Connected ideal space and continuous function types

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Abstract

In this search we will study
1) The continuous image of *-connected ideal space.
2) The S-connected ideal space.
3) The S*-connected ideal space.
4) The S**-connected ideal space.

And we get this result: the (continuous, the S*-continuous) image of *-connected ideal space is connected space.

But the (S-continuous, S**- continuous) image of *- connected ideal space is not necessary be connected space.

1. Introduction:

The notion of S- open set was introduced in 1963 [5]. The concepts (S-continuous function, S*- continuous function, S**-continuous) was introduced by Mustafa in 2001 [4]. *- Connected ideal space was introduced by E kici, E., noiri, T. [1].

In section 2, the connected space, *-connected ideal space were introduced, also study the relation between these types, and study the relation between continuous function, S- continuous function, S*- continuous function S**-continuous function.

And we will recall some important theorems and remarks will be used in section 3. in section 3, the main results will be studied.

2. Preliminaries:

In this section, the connected space and connected ideal space was defined and study the relation between connected space and connected ideal space and defined continuous, S-continuous, S*-continuous and S**-continuous functions and study the relation between them.

Definition (2-1) [5]

A sub set A of a topological space x is (s-open) if $A \subseteq$ int (A).

Definition (2-2) [6]

A space X is (connected space) iff X can not is decomposed in to the union of two disjoint non empty open sets, other wise X is called disconnected.

Definition (2-3) [1]

An ideal topological space (X,T,I) is a topological space (X,T) and an ideal I on (X,T) and will be denoted by I T S ideal topological space.

Definition (2-4) [1]

An ideal space (X,T,I) is called *-connected ideal space if X can not be written as the disjoint union of non empty open set and non empty *-open set.

Remark (2-5) [1]

The following implications hold for an ideal space (X,T,I) these implications are most reversible.

Diagram (1) | (X, T, I) is *- connected | ⇒ | (X, T) is connected

Definition (2-6) [6]

Let X, Y be two I.T.S. The functions f: x→y is continuous iff the inverse image of open set in y is open set in X.

Definition (2-7) [4]

Let X, Y be two I.T.S. The function f: x→y is S-continuous iff the inverse image of each open set in y is s-open set in X

Definition (2-8) [4]

Let X, Y be two I.T.S. The function f: x →y is S*- continuous iff the inverse image of each S- open set in y is open set in X.

Definition (2-9) [4]

Let X, Y be two I.T.S. The function f: x → y is S**- continuous iff the inverse image of each S-open set in y is S-open set in X.

Remark (2-10) [3]

Let X, Y be two I.T.S. Diagram (2) show the relation between continuous functions types where the converse is not necessary true

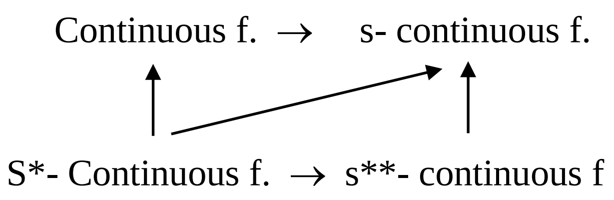


Diagram (2)

Preposition (2-11) [6]

The continuous image of a connected space is connected.

Preposition (2-12) [3]

The S-continuous image of a connected space is not necessary connected.

Preposition (2-13) [3]

The S*-continuous image of a connected space is connected.

Preposition (2-14) [3]

The S**-continuous image of connected space is not necessary connected.

Preposition (2-15) [3]

The continuous image of connected space is not necessary S-connected.

Preposition (2-16) [3]

The continuous image of connected space is not necessary S *-connected.

Preposition (2-17) [3]

The S*-continuous image of a connected space is S- connected space.

Preposition (2-18) [3]

The S**-continuous image of a connected space is not necessary S-connected space.

Remark (2-19) [3]

1- Open set $\Rightarrow$ S-open set
2- S-connected $\Rightarrow$ connected space

3. Results and Discussion

In this section, the {continuous, S-continuous S*-continuous, S**-continuous} image of *-connected ideal space will be study.

Theorem (3-1)

The surjective continuous image of *-connected ideal space is connected.

Proof

Let f: x $\to$ y be continuous function, x is *-connected ideal space.
by remark (2-5) $\to$ x is connected space $\to$ by preposition (2-11) and since f is surjective
$\to$ y = f(x) is connected space.

Theorem (3-2)

The surjective continuous image of *-connected ideal space is not necessary S-connected.

Proof

Let f: x$\to$y be continuous function, let x be *-connected ideal space.
$\to$ by Remark (2-5) x is connected space.
$\to$ by preposition (2-11) and since f is surjective.
$\to$ y = f(x) is not necessary S-connected space
Since by remark (2-19) (3).

Theorem (3-3)

The surjective S-continuous image of *-connected ideal space is not necessary connected.

Proof

Let f: x$\to$ y be continuous function
let x be *-connected ideal space
$\to$ by remark (2-5) x is connected space

$\rightarrow$ by preposition (2-12) and f is surjective $y = f(x)$ is not necessary connected space.

Theorem (3-4)

The surjective S-continuous image of *-connected ideal space is not necessary S-connected.

Proof

Let f: x→y be S-continuous function, let x be *-connected ideal space.

→ by remark (2-5) x is connect space.

→ by preposition (2-13) and since f is surjective → y=f(x) is not necessary S-connected space.

Theorem (3-5)

The surjective S*-continuous image of *-connected ideal space is connected.

Proof

Let f: x →y be S*-continuous function, x be *-connected ideal space,

→ by remark (2-5) x is connected space

→ by and since f is surjective preposition (2-14)

f(x) = y will be connected space.

Theorem (3-6)

The surjective S*-continuous image of *-connected ideal space is S-connected space.

Proof

Let f: x→y be S*-continuous function, x be *-connected ideal space.

→ by remark (2-5) x is connected space.

→ by preposition (2-15) and since f is surjective

→ f(x) =y will be S-connected space.

Theorem (3-7)

The surjective S**-continuous image of *-connected ideal space is not necessary connected.

Proof

Let f: x → y be S**-continuous function, x be *-connected ideal space

→ by remark (2-5) x is connected space.

→ by preposition (2-16) and since f is surjective y =f(x) is not necessary connected.

Theorem (3-8)

The surjective S**-continuous image of *-connected ideal space is not necessary S-connected.

Proof

Let f: x→y be S**-continuous function, let x be *-connected ideal space.

→ by remark (2-5) x is connect space.

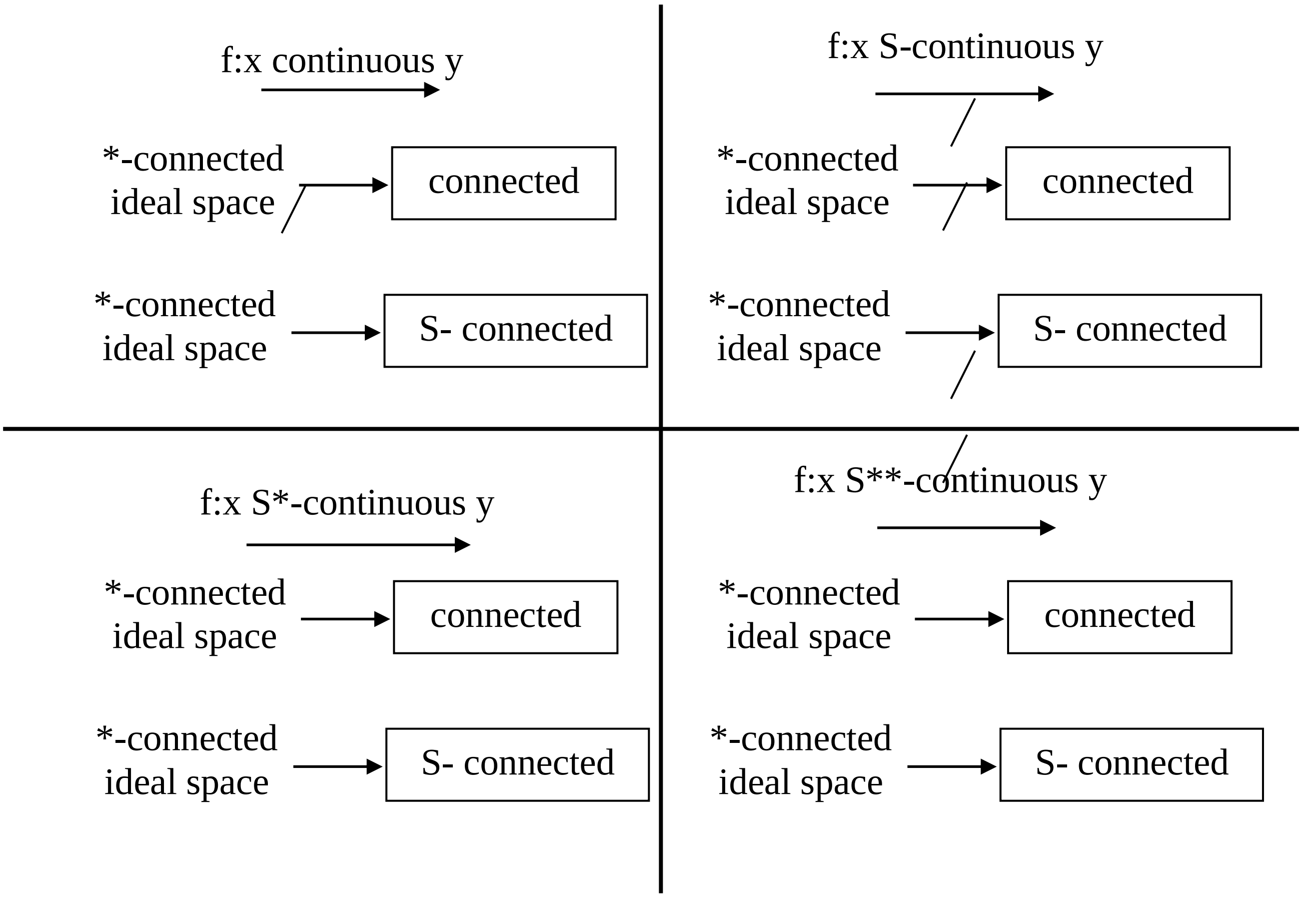
→ by preposition (2-17) and since f is surjective, f(x)=y

y is not necessary S-connected space

Conclusions

Remark:

$\nrightarrow$ Means not necessary.



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أنواع الدوال المستمرة للفضاء المثالي المتصل

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<u>الخلاصة</u>

في هذا البحث سوف ندرس:

1) الصورة المستمرة للفضاء المثالي المتصل-*.

2) الصورة المستمرة للفضاء المثالي المتصل-S.

3) الصورة المستمرة للفضاء المثالي المتصل-*S.

4) الصورة المستمرة للفضاء المثالي المتصل-**S.

وقد حصلنا على النتيجة التالية:

إن الصورة (المستمرة، المستمرة-*S) للفضاء المثالي المتصل-* هي فضاء متصل.

ولكن الصورة (المستمرة-S، المستمرة-**S) للفضاء المتصل المثالي-* ليستا بالضرورة أن يكون فضاءً متصل.