BORNOLOGICAL ON THE SPACE ALL ELEMINT REPRESENTED BY DIRICHLET SERIES

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Abstract

In this paper, we study and deal with the fundamental bornological constructions for bornology defines on the space of entire function all element represented by dirichlet series, such as bornological subspace, product bornologies and Quotient bornologies with adding some properties to them.

1. Introduction

The space of entire functions over the complex field C was introduced by [5] who defined a metric on this space by introducing a real-valued map on it. In (1971), H.Hogbe-Nlend introduced the concepts of bornology on a set. In(1981) M.D. Patwardhan extended this idea to a space of entire functions. However, we shall study the bornological constriction of the space which all element represented by dirishlet seres.

In this paper, we consider the space of entire function $\alpha(s)$ represented by Dirichlet series is $\alpha(s)=\sum_{n=1}^{\infty} a_n \exp(s\lambda_n)=a_1e^{s\lambda}+a_2e^{s\lambda_2}+...+a_ne^{s\lambda n}$ … (1)

Where $a_i \in c,\ i\geq 1,\ \lambda_{n+1}\succ\lambda_n,\ \lambda_1\geq 0,\ \lim_{n\to\infty}\lambda_n=\infty,\ s=\sigma+it(\sigma,t\ real\ variables).and\{a_n\}$ is any sequence of complex number set?

For an entire function α (s), define the number $|\alpha|$ by

$|\alpha|=l.u.b\ |\mathbf{a_n}|^{\frac{1}{\lambda_n}}, n\geq 1$ …(2)

It is easily verified that $|\alpha|$ satisfies the following conditions…(3)

(i) $|\alpha|\geq 0$ and $|\alpha|=0$ if and only if $\alpha=\theta$, the identically zero function;

(ii) $|\alpha+\beta|\leq|\alpha|+|\beta|$

(iii) $|\omega\alpha|\leq \mathbf{A(\omega)}|\alpha|$ Where A (ω) = max $(1,|\omega|),\omega$ being any complex number

It follows from (i) and(ii) of (3) that $|\alpha-\beta|$ defines a distance in the class of entire function.

Then $\Gamma=\left\{\alpha:\alpha=\sum_{n=0}^{\infty}a_n\exp(s\lambda_n),a_n\in c \text{ and } \lim_{n\to\infty}|a_n|^{\frac{1}{\lambda_n}}=0\right\}$, the vector space of all entire function all element represented by Dirichld series.

Lister we define a bornology of the space Γ

The aim of the present paper is study and deals with the fundamental Bornological constructions of Γ and with adding some properties to them, such as Bornological subspace, product bornologies and Quotient bornologies.

Keywords: Bornological spaces, Entire functions, Dirichld series

2. Bornological Spaces

In this section we recall several basic notions from the theory of Bornological linear spaces.

(2.1)Definition

A Bornology B on a set X is a family of subsets of X such that B is a covering of X; finite unions of elements of B are in B ; any subset of an element of B is also in B . The elements of B are called bounded sets.[1]

(2.2)Definition

A Bornological linear space is a linear space over the field **K** (the real or complex field) together with a Bornology on underlying set of vectors such that the sum of vectors and the product of elements of **K** by vectors are bounded operations, i.e. the sets **A+B** and **C .B** are bounded sets whenever **A** and **B** are bounded subsets of **X** and **C** is a bounded subset of **K** by [2]

(2.3)Definition

Let (X, B) be a Bornological space and let $Y\subseteq X$. Then the collection B_y = $\{B\cap Y:B\in B\}$is a Bornology on Y and he Bornological space(Y,B_Y) is called relative Bornology on Y .**[3]**

(2.4)Definition

Let $(X_i, B_i)_{i\in I}$ be a family of Bornological space indexed by a non-empty set I and let $X=\prod_{i\in I}X_i$ be the product of the sets X_i. For every $i\in I$, let $p_i:X\to X_i$ be the canonical projection then. The ***product Bornology*** on X is the initial Bornology on X for the maps p_i. The set X, endowed with the product bornology is called ***the Bornological product*** of the space (X_i, B_i).[**3]**

(2.5) Definition

Let (X, B) be a Bornological space and Let $\theta : X \rightarrow Y$ be a map of X onto a set Y. Then $B_\theta = \left\{ \theta(B) \subseteq Y : B \underset{\text{bouned}}{\subseteq} X \right\}$ is the quotient bornology of B under θ. Y is called a quotient space of X, θ is called a quotient map (Final Born logy). **[3]**

(2.6) Definition

A base of a Bornology B on X is any subfamily B_0 of B such that every element of B is contained in an element of B_0 .**[1]**

3.Bornological Constriction of the space Γ .

In this section study the fundamental Bornological constrictions of the space Γ which all element represent by Dirilets series such as Bornological subspace, product Bornologies and Quotient Bornologies.

3.1 Bornological Subspace of the Space Γ .

All Definition given in (2) can be applied for the special case where

X= Γ and $\Gamma = \left\{ \alpha : \alpha = \sum_{n=0}^{\infty} a_n \exp(s\lambda_n), a_n \in c \text{ and } \lim_{n\to\infty} |a_n|^{\frac{1}{\lambda_n}} = 0 \right\}$**,** thevector space of all entire function all element represented by Dirichld series

(3.1.1)Theorem

Let Γ be a vector space of all entire functions over the complex field C, let (Γ ,B) be a Bornological vector space and Y be a vector subspace of Γ . Then the collection B_y = $\{B \cap Y : B \in B\}$ is a vector bornology on Y.

Proof: (by definition of bornology) B_Y is a bornology on y

To prove B_Y is a vector bornology on Y, $\forall A, B \in B_Y$ and $\lambda \in C$

Let A=$V \cap Y$, $B = U \cap Y$ where {U, V $\in B$ }

$$\left\{ \begin{array}{l} \because A \subseteq V, \because V \in B, \quad \therefore A \in B \\ \because B \subseteq U, \ \because U \in B, \quad \therefore B \in B \end{array} \right\}$$

I.e. every element of B_Y is an element of B

Since $\mathcal{B}$ is a vector bornology on Γ, then $\left\{\begin{matrix} A+B \\ \lambda A \\ \bigcup_{|\lambda|<1} \lambda A \end{matrix}\right\} \in \mathcal{B}$

$\because$ Y be vector subspace of $\Gamma \therefore \left\{\begin{matrix} A+B \\ \lambda A \end{matrix}\right\} \subseteq Y$, then $\left\{\begin{matrix} A+B \\ \lambda A \end{matrix}\right\} \in \mathcal{B}_Y$

I.e. $\mathcal{B}_Y$ is stable under vector addition, homothetic transformation and circled hull.

Then (Y, $\mathcal{B}_Y$) is a Bornological vector subspace of (Γ, $\mathcal{B}$).

(3.1.2)Remark

It is clear that every bounded subset of Y is also a bounded subset of Γ, i.e. $\mathcal{B}_Y \subseteq \mathcal{B}$, and every subspace of discrete space is discrete space.

(3.1.3) Proposition

Let $W \subseteq Y \subseteq \Gamma$ Then if $\mathcal{B}_W$ is the vector bornology on W w.r.t (Γ, $\mathcal{B}$ **)** and $\mathcal{B}_W^{'}$ is the vector bornology on W w.r.t (Y, $\mathcal{B}_Y$). Then $\mathcal{B}_W = \mathcal{B}_W^{'}$

Proof: Let the bornology $\mathcal{B}$ (Γ) given to W from Γ**.** And from Y to be $\mathcal{B}$ (Y) (Since every bounded subset of Y is also a bounded subset of X), then it is clear that $\mathcal{B}$ (Y) $\subseteq \mathcal{B}$ (Γ)… (1)

Let $V \in \mathcal{B}$ (Γ), then $\exists v^{'} \subseteq \Gamma$ such that

$V = W \bigcap v^{'} = (W \bigcap Y) \bigcap V^{'} = W \bigcap (Y \bigcap V^{'}), V^{'} \underset{bounded}{\subseteq} \Gamma$, then $Y \bigcap V^{'} = V^{''} \underset{bounded}{\subseteq} Y$

Then $V = W \bigcap V^{''}, V^{''} \subseteq Y$ and $V \in \mathcal{B}$ (Y)

Then $\mathcal{B}$ (Γ) $\subseteq \mathcal{B}$ (Y)… (2)   From (1) and (2), then $\mathcal{B}_W = \mathcal{B}_W^{'}$

(3.1.4)Example

Let Γ denotes the vector space of all entire function all element represented by Dirichld series, and $Y \subseteq \Gamma$ such that

$Y = \left\{ \alpha = \sum_{n=0}^{\infty} \frac{(ke^{s\lambda})^{2n+1}}{(2n+1)!} = \sin(ke^{s\lambda}) : \lim_{n\to\infty} \left| \frac{k^{2n+1}}{(2n+1)!} \right|^{\frac{1}{\lambda_n}} = 0, k \text{ is real} \right\}$ and let $\mathcal{B} = \{ B : B \subseteq \Gamma \}$,

$B = \left\{ \alpha : \|\alpha\| = \sup\{|k|, \left\| \frac{k^{2n+1}}{(2n+1)!} \right\|^{1/n}, n \geq 1 \right\}$, then $\mathcal{B}_y = \{ A = B \bigcap Y : B \in \mathcal{B} \}$,

$A = \alpha(s) : \|\alpha(s)\| = l.u.p \left\{ |a_n|^{\frac{1}{\lambda_n}} \right\}$

Now to define the base of the subspace of Bornological vector space (Γ, $\mathcal{B}$). We denote by B_r the set of all $\{\alpha(s):\|\alpha(s)\|\le r\}$. Then the family $\mathcal{B}_0=\{B_r : r=1,2...\}$ forms a base for a bornology $\mathcal{B}$ on Γ, then the base of the subspace is $\mathcal{B}_0'=\{A=B_r\cap Y : B_r\in\mathcal{B}_0\}$ such that A= $\{\alpha(s):\|\alpha(s)\|\le r_1, r_1\le r\}$ Where r integer.

3-2Product Bornology of the Space Γ

(3.2.1)Definition

Let $(\Gamma_i, \mathcal{B}_i)_{i\in I}$ be a family of Bornological vector space indexed by a non-empty set I and let $\Gamma=\prod_{i\in I}\Gamma_i = \left\{\prod_{i\in I}\alpha_i(s):\forall\alpha_i(s)\in\Gamma_i, i\in I\right\}$ be the product of the vector space Γ_i. For every $i\in I$, let $p_i:\Gamma\to\Gamma_i$ be the canonical projection then the product bornology on Γ is the coarsest bornology on Γ for which each map p_i is bounded.

I.e. the product bornology is $\mathcal{B}=\left\{\prod_{i\in I}B_i : \forall B_i\in\mathcal{B}_i, i\in I\right\}$,

Bi= $\{\alpha(s):\|\alpha(s)\| = l.u.p\left\{|a_n|^{\frac{1}{\lambda_n}}\right\}\}$

Now to define the base of the product bornologies of Bornological vector space $(\Gamma_i, \mathcal{B}_i)_{i\in I}$.

(3.2.2) Definition

The product bornology on Γ has a base consisting of sets of the form B= $\prod B_{0i}$ where $B_{0i}\in\mathcal{B}_{0i}$ for all $i\in I$ where $\mathcal{B}_{0i}$ is a base for Γ_i,

$$\prod_{i\in I}B_i=\left\{\prod_{i\in I}\boldsymbol{\alpha}_i : \|\boldsymbol{\alpha}_i\|\le r_i, r_i=1,2,..., i\in I\right\}$$

3-3 Quotient Bornlogy of the space Γ.

Definition(3.2.3)

Let F is a linear subspace of a Bornological vector space Γ, ***the linear*** Bornological ***quotient space*** is the quotient space Γ/F. with the quotient Bornology on E/F such that i.e $\mathcal{B}$ ={ θ(B): θ(B)= B+F, B is bounded in Γ

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حول التراكيب البرنولوجية لفضاء كل عناصره ممثلة بواسطة متسلسلة داشليت

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المستخلص

في هذا البحث قمنا بدراسة تراكيب برنولوجية أساسية لبرنولجي معرف على فضاء الدوال الكليه التي كل عناصره ممثله بواسطة سلسلة دشليت وإضافة بعض الخواص لها مثل الفضاء البرنولوجي الجزئي، وفضاء الجداء البرنولوجي، وفضاء القسمة