τ_1, τ_2) ⟶ (Y,σ_1,σ_2) be a (1,2)*-M**-πgb-closed mapping, then *f* is

(i) $\tau_1\tau_2$-closed map.
(ii) (1,2)*-g-closed map.
(iii) (1,2)*-sg*-closed map.
(iv) (1,2)*-πgb-closed map.
(v) (1,2)*-M-πgb-closed map.

Proof:-

Since *f* is(1,2)*-M**-πgb-closed map. Then by using Proposition (3.16) we get *f* is (1,2)*-M-πgb-closed map and by Proposition (3.2) we have *f* is $\tau_1\tau_2$-closed map (resp. (1,2)*-g-closed, (1,2)*-sg*-closed, (1,2)*-πgb-closed and M-(1,2)*-πgb-closed map.

The converse of Corollary (3.17) need not be true in general.

Example (3.7):-

Let X={a,b,c}, τ_1={X,ϕ} and τ_2={X,ϕ,{a}}. Then $\tau_1\tau_2$-open sets in (X, τ_1, τ_2) are {X,ϕ,{a},{b},{a,b},{b,c}}, $\tau_1\tau_2$-closed set in (X, τ_1, τ_2) are {X,ϕ,,{b,c}} and (1,2)*-RC(X, τ_1, τ_2)={X,ϕ}. Define *f* : (X, τ_1, τ_2) ⟶ (X, τ_1, τ_2) by *f* (a) = a, *f* (b) = b and *f* (c) = c. Then, *f* is $\tau_1\tau_2$-closed (resp. (1,2)*-g-closed, (1,2)*-sg*-closed, (1,2)*-πgb-closed and M-(1,2)*-πgb-closed) map, but *f* is not (1,2)*-M**-closed map, since for (1,2)*-πgb-closed set A = {a,c} in (X, τ_1, τ_2) .*f* (A) = *f* ({a,c}) = {a,c} is not (1,2)*-regular closed set in (X,τ_1,τ_2).

Proposition (3.18): Let *f*: (X, τ_1, τ_2) ⟶ (Y,σ_1,σ_2) and g: (Y,σ_1,σ_2) ⟶ (Z,μ_1,μ_2).are both (1,2)*-M**-πgb-closed mappings. Then g∘*f*: (X, τ_1, τ_2) ⟶ (Z,μ_1,μ_2).is also (1,2)*-M**-πgb-closed mapping

Proof:-

Let A be (1,2)*-πgb-closed set in (X, τ_1, τ_2) , thus *f* (A) is (1,2)*-regular closed set in (Y,σ_1,σ_2) , (since every (1,2)*-regular closed set is (1,2)*-closed set, [4]). Hence, *f* (A) is $\sigma_1\sigma_2$-closed set in (Y,σ_1,σ_2) and by Remark (2.7) Part (i) we get *f* (A) is (1,2)*-πgb-closed set in (Y,σ_1,σ_2) . Also, since g is (1,2)*-M**-πgb-closed map. Thus g(*f* (A)) is (1,2)*-regular closed set in (Z,μ_1,μ_2).But g (*f* (A)) = g∘*f* (A). Thus, g∘*f* (A) is (1,2)*-regular closed set in (Z,μ_1,μ_2). Therefore , g∘*f*: (X, τ_1, τ_2) ⟶ (Z,μ_1,μ_2), is (1,2)*-M**-πgb-closed mapping

The proof of the following Propositions are clear. Thus we omitted it.

Proposition (3.19): Let *f*: (X, τ_1, τ_2) ⟶ (Y,σ_1,σ_2) is (1,2)*-M**-πgb-closed mapping and g: (Y,σ_1,σ_2) ⟶ (Z,μ_1,μ_2).be any mapping. Then g∘*f*: (X, τ_1, τ_2) ⟶ (Z,μ_1,μ_2).is (1,2)*-πgb-closed (resp. M-(1,2)*-πgb-closed) map if g is

(i) (1,2)*-πgb-closed map.

(ii) M-(1,2)*-πgb-closed map.

(iii) (1,2)*-M*-πgb-closed map.

Proposition (3.20): Let *f*: (X, τ_1, τ_2) ⟶ (Y,σ_1,σ_2) be any and g: (Y,σ_1,σ_2) ⟶ (Z,μ_1,μ_2).be (1,2)*-M*-πgb-closed mapping. Then g∘*f*: (X, τ_1, τ_2) ⟶ (Z,μ_1,μ_2).is (1,2)*-M*-πgb-closed map iff is

(i) M-(1,2)*-πgb-closed map.

(ii) (1,2)*-M*-πgb-closed map.

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بعض انواع من التطبيقات المغلقة *(1,2)− من النمط M −πgb

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المستخلص :

في هذا البحث، قمنا بدراسة أنواع جديدة من التطبيقات المغلقة M-*(1,2)-πgb وتدعى (التطبيقات المغلقة *(1,2)−من النمط *πgb- M والتطبيقات المغلقة *(1,2)−من النمط -πgb- **M) في ثنائية التبولوجي، درسنا بعض تطبيقاته وصفاته.