Best approximation in metric space for contractive mapping of integral type and Applications of fixed point to Approximation theory

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Abstract :

We obtain generalization of best approximation for contractive condition of integral type and we next consider two results on invariant approximation also analyze the existence the application of fixed point and extend some known results of Habinik [7] ,Sahab [10], Kumar [5] Branciari [1], Singh [12].

1. Introduction and preliminaries

The purpose of this paper is to study and analogous the invariant best approximation in the setting contractive condition of integral inequality in metric space and analyze the existence the application of fixed point. The generalization of applications on best approximation obtaining fixed point also common fixed point. Recently Sahab [10] have obtained some results on approximation theory in the setting contraction mapping without integral type. We prove the results on best approximation theory for mappings satisfying a general contractive condition of integral type. These results unify and extend some results in Habiniak [7], Sahab [10] , Kumar [5] Branciari [1].

The following definition and results will be needed :

Let C be anonempty subset of a metric space (X, d). A mapping T: C→ C is acompact mapping [4] if for every bounded subset K of C, $\overline{T(K)}$ (closure of $T(K)$) is compact. The restriction $T|_C : C \to X$ where $T : X \to X$. A mapping $T : C \to C$ is called contraction mapping [3] if $d(Tx, Ty) \leq \lambda d(x, y)$ for all $x, y \in C$, $\lambda \in [0, 1)$. If $\lambda = 1$ then T is called non expansive mapping. Let $T, f : C \to C$ be a mapping T is called f contraction [6] , [11] if $d(Tx, Ty) \leq \lambda d(fx, fy)$ for all $x, y \in C$.

$\lambda \in [0,1)$ if $\lambda = 1$, then T is called f-nonexpansive mapping. If $T(C) \subseteq C$ then C is called a T– invariant subset of X .

Definition 1.1 : [8] , [4]

Let C be a subset of a metric space (X, d), for any $x \in X$ we denote

i - $d(x, C) = \inf\{d(x,y); y \in C\}$

ii- If any $y \in C; d(x,y) = d(x,C) = \inf\{d(x,z); z \in C\}$ where $x \in X$ is called y approximation to x by C .

iii- $P_c(x) = \{y \in C : d(x,y) = d(x,C)\}$ such that $P_c(x)$ is called the set of all best approximation of x from C .

iiii- If for each $x \in X$, $P_c(x)$ is nonempty, then C is called proximinal. $P_c(x)$ is bounded subset of X also if C is closed then $P_c(x)$ is closed .

Definition 1.2 : [8]

Let f and T be two self maps on a set X. mapps f and T are said to be commuting if $fTx = Tfx$ for all $x \in X$.

If $fx = Tx$ for some $x \in X$ then x is called concidence point of f and T.

Definition 1.3 : [13]

Let (X, d) be a metric space and $C \subseteq X$ and $f,T: C \to C$, be a mappings , then :

i- T and f are called compatible if $fx_n, Tx_n \in C$ for all n and $\lim_{n\to\infty} d(Tfx_n, fTx_n) = 0$, whenever (x_n) is a sequence such that: $\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} fx_n = t$ for some t in C.

ii- f and T are called weakly compatible if they commute at there coincidence points (i. e.) $Tfx = fTx$ whenever $Tx = fx$.

Remark 1.4 : [2]

i- Every compatible is weakly compatible but the converse is not true.

ii- Every commute mappings is compatible mapping but the converse is not true.

Definition 1.5 : [9]

Let C be a subset of metric space (X,d) and $\Delta = \{f_\alpha\}_{\alpha \in C}$ a family of functions from [0,1] into C such that $f_\alpha(1) = \alpha$ for each $\alpha \in C$. The family Δ is said to be contractive if whenever there exists a function $\Psi : (0,1) \to (0,1)$ such that for all $\alpha, \beta \in C$ and $s \in (0,1)$ we have

$d(f_\alpha(s), f_\beta(s)) \leq \Psi(s) d(\alpha, \beta)$.

The family is said to be jointly continuous if $s \to s_0$ in $[0,1]$ and $\alpha \to \alpha_0$ in C imply that $f_\alpha(s) \to f_{\alpha_0}(s)$ in C.

Hence property (w) on contractive jointly continuous family Δ can now be defined as :

Definition 1.6 : [9]

Let T be a self map of the set C having a family of function $\Delta = \{f_x\}, x \in C$ as defined above then T is said to satisfy the property (w) if $T(f_x(s)) = f_{Tx}(s)$, for all $x \in C$ and $s \in [0,1]$.

Definitiozn 1.7 : [1]

Let (X,d) be a metric space, $T: X \to X$ is called general contractive inequality of integral type if there exist a real number $\lambda \in (0,1)$ such that

for each $x, y \in X$, we have $\int_{0}^{d(Tx,Ty)} \varphi(t)dt \leq \lambda \int_{0}^{d(x,y)} \varphi(t)dt$

where $\varphi:[0,+\infty) \to [0,+\infty)$ is a Lebesgue integrable mapping which is summble on each compact subset of $[0,+\infty)$ nonnegative and such that for each $\varepsilon > 0$, $\int_{0}^{\varepsilon} \varphi(t)dt > 0$.

The generalization of definition 1.5 by

$$\int_{0}^{d(f_x(s),f_y(s))} \varphi(t)dt \leq \Psi(s) \int_{0}^{d(x,y)} \varphi(t)dt.$$

The following result would also be used in the sequel :

Theorem 1.8 : [5]

Let T and f be compatible self maps of a complete metric space

(X,d) satisfying the following conditions: $T(X) \subseteq f(X)$, f is continuous

$\int_{0}^{d(Tx,Ty)} \varphi(t)dt \leq \lambda \int_{0}^{d(fx,fy)} \varphi(t)dt$ for each $x,y \in X$, $\lambda \in [0,1)$, where $\varphi: R^+ \to R^+$ is a Lebesgue – integrable function, which is summable on each compact subset of R^+, non-negtive , and such that for each

$\varepsilon > 0, \int_{0}^{\varepsilon} \varphi(t)dt > 0$. Then T and f have a unique common fixed point.

2- Main Results

In this section we introduce two results best approximation and one result of fixed point we need the result fixed point the following.

Theorem 2.1

Let T and f be weakly compatible self maps of a subset X of metric space (X, d) satisfying the following condition : $T(X) \subset f(X)$,

$$\int_0^{d(Tx,Ty)} \phi(t)dt \leq \lambda \int_0^{d(fx,fy)} \phi(t)dt$$

$T(X)$ or $f(X)$ is complete, , for each $x,y \in X$, $\lambda \in [0,1)$, where $\phi:[0,+\infty) \to [0,+\infty)$ is a Lebesgue-integrable function which is summable on each compact subset $[0,+\infty)$ non-negative,

and such that for each $\varepsilon > 0$, $\int_0^\varepsilon \phi(t)dt > 0$

Then T and f have a unique common fixed point

Proof:

Since $T(X) \subset f(X)$, let $x_0 \in X$ such that $fx_1 = Tx_0$, $x_1 \in X$

In general $y_n = fx_{n+1} = Tx_n$, $n = 0,1,2,3,...$

Now

$$\int_0^{d(Tx_n,Tx_{n+1})} \phi(t)dt \leq \lambda \int_0^{d(fx_n,fx_{n+1})} \phi(t)dt = \lambda \int_0^{d(Tx_{n-1},Tx_n)} \phi(t)dt$$

$$\leq \lambda^2 \int_0^{d(fx_{n-1},fx_n)} \phi(t)dt$$

$$= \lambda^2 \int_0^{d(Tx_{n-2},Tx_{n-1})} \phi(t)dt$$

$$\vdots$$

$$= \lambda^n \int_0^{d(Tx_0,Tx_1)} \phi(t)dt \to 0$$

Therefore $\int_0^{d(Tx_n,Tx_{n+1})} \phi(t)dt \to 0$ as $n \to \infty$ hence $d(Tx_n,Tx_{n+1}) \to 0$

We now to show that (Tx_n) is a Cauchy sequence suppose that (Tx_n) is not Cauchy sequence then there exist an $\varepsilon > 0$ and a sub sequence $(m(v)_p)$ and $(n(v)_p)$ such that for each positive integer p ; $n(v)_p$ is minimal in the sense that

$$d\left(Tx_{m(v)_p}, Tx_{n(v)_p}\right) \geq \varepsilon \ , \quad d\left(Tx_{m(v)_p}, Tx_{n(v)_{p-1}}\right) < \varepsilon$$

If : $d\left(Tx_{m(v)_{p-1}}, Tx_{n(v)_{p-1}}\right) > \varepsilon$ then:

$$d\left(Tx_{m(v)_{p-1}}, Tx_{n(v)_{p-1}}\right) \leq d\left(Tx_{n(v)_{p-1}}, Tx_{m(v)_p}\right) + d\left(Tx_{m(v)_p}, Tx_{n(v)_{p-1}}\right)$$

Thus : $d\left(Tx_{n(v)_{p-1}}, Tx_{n(v)_{p-1}}\right) < d\left(Tx_{n(v)_{p-1}}, Tx_{m(v)_p}\right) + \varepsilon$

Since : $d(Tx_{n+1}, Tx_n) \to 0$ then $d\left(Tx_{n(v)_{p-1}}, Tx_{n(v)_{p-1}}\right) \to \varepsilon$ as $p \to \infty$ also

$$\int_0^\varepsilon \phi(t)dt \leq \int_0^{d(Tx_{m(v)_p},Tx_{n(v)_p})} \phi(t)dt \leq \lambda \int_0^{d(fx_{m(v)_p},fx_{n(v)_p})} \phi(t)dt$$

=

$$\lambda \int_0^{d\left(Tx_m\cdot y_{p-1},\ Tx_n\cdot y_{p-1}\right)} \varnothing(t)dt$$

When $p \to \infty$ then

$$\int_0^\varepsilon \varnothing(t)dt \leq \lim \int_0^{d\left(Tx_m\cdot y_{p-1},\ Tx_n\cdot y_{p-1}\right)} \varnothing(t)dt = \lambda \int_0^\varepsilon \varnothing(t)dt < \int_0^\varepsilon \varnothing(t)dt$$

Which is contradiction thus : $d\left(Tx_m\cdot y_{p-1},\ Tx_n\cdot y_{p-1}\right) \leq \varepsilon$, also

$$\int_0^\varepsilon \varnothing(t)dt \leq \int_0^{d\left(Tx_m\cdot y_p,\ Tx_n\cdot y_p\right)} \varnothing(t)dt \leq \lambda \int_0^{d\left(fx_m\cdot y_p,\ fx_n\cdot y_p\right)} \varnothing(t)dt$$

$$= \lambda \int_0^{d\left(Tx_m\cdot y_{p-1},\ Tx_n\cdot y_{p-1}\right)} \varnothing(t)dt$$

$$\leq \lambda \int_0^\varepsilon \varnothing(t)dt < \int_0^\varepsilon \varnothing(t)dt$$

Which is contradiction thus (Tx_n) is a Cauchy sequence. By definition of (fx_{n+1}) is also Cauchy sequence. Since $f(X)$ or $\overline{T(X)}$ is complete suppose that $\overline{T(X)}$ is complete then $Tx_n \to u \in \overline{T(X)}$ and by definition of (fx_{n+1}) obtain $fx_{n+1} \to u$. Since $u \in \overline{T(X)} \subset f(X)$, then $v \in X$ such that $u = Tv$

Also $u = fz$, $z \in X$

Now : if $n \to \infty$

$$\int_0^{d(Tz,Tx_{n+1})} \varnothing(t)dt \leq \lambda \int_0^{d(fz,fx_{n+1})} \varnothing(t)dt = \lambda \int_0^{d(u,fx_{n+1})} \varnothing(t)dt \to 0$$

Thus $Tx_n \to Tz = u = fz$

As (T,f) is weakly compatible and $Tz = fz$ then

$Tu = Tfz = fTz = fu$, therefore $Tu = fu$

$$\int_0^{d(u,Tu)} \varnothing(t)dt = \int_0^{d(Tz,Tu)} \varnothing(t)dt \leq \lambda \int_0^{d(fz,fu)} \varnothing(t)dt = \lambda \int_0^{d(u,Tu)} \varnothing(t)dt < \int_0^{d(u,Tu)} \varnothing(t)dt$$

Then $\int_0^{d(u,Tu)} \varnothing(t)dt < \int_0^{d(u,Tu)} \varnothing(t)dt$ Hence $Tu = u$

Thus $Tu = u = fu$

If $s \in C$ such that $s = Ts = fs$, then

$$\int_0^{d(u,s)} \varnothing(t)dt = \int_0^{d(Tu,Ts)} \varnothing(t)dt \leq \lambda \int_0^{d(fu,fs)} \varnothing(t)dt = \lambda \int_0^{d(u,s)} \varnothing(t)dt$$

Then $\int_0^{d(u,s)} \varnothing(t)dt \leq \lambda \int_0^{d(u,s)} \varnothing(t)dt < \int_0^{d(u,s)} \varnothing(t)dt$ Hence $s = u$

There for the unique common fixed point.

Theorem 2.2 :

Let (X,d) be a metric space and $T : X \to X$ with fixed point $u \in X$

satisfying $\int_0^{d(Tx,Ty)} \varnothing(t)dt \leq \lambda \int_0^{d(x,y)} \varnothing(t)dt$, for each $x,y \in X, \lambda \in [0,1)$

where $\varnothing:(0,+\infty) \to [0,+\infty)$ is a Lebesgue-integrable function which assumable on each compact subset of $(0,+\infty)$, nonnegative, and such that

for each $\varepsilon > 0 ; \int_0^\varepsilon \varnothing(t)dt > 0.$ If C closed T invariant subset of X, further the restriction $T|_C$ is a compact, then the set $P_C(u)$ of best a pproximation is nonempty.

Proof:

Let $\varepsilon = d(u,C)$ $,\varepsilon > 0$ then there exist a minimizing sequence (y_n) in C such that $\varprojlim_{n\to\infty} d(u,y_n) = \varepsilon$

Since (y_n) is bounded sequence. Since $T|_C$ is compact

Then $\overline{(Ty_n)}$ is a compact subset of C and so (Ty_n) has a convergent sub sequence $(Ty_{ni} : i \geq 1)$ with $\varprojlim_{n\to\infty} Ty_{ni} = x$ where x in C

Now :

If $d(u,C) = \varepsilon = \varprojlim_{n\to\infty} d(u,y_n),$ then $\int_0^{d(u,x)} \varnothing(t)dt = \varprojlim_{n\to\infty} \int_0^{d(u,y_n)} \varnothing(t)dt$

Such that $\varepsilon > 0$ then $\int_0^{d(u,x)} \varnothing(t)dt = r ,$ $r > 0$

Suppose that $d(u,x) \geq \varepsilon$, now: either $d(u,x) = \varepsilon$, then

$d(u,x) = d(u,C) = \varepsilon ; \int_0^{d(u,x)} \varnothing(t)dt = r$

or $d(u,x) > \varepsilon$ then :

$r < \int_0^{d(u,x)} \varnothing(t)dt = \varprojlim_{n\to\infty} \int_0^{d(Tu,Ty_{ni})} \varnothing(t)dt \leq \varprojlim_{n\to\infty} \lambda \int_0^{d(u,y_{ni})} \varnothing(t)dt$

$= \varprojlim_{n\to\infty} \lambda \int_0^{d(u,y_{ni})} \varnothing(t)dt$

$< \varprojlim_{n\to\infty} \int_0^{d(u,y_n)} \varnothing(t)dt = r$

Therefore $r < \int_0^{d(u,x)} \varnothing(t)dt < r$ hence $d(u,x) = \varepsilon$

Thus $d(u,x) = \varepsilon = d(u,C)$ then x is best approximation to u by C

Therefore $P_C(u)$ is nonempty.

Theorem 2.3 :

Let (X,d) be a metric space, X has family Λ and $T,g : X \to X$ be two mappings and $C \subseteq X$ such that C be a T-invariant subset of X ;

$x_0 \in F(T) \cap F(g).$ Let g satisfies property (w) , $Tg = gT$ on $D = P_C(x_0).$ If D is nonempty and $g(D) \subseteq D$ and T,g satisfying

$$\int_0^{d(Tx,Ty)} \emptyset(t)dt \leq \lambda \int_0^{d(gx,gy)} \emptyset(t)dt, \lambda \in [0,1)$$

for each $x,y \in D' = D \cup \{x_0\}$ Where $\emptyset(t) : [0,+\infty) \to [0,+\infty)$ is a Lebsgue-integrable function which is summable, nonnegative and such that for each $\epsilon > 0, \int_0^\epsilon \emptyset(t)dt > 0$ then

i- If $\overline{T(D)}$ is compact and T is continuous and $g(D) = D$, D is closed then $D \cap F(T) \cap F(g) \neq \emptyset$

ii- If D is compact and g is continuous also the range of t_0 is contained in g(D) then $D \cap F(T) \cap F(g) \neq \emptyset$

iii- If $\overline{T(D)}$ is compact on D and g is continuous $g(D) = D$, D is closed then $D \cap F(T) \cap F(g) \neq \emptyset$

Proof :

Let $y \in D$ then $gy \in D$ since $g(D) \subseteq D$, Further $y \in C$ and then $Ty \in C$ since C be T-invariant subset of X

let $d(x_0, gy) = d(x_0, C) = \epsilon > 0 ; \int_0^\epsilon \emptyset(t)dt = r > 0$

suppose that $d(x_0, Ty) \geq \epsilon$

Now : either $d(x_0, Ty) = \epsilon$ then $Ty \in D$

or $d(x_0, Ty) > \epsilon$, then

$$r < \int_0^{d(x_0,Ty)} \emptyset(t)dt = \int_0^{d(Tx_0,Ty)} \emptyset(t)dt \leq \lambda \int_0^{d(gx_0,gy)} \emptyset(t)dt = \lambda \int_0^{d(x_0,gy)} \emptyset(t)dt$$
$$< \int_0^{d(x_0,gy)} \emptyset(t)dt$$
$$= \int_0^\epsilon \emptyset(t)dt = r$$

Therefore $r < \int_0^{d(x_0,Ty)} \emptyset(t)dt < r$ hence $d(x_0,Ty) = \epsilon$

Hence $Ty \in D$, then $T : D \to D$. Choose $\lambda_n \in (0,1)$

Such that $\lambda_n \to 1$, as $n \to \infty$, then define T_n as $T_n(x) = t_{Ty}(\lambda_n)$

For all $x \in D$ T_n is well defined map from D into D for each n and $T_n(D) \subseteq D$.

Now T commutes with g and g satisfying property(w) then

$T_n(gx) = t_{T(gx)}(\lambda_n) = g(t_{Tx}(\lambda_n)) = gT_n(x)$

Thus $T_n g = gT_n$ for all $n \in N$ and for all $x \in D$, therefore

$$\int_0^{d(T_n(x),T_n(y))} \emptyset(t)dt = \int_0^{d(t_{Tx}(\lambda_n),t_{Ty}(\lambda_n))} \emptyset(t)dt \leq \Psi(\lambda_n) \int_0^{d(Tx,Ty)} \emptyset(t)dt$$

$$\leq \lambda \Psi(s) \int_0^{d(gx,gy)} \varnothing(t)dt$$

For all $x, y \in D$

Thus $\int_0^{d(T_n x, T_n y)} \varnothing(t)dt \leq \lambda \Psi(s) \int_0^{d(gx,gy)} \varnothing(t)dt$ for all $x, y \in D$

Since T_n commutes with g then T_n and g are weakly compatible. If (i) $\overline{T(D)}$ is compact on D , then $\overline{T_n(D)}$ is compact also $\overline{T_n(D)}$ is complete and $T_n(D) \subseteq D = g(D)$ also D is closed $\overline{T_n(D)} \subseteq D = g(D)$ thus $\overline{T_n(D)} \subseteq g(D)$ then by theorem (2.1) T_n and g have common fixed point, $x_n = T_n x_n = g x_n$ for all $n \in N$ and $x_n \in D$.

Since $\overline{T(D)}$ is compact then the sequence (Tx_n) has a subsequence (Tx_{ni}) converging to $y \in \overline{T(D)} \subseteq D = g(D)$
then $y = gv$ where $v \in D$ and $x_{ni} = T_{ni}x_{ni} = g x_{ni}$
Now : $x_{ni} = g x_{ni} = T_{ni}x_{ni} = t_{Tx_{ni}}(x_{ni}) \to t_y(1) = y$ as $n_i \to \infty$
Thus $x_{ni} \to y$ and $Tx_{ni} \to Ty$ by T is continuous then $Ty = y$
Also :

$$\int_0^{d(Tx_{ni},Ty)} \varnothing(t)dt \leq \lambda \int_0^{d(gx_{ni},gy)} \varnothing(t)dt$$

$$= \lambda \int_0^{d(gx_{ni},y)} \varnothing(t)dt \to 0 \ as \ n_i \to \infty$$

Hence $\int_0^{d(Tx_{ni},Ty)} \varnothing(t)dt \to 0 \ then \ Tx_{ni} \to Ty = Ty = y = gv$
Therefore $Tv = gv,$ then $Ty = Tgy = gTv = gv$
Hence $y = Ty = gy,$ then $D \cap F(T) \cap F(g) \neq 0$
If (ii) (D is compact and g is continuous and range of t_x is contained in $g(D)$ then $T_n(D) \subseteq g(D)$.
Since T_n commute with g then (T_n, g) are compatible then by theorem (1.8) then $x_n = T_n x_n = g x_n$
Where $x_n \in D$, but D is compact then (x_n) has a subsequence (x_{ni}) converging to $y \in D$
Now : $x_{ni} = T_{ni}x_{ni} = g x_{ni} \to y$ as $n_i \to \infty$
Since g is continuous and $x_{ni} \to y$ then $gx_{ni} \to gy$; but $gx_{ni} \to y$
then $gy = y$, also

$$\int_0^{d(Tx_{ni},Ty)} \varnothing(t)dt \leq \lambda \int_0^{d(gx_{ni},gy)} \varnothing(t)dt \to 0 \ as \ n_i \to \infty$$

thus :

$$\int_0^{d(Tx_{n_i}, Ty)} \varnothing(t)dt \to 0 \text{ then } Tx_{n_i} \to Ty \quad \text{as } n_i \to \infty$$

Now :

$x_{n_i} = T_{n_i} x_{n_i} = f_{Tx_{n_i}}(x_{n_i}) \to f_{Ty}(1) = Ty$

Therefore $x_{n_i} \to Ty$ and $x_{n_i} \to y$ then $Ty = y$ thus $Ty = gy = y \in D$

Hence $D \cap F(T) \cap F(g) \neq \varnothing$

iii- $\overline{T(D)}$ is compact and D is closed also $g(D) = D$, g is continuous, then by prove part (i) if $\overline{T(D)}$ is compact we have

$x_{n_i} = T_{n_i} x_{n_i} = g x_{n_i} \to y$ as $n_i \to \infty$

Since g is continuous then $g x_{n_i} \to gy$ also $g x_{n_i} \to y$ and $x_{n_i} \to y$

Hence $gy = y$, also $\int_0^{d(Tx_{n_i}, Ty)} \varnothing(t)dt \leq \lambda \int_0^{d(gx_{n_i}, gy)} \varnothing(t)dt \to 0$

Therefore

$$\int_0^{d(Tx_{n_i}, Ty)} \varnothing(t)dt \to 0 \text{ then } Tx_{n_i} \to Ty \quad \text{as } n_i \to \infty$$

Now

$x_{n_i} = T_{n_i} x_{n_i} = f_{Tx_{n_i}}(x_{n_i}) \to f_{Ty}(1) = Ty \quad \text{as } n_i \to \infty$

Thus $x_{n_i} \to Ty$ then $Ty = y = gy$ and $y \in g(D) = D$

Then $D \cap F(T) \cap F(g) \neq \varnothing$

Corollary 2.4:

Let (X, d) be a metric space , X having family Δ , let $T : X \to X$ is continuous , $C \subseteq X$ such that $T(C) \subseteq C$ and $x_0 \in F(T)$ if D nonempty, compact

and $\int_0^{d(Tx, Ty)} \varnothing(t)dt \leq \lambda \int_0^{d(x,y)} \varnothing(t)dt$ for all

$x, y \in D' = D \cup \{x_0\}$, where $\varnothing(t): R^+ \to R^+$ is Lebsgue–integrable function

which is summble, nonnegative and $\varepsilon > 0 ; \int_0^\varepsilon \varnothing(t)dt > 0$ then

$D \cap F(T) \neq \varnothing$

Proof :

If g is identity mapping in theorem (2.3) then $D \cap F(T) \neq \varnothing$

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التقريب الافضل في الفضاء المتري لتطبيق الانكماش من النوع التكاملي وتطبيقات النقطة الصامدة لنظرية التقريب

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جامعة بغداد

الخلاصة :

نحن نحصل على تعميم للتقارب الافضل لشرط الانكماش العام من النوع التكاملي في الفضاء المتري ونحن نضع نتيجتين على التقرب الافضل . ايضاًنعمل

على ايجاد تطبيقات النقطة الصامدة و نوسع بعض النتائج المعروفة للباحثين

هابنيك [7] وصاحب [10] وكمر [5] وبرنكير [1] وسنها [12] .