Wilcoxon Signed-Rank Test for Serum Thyroid Hormones

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Abstract:

In this work, we apply Wilcoxon singed-rank test upon three nonparametric random samples taken from serum thyroxin hormones which are T.S.H, T_3,and T_4 respectively with each sample size n=25, to test the hypothesis about population mean and tabulate the data. After applying the test we conclude that the chosen population means are accepted for all samples.

Introduction:

Biologically, Thyroxin hormones T_3, T_4 secreted from thyroid gland which situated in the anterior aspect of the neck. These hormones affect all vital activities of the body. This decrease leads to what we call hypothyroidism that is characterized by low response decrease in ability of thinking, in the contrary increases in these hormones leads to what we call hyperthyroidism which characterized tachycardia, increase blood pressure, hot temperature and exophthalmoses. These two hormones under the affect of Thyroid Stimulating Hormone (T.S.H) which is secreted by the pituitary gland a small gland situated at the base of the brain[5]. There is a reciprocal affect between these hormones when there is a low secretion of T_3, T_4 there is increase in T.S.H to stimulate more release of T_3, T_4 on the contrary if there is increase in T_3, T_4 the pituitary gland will lower its secretion of T.S.H. The thyroid gland uses iodine as precursor to manufacture thyroxin therefore any decrease in iodine intake lead to enlargement of this gland to compensate for this deficiency therefore whenever we want to do scanning for this gland we give a dos of radioactive iodine RAI this will be picked up by the thyroid gland after that we do x ray scan to see the gland [1].

Statistically, most of the hypothesis-testing procedures based on the assumption that they are working with random samples from normal distribution, these procedures called parametric methods. Otherwise the non parametric or distribution free methods make no assumption about the distribution of the underlying population other than that it is continuous with actual level of significance α [3].

Frank Wilcoxon devised a test procedure that uses both direction (sign) and magnitude. This procedure called the Wilcoxon signed rank-test [4]. The Wilcoxon signed-rank test applies to the case of symmetric continuous distribution[6]. Under these assumptions the mean equals the median. In this paper we use this procedure to test the hypothesis about population mean of three types of hormones in human blood serum.

DESCRIPTION OF WILCOXON SIGNED-RANK TEST:

It is interested in testing $H_0: \mu = \mu_0$ against the usual alternatives. Assume that $X_1, X_2, \ldots, X_n$ is a random sample from a continuous and symmetric distribution with mean (and median) μ. Differences $|X_i - \mu_0|$, i=1,2,…,n in ascending order, and then give the ranks the signs of their corresponding differences. Let W^+ be the sum of the positive ranks and W^- be the absolute

value of the sum of the negative ranks, and let W=min(W^+, W^-) . Appendix

Table VIII [4] contains critical values of W, say W_α^* . If the alternative hypothesis is $H_1: \mu \neq \mu_0$ then if the observed value of the statistic $W \le W_\alpha^*$, the null hypothesis $H_0: \mu = \mu_0$ is rejected. Appendix Table VIII [4] provides significance levels of $\alpha = 0.10, \alpha = 0.05, \alpha = 0.02, \alpha = 0.01$ for the two-sided test.

For one-sided test, if the alternative is $H_1: \mu > \mu_0$, reject $H_0: \mu = \mu_0$ if $W^- \le W_\alpha^*$ and if the alternative is $H_1: \mu < \mu_0$, reject $H_0: \mu = \mu_0$ if $W \le W_\alpha^*$. The significance levels for one-sided test provided in Appendix Table VIII [4] are $\alpha = 0{,}05, 0.025, 0.01$, and 0.005. We will apply Wilcoxon signed-rank test to the selected hormones data as in the following examples assuming that the underling distribution is a continuous and symmetric:

Numerical Examples:

EXAMPLE (1): The selected data for this example from T.S.H hormone levels in blood with n=25

Table (1): T.S.H Data (normal 0.25–5 Uv/mL)

Observation (i)	Random TSH (x_i)	Difference($x_i - \mu$)
1	9.1	+7.57
2	0.15	-1.38
3	0.07	-1.46
4	0.23	-1.3
5	1.1	-0.43
6	1.43	-0.1
7	1.57	+0.04
8	0.28	-1.25
9	1.99	-0.46
10	1.26	-0.27
11	26.06	+24.53
12	1.63	+0.1
13	1.53	0
14	1.61	+0.08
15	1.79	+0.26
16	1.4	-0.13
17	3.01	+1.48
18	3.28	+1.75
19	0.04	-1.49
20	0.94	-0.59
21	33.71	+32.18
22	2.05	+0.52
23	42.68	+41.15
24	0.77	-0.76
25	0.74	+0.79

The eight- step procedure is applied as follows:

1-The parameter of interest is the mean (or median) of Serum T.S.H

2-The null hypothesis $H_0 : \mu = 1.53$

3- The alternative hypothesis is $H_1 : \mu \neq 1.53$ (two sided trail)

4- The actual level of significance $\alpha = 0.05$

5- Reject H_0 if $W \le W_\alpha^* = 89$ from Appendix Table VIII [4].

6- Computations: The signed ranks from Table (1) are shown in the following table:

Table (2): T.S.H Signed Rank Data

Observation (i)	Random TSH (x_i)	Difference($x_i - \mu$)	Signed-Rank
13	1.53	0	+1
7	1.57	0.04	+2
14	1.61	0.08	+3
6	1.43	-0.1	-4
12	1.63	0.1	+5
16	1.4	-0.13	-6
15	1.79	0.26	+7
10	1.26	-0.27	-8
5	1.1	-0.43	-9
9	1.99	0.46	+10
22	2.05	0.52	+11
20	0.94	-0.59	-12
24	0.77	-0.76	-13
25	0.74	-0.79	-14
4	0.23	-1.3	-15
8	0.28	-1.25	-16
2	0.15	-1.38	-17
3	0.07	-1.46	-18
17	3.01	1.48	+19
19	0.04	-1.49	-20
18	3.28	1.75	+21
1	9.1	7.57	+22
11	26.06	24.53	+23
21	33.71	32.18	+24
23	42.68	41.15	+25

7-The test statistic is W=min(W^{+}, W^{-}). The sum of the positive ranks is and the sum of the absolute values of the negative ranks is Therefore, the test statistic W= min (162,163) = 162

8. Since w=162 is not less than or equal to the critical value $W_{0.05} = 89$, we cannot reject the null hypothesis that the mean(or median, since the population is assumed to be symmetric) so the population mean is 1.53

EXAMPLE (2): The selected data for the this example is T_3 hormone levels in blood with n=25

Table (3): T_3 Data (normal 0.92-2.33 nmoL/L)

Observation (i)	Random T3 (x_i)	Difference($x_i - \mu$)
1	1.3	0.25
2	1.55	0
3	1.65	0.1
4	2.42	0.87
5	1.87	0.32
6	1	-0.55
7	1.9	0.35
8	1.98	0.43
9	1.45	-0.1
10	0.69	0.87
11	0.59	-0.96
12	1.22	-0.33
13	1.37	-0.18
14	1.46	-0.09
15	1.35	-0.2
16	1.16	-0.39
17	2.42	0.87
18	1.43	-0.12
19	2.7	1.15
20	0.59	-0.96
21	2.4	0.85
22	1.88	0.33
23	1.84	0.29
24	2.82	1.27
25	4.08	2.53

The eight- step procedure is applied as follows:
1-The parameter of interest is the mean (or median) of Serum T_3
2- The null hypothesis is $H_0 : \mu = 1.55$
3- The alternative hypothesis is $H_1 : \mu \neq 1.55$ (two sided trail)
4- The actual level of significance $\alpha = 0.05$
5- Reject H_0 if $W \leq W_{\alpha}^{*}$=89 from Appendix Table VIII [4].
6- Computations: The signed ranks from Table (3) are shown in the following table:

Table (4): T_3 Signed Rank Data

Observation (i)	Random T3(x_i)	Difference($x_i - \mu$)	Signed Rank
2	1.55	0	+1
14	1.46	-0.09	-2
3	1.65	0.1	+3
9	1.45	-0.1	-4
18	1.43	-0.12	-5
13	1.37	-0.18	-6
15	1.35	-0.2	-7
1	1.3	0.25	+8
23	1.84	0.29	+9
5	1.87	0.32	+10
12	1.22	-0.33	-11
22	1.88	0.33	+12
7	1.9	0.35	+13
16	1.16	-0.39	-14
8	1.98	0.43	+15
6	1	-0.55	-16
21	2.4	0.85	+17
4	2.42	0.87	+18
10	0.69	0.87	+19
17	2.42	0.87	+20
11	0.59	-0.96	-21.5
20	0.59	-0.96	-21.5
19	2.7	1.15	+22
24	2.82	1.27	+23
25	4.08	2.53	+24

7- The test statistic is W=min(W^{+},W^{-}).The sum of the positive ranks is $W^{+}=(1+3+8+9+10+12+13+15+17+18+19+20+22+23+24)=214$

And the sum of the absolute values of the negative ranks is

Therefore, W=min (214,108) =108

8. Since w=108 is not less than or equal to the critical value $W_{0.05}=89$, we cannot reject the null hypothesis that the mean (or median, since the population is assumed to be symmetric) the mean is 1.55

EXAMPLE (3): The selected data for the this example is T_4 hormone levels in blood with n=25

Table (5): T_4 Data (normal: 60–120 nmoL /L)

Observation (i)	Random T4 (x_i)	Difference($x_i-\mu$)
1	61.9	-17.39
2	88.37	9.08
3	128.66	49.37
4	76.44	-2.85
5	96.9	17.61
6	82.36	3.07
7	97.28	17.99
8	77.32	-1.97
9	66.49	-12.8
10	115.66	36.37
11	85.22	5.93
12	72.84	-6.45
13	75.42	-3.87
14	100.22	20.93
15	74.11	-5.18
16	68.02	-11.27
17	79.29	0
18	98.08	18.79
19	75.8	-3.49
20	67.75	-11.54
21	82.45	3.16
22	102.7	23.41
23	86.88	7.59
24	68.28	-11.01
25	60.6	-18.69

The eight- step procedure is applied as follows:
1-The parameter of interest is the mean (or median) of Serum T_4
2- The null hypothesis is $H_0 : \mu = 79.29$
3- The alternative hypothesis is $H_1 : \mu \neq 79.29$ (two sided trail)
4- The actual level of significance $\alpha = 0.05$
5-We will reject H_0 if $W \leq W_\alpha^*$=89 from Appendix Table VIII [4].
6-Computeations: The signed ranks from Table (5) are shown in the following table:

Table (6): T_4 Signed Rank Data

Observation (i)	Random T4 (x_i)	Difference($x_i - \mu$)	SIGNED RANK
17	79.29	0	+1
8	77.32	-1.97	-2
4	76.44	-2.85	-3
6	82.36	3.07	+4
21	82.45	3.16	+5
19	75.8	-3.49	-6
13	75.42	-3.87	-7
15	74.11	-5.18	-8
11	85.22	5.93	+9
12	72.84	-6.45	-10
23	86.88	7.59	+11
2	88.37	9.08	+12
24	68.28	-11.01	-13
16	68.02	-11.27	-14
20	67.75	-11.54	-15
9	66.49	-12.8	-16
1	61.9	-17.39	-17
5	96.9	17.61	+18
7	97.28	17.99	+19
25	60.6	-18.69	-20
18	98.08	18.79	+21
14	100.22	20.93	+22

22	102.7	23.41	+23
10	115.66	36.37	+24
3	128.66	49.37	+25

7- The test statistic is W=min(W^+, W^-). The sum of the positive ranks is

$$W^+ = (1+4+5+9+11+12+18+19+21+22+23+24+25) = 194$$

And the sum of the absolute values of the negative ranks is

$$W^- = (2+3+6+7+8+10+13+14+15+16+17+20) = 131$$

Therefore, W=min (194, 131) =131

8. Since w=131 is not less than or equal to the critical value $W_{0.05} = 89$, we cannot reject the null hypothesis that the mean (or median, since the population is assumed to be symmetric) so the mean is 79.29

Conclusions:

[1] Since the signed-ranks in each example is not less than or equal to the critical value $W_{0.05} = 89$ we cannot reject the null hypothesis that the mean (or median, since the population is assumed to be symmetric) this mean that the chosen population mean is accepted in each example.

[2] In all examples there is no directional difference suggested for the value of μ; that μ might be either larger or smaller than the median if H_1 is true. This type of test is called two-tailed test of hypothesis.

[3] There is no rule to determine whether the sum of positive or negative ranks is greater than the other for example in T.S.H example w^- is greater than w^+ on the other hand in T_3, T_4 examples w^+ is greater than the w^-.

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اختبار ويلكسن لهرمونات الثايروكسين الموجودة في مصل الدم

الخلاصة:

في هذا البحث قمنا بتطبيق طريقة ويلكسن لأختبار الوسط الحسابي على عينات عشوائية غير خاضعة لتوزيع معين لثلاث انواع من هرمونات الثايروكسين الموجودة في مصل الدم وهي T.S.H و T_3 و T_4 بفضاء عينة n=25 وقمنا بجدولة هذه البيانات وبعد تطبيق الأختبار وجدنا ان الوسط الحسابي المفروض لكل مجتمع مقبول.