The Collocation Method For Fourth Order Fuzzy Boundary Value Problems

طريقة الحشد لحل المعادلات التفاضلية الحدودية الضبابية من الدرجة الرابعة

By Rasha H.Ibraheem, Department of Mathematics , College of Basic Education ,Al- mustansiriyah University, Baghdad- Iraq.

Abstract

In this paper, the collocation method is considered to solve the nonhomogeneous for fourth order fuzzy boundary value problems, In which the fuzziness appeared together in the boundary conditions and in the nonhomogeneous term of the differential equation. The method of solution depends on transforming the fuzzy problem to equivalent crisp problems using the concept of α-level sets.

Keywords: the collocation method ; Fuzzy sets; α-level sets ; fourth order fuzzy boundary value problems.

1-Introduction

A finite element method involving collocation method as basis functions has been developed to solve fourth order boundary value problems. The fourth order and third order derivatives for the dependent variable are approximated by the central differences of second order derivatives. The basis functions are redefined into a new set of basis functions which in number match with the number of collocated functions are redefined into a new set of basis functions which in number match with the number of collocated points selected in the space variable domain.

Fuzzy set had been introduced by Zadeh in1965, in which, Zadeh s original definition of Fuzzy set is as follows a fuzzy set (denoted by $\tilde{A}$) is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) Function $\mu_{\tilde{A}}$:X $-\!\!\rightarrow$[0, 1], where X is the universal set, which assigns to each object a Grade of membership ranging between zero and one, i.e., the fuzzy set may be given by,[1],[2]:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)),\ x \in X,\quad 0 \le \mu_{\tilde{A}} \le 1\}$$

Convex fuzzy sets are of great importance in defining fuzzy numbers. This property is Viewed as a generalization of the classical concept of convexity of nonfuzzy sets. The Definition of convexity for fuzzy set does not necessarily mean that the membership function

Of a convex fuzzy set is also convex function, where a fuzzy set $\tilde{A}$ On is convex if, [7]:

$\mu_{\tilde{A}}\ (\lambda x_1 + (1-\lambda)x_2\ ) \geq \text{Min}\{\mu_{\tilde{A}}\ (x_1)\ , \mu_{\tilde{A}}(x_2)\ \}$

For all $x_1, x_2 \in X$, and all $\lambda \in [0, 1]$. Also, a non-empty fuzzy set $\tilde{A}$ Can always be normalized (i.e., the greatest membership value) by dividing $\mu_{\tilde{A}}(x)$,$\forall\ x \in X$ by $\mu_{\tilde{A}}(x)$ and as a matter of convenience, we will generally assume that fuzzy sets are

Normalized, [5]. Among the basic concepts in fuzzy set theory which will play a central role in solving fuzzy differential equations is the concept of an α-level, where if we are given a fuzzy set $\tilde{A}$ Defined on the universal set X and any number $\alpha \in [0, 1]$,the α-level, A_α is the crisp set (non fuzzy set) that contains all elements of X whose membership grades in $\tilde{A}$ Are greater than or equal to a pre specified value of α, i.e. $A_\alpha = \{x : \mu_{\tilde{A}}\ (x) \geq \alpha, \forall\ x \in X\}$.

Kandel in 1986 [5] applied the concept of fuzzy differential equations to the analysis of fuzzy dynamical problems

Pearson in 1997 [3], introduced the analytical method for solving linear system of fuzzy differential equations with the cooperation of complex numbers while there is no such study for evaluating the analytical solution of fuzzy boundary value problems explicitly, except of the work of Al-Saedy A. J. in 2006 [6]

Fourth order fuzzy differential equations appear in several branches of applied mathematics and engineering. For example, fourth order fuzzy deferential equations are used to describe deformable systems. These systems include arches, beams, load bearing members like street lights in electrical engineering to robotic arms in other multi-purpose engineering systems where elastic members serve as key elements for shedding or transmitting loads. Because of the pervasive presence of deformable systems in the development and application of latest technologies, there has been a continuous interest in this area of research. Solving such type of boundary value problems analytically is possible only in very rare cases.

The objective of this paper is to present a simple technique in solving a fourth order Fuzzy boundary value problem , using the collocation method.

2-Fuzzy Number

In this section, we shall give some basic concepts for fuzzy numbers and fuzzy functions before we present the modified approach for solving fuzzy boundary value problems using the collocation method in order to make our paper of self contents.

First, we will give the definition of a fuzzy number and its representation using two approaches as a α-level sets (which will be in this case as a closed subsets of the real line).

Definition (1)[4]:

A fuzzy number $\widetilde{M}$ Is a convex normalized fuzzy set $\widetilde{M}$ Of the real line R, such that:

1. There exists exactly one $x_0 \in$ R, with $\mu_{\widetilde{M}}(x_0) = 1$ (x_0 Is called the mean value of $\widetilde{M}$).
2. $\mu_{\widetilde{M}}(x)$ Is piecewise continuous.

Now, in applications, the representation a fuzzy number in terms of its membership function is so difficult to use, therefore two approaches are given for representing the fuzzy number in terms of its α-level sets, as in the following remark:

Remark (1)[3]:

A fuzzy number $\widetilde{M}$ may be uniquely represented in terms of its α-level sets, as the following closed intervals of the real line:

$M_\alpha = [m - \sqrt{1-\alpha}\,, m + \sqrt{1-\alpha}]$ or$M_\alpha = [\alpha m, \frac{1}{\alpha} m]$

Where m is the mean value of $\widetilde{M}$ and α ∈ [0, 1]. This fuzzy number may be written as

$M_\alpha = [\underline{\widetilde{M}}\,, \underline{\widetilde{M}}\,]$ where $\underline{\widetilde{M}}$ refers to the greatest lower bound of M_α and $\underline{\widetilde{M}}$ to the least upper bound of M_α.

Remark (2):

Similar to the second approach given in remark (1), one can fuzzyfy any crisp or nonfuzzy function f, by letting:

$\underline{f}(x) = \alpha f(x), \quad \underline{f}(x) = \frac{1}{\alpha} f(x) \quad , x \in X$, α ∈ (0, 1]

And hence the fuzzy function $\tilde{f}$ in terms of its α-levels is given by $f_\alpha = [\underline{f}, \underline{f}]$.

3-The Collocation Method for Solving FUZZY BOUNDARY VALUE PROPLEMS

In this paper, we consider the general fourth order linear boundary value problem :

c_0(x) $y^{(4)}$(x)+ c_1(x)y (x)+ $c_2(x)y$ (x)+ $c_3(x)y$ (x)+ $c_4(x)$y(x) = $f(x)$, a ≤ x ≤ b (1)

Subject to fuzzy boundary conditions

Y(a) = A_0 , y(b) = B_0 , y (a) = A_1 , y (b) = B_1 ,

Where A_0, B_0, A_1, B_1 Are finit fuzzy constant and c_0, c_1, c_3, c_4 Are continuous functions defined on the [a, b].

Where f is the fuzzyfying function of the crisp function f which may be written in terms of its α-levels as $f_\alpha = [\underline{f}\,, \underline{f}]$, $\underline{f}$(x)=α $f(x)$,$\underline{f}$(x)= $\frac{1}{\alpha} f(x)$, x∈ $X, \alpha \in (0,1], \forall x \in [a,b]$; *with certain fuzzy bounday conditions.*

Let y(x) be the approximate solution of eq.(1), defined by:

Y(x)=w(x)+$\sum_{i=1}^{N} \quad a_i B_i(x)$………… (2)

Where w(x) is a function which satisfies non homogeneous boundary conditions,B_{i}, $\forall\, i = 1,2,\ldots,N$; is sequence of functions, which satisfies the homogeneous conditions and, $\forall\, i = 1,2,\ldots,N$; are fuzzy numbers to be determined.

To find the approximate solution y , substitute y in the differential equation (1) and hence the problem is reduced to the problem of evaluating of the constants a_i , for all i = 1, 2, …, N; which gives residue function:

$$R(y,x)=c_4\left\{w(x)+\sum_{i=1}^{N} a_i\,B_i\,(x)\right\}^4+c_3\left\{w(x)+\sum_{i=1}^{N} a_i\,B_i\,(x)\right\}'''+c_2\left\{w(x)+\sum_{i=1}^{N} a_i\,B_i\,(x)\right\}''+c_1\left\{w(x)+\sum_{i=1}^{N} a_i\,B_i\,(x)\right\}'+c_0\left\{w(x)+\sum_{i=1}^{N} a_i\,B_i\,(x)\right\}-f(x)$$

$$=c_4\left\{w^{(4)}(x)+\sum_{i=1}^{N} a_i\,B_i^{(4)}\,(x)\right\}+c_3\left\{w'''(x)+\sum_{i=1}^{N} a_i\,B_i'''\,(x)\right\}+c_2\left\{w''(x)+\sum_{i=1}^{N} a_i\,B_i''(x)\right\}+c_1\left\{w'(x)+\sum_{i=1}^{N} a_iB_i'(x)\right\}+c_0\left\{w(x)+\sum_{i=1}^{N} a_i\,B_i\,(x)\right\} \quad \ldots\ldots..(3)$$

Therefore, $R(y,x)$ is now a function of the unknowns $a_1,a_2,\ldots,a_N$ Which may be rewritten as $R(a_1\,,a_2\,,\ldots.a_N;x)$ and therefore $R(a_1\,,a_2\,,\ldots.a_N;x)\cong 0$, for all x∊[a, b]

Hence eq.(3) may be rewritten for the approximate solution as:

$$R(a_1\,,a_2\,,\ldots.a_N;x)=\sum_{i=1}^{N}\left\{c_4a_iB_i^{(4)}(x)+c_3a_iB_i'''+c_2a_iB_i''(x)+c_1a_iB_i'(x)+c_0a_iB_i(x)\right\}+c_4w^{(4)}(x)+c_3w'''(x)+c_2w''(x)+c_1w'(x)+c_0\,w(x)-f(x)\ \ldots\ldots\ldots(4)$$

To evaluate the coefficients a_i 's, $i = 1,2,\ldots,N$; we evaluate eq.(4) at n-distinct point $x_1,x_2\,\ldots,x_N \in [a,b]$, which will produce the following linear system:

$R(a_1\,,a_2\,,\ldots,a_N;x_1)=0$
$R(a_1\,,a_2\,,\ldots,a_N;x_2)=0$

•

$R(a_1\,,a_2\,,\ldots,a_N;x_n)=0$

Therefore, we have:

$\sum_{i=1}^{N} \Big\{c_4 a_i B_i^{(4)}(x) + c_3 a_i B_i''' + c_2 a_i B_i''(x) + c_1 a_i B_i'(x) + c_0 a_i B_i(x)\Big\} = f(x) - c_4 w^{(4)}(x) - c_3 w'''(X) - c_2 w''(x) - c_1 w'(x) - c_0\, w(x)$, i=1,2,….N

Or in matrix form:

A a = D ………………..(5)

Where:

A=[$a_{11}\ a_{12}\ \ .\ a_{1N}\ a_{21}\ a_{22}\ \ .a_{2N}\ ..\ a_{N1}\ a_{N2}\ \ .a_{NN}$]

Where:

$$a_{11} = c_4 B_1^{(4)}(x_1) + c_3 B_1'''(x_1) + c_2 B_1''(x_1) + c_1 B_1'(x_1) + c_0 B_1(x_1)$$

$$a_{12} = c_4 B_2^{(4)}(x_1) + c_3 B_2'''(x_1) + c_2 B_2''(x_1) + c_1 B_2'(x_1) + c_0 B_2(x_1)$$

.

$$a_{1N} = c_4 B_N^{(4)}(x_1) + c_3 B_N'''(x_1) + c_2 B_N''(x_1) + c_1 B_N'(x_1) + c_0 B_N(x_1)$$

$$a_{21} = c_4 B_1^{(4)}(x_2) + c_3 B_1'''(x_2) + c_2 B_1''(x_2) + c_1 B_1'(x_2) + c_0 B_1(x_2)$$

$$a_{22} = c_4 B_2^{(4)}(x_2) + c_3 B_2'''(x_2) + c_2 B_2''(x_2) + c_1 B_2'(x_2) + c_0 B_2(x_2)$$

.

$$a_{2N} = c_4 B_N^{(4)}(x_2) + c_3 B_N'''(x_2) + c_2 B_N''(x_2) + c_1 B_N'(x_2) + c_0 B_N(x_2)$$

$$a_{N1} = c_4 B_1^{(4)}(x_N) + c_3 B_1'''(x_N) + c_2 B_1''(x_N) + c_1 B_1'(x_N) + c_0 B_1(x_N)$$

$$a_{N2} = c_4 B_2^{(4)}(x_N) + c_3 B_2'''(x_N) + c_2 B_2''(x_N) + c_1 B_2'(x_N) + c_0 B_2(x_N)$$

.

.

$$a_{NN} = c_4 B_N^{(4)}(x_N) + c_3 B_N'''(x_N) + c_2 B_N''(x_N) + c_1 B_N'(x_N) + c_0 B_N$$

And:

D=$\Big[f(x_1) - c_4w^{(4)}(x) - c_3w'''(X) - c_2w''(x) - c_1w'(x) - c_0\, w(x)\; f(x_2) - c_4w^{(4)}(x) - c_3w'''(X) - c_2w''(x) - c_1w'(x) - c_0\, w(x)\; .f(x) - c_4w^{(4)}(x) - c_3w'''(X) - c_2w''(x) - c_1w'(x) - c_0\, w(x)\;\Big]$,

$\tilde{a}$= $[\tilde{a}_1\; \tilde{a}_{2\,..}\; \tilde{a}_N\,]$

In order to solve the resulting system (5), one must first use remark (1) to rewrite the fuzzy numbers $\tilde{a}_i$, ∀ i = 1, 2, …, N; in terms of its α-level sets as a_{i_α} = [$\underline{a}_i$, $\underline{a}_i$] , f_α = [$\underline{f}$, $\underline{f}$], ∀ α ∈ (0,1]; and similarly for the fuzzy boundary conditions. Then solving the related non fuzzy linear systems for the lower and upper values of the α-level sets $\underline{a}_i$ And $\underline{a}_i$, ∀ i = 1, 2, …, N, respectively.

4-Illustrative Example

Consider the fourth order fuzzy boundary value problems

$y^{(4)}(x) - y''(x) - y(x) = \tilde{f}\,(x)$, x ∈ [0, 1] …………..(6a)

With fuzzy boundary conditions:

$y(0) = \tilde{1}$, $y'(0) = \tilde{0}$, $y(1) = \tilde{0}$, $y'(1) = -\tilde{e}$ …………….(6b)

Where $\tilde{f}$ is a fuzzyfying function of the crisp function $f(x) = e^x(x - 3)$.

In order to solve the fuzzy boundary value problem (6a) and (6b), use remark (1) to rewrite first the fuzzy function $\tilde{f}$ in terms of its α-levels as f_α = [$\underline{f}$, $\underline{f}$], where and $\underline{f}(x) = e^x\alpha(x-3)$, $\underline{f}(x) = \frac{e^x(x-3)}{\alpha}$, α ∈ (0, 1] and the fuzzy boundary conditions in terms of its α-levels, as:

$y_\alpha(0)$ = [1− $\sqrt{1-\alpha}$, 1+ $\sqrt{1-\alpha}$], $y'_\alpha(0)$ = [0− $\sqrt{1-\alpha}$, 0+ $\sqrt{1-\alpha}$]

And

$y_\alpha(1)$ = [0− $\sqrt{1-\alpha}$, 0+ $\sqrt{1-\alpha}$], $y'_\alpha(1)$ = [-e− $\sqrt{1-\alpha}$, -e+ $\sqrt{1-\alpha}$]

Therefore, to solve this problem using the collocation method, consider the fuzzy approximate solution $\tilde{y}$With α-levels: $y_x(x)$ = [$\underline{y}$, $\underline{y}$], α ∈ (0, 1], Hence, to find the solution in the lower case of solution $\underline{y}$, consider the problem:

$\underline{y}^{(4)}(x) - \underline{y}''(x) - \underline{y}(x) = e^x\alpha(x-3)$, ……………………………..(7)

With lower bound of boundary conditions:

$\underline{y}(0) = 1- \sqrt{1-\alpha}$ $\quad \underline{y}'(0)=0-\sqrt{1-\alpha}$, $\underline{y}(1)= 0-\sqrt{1-\alpha}$, $\underline{y}(1) = -e-\sqrt{1-\alpha}$,α∈(0,1]

Now, let:

$\underline{y}(x)$ = w(x) +$\sum_{i=1}^{3} \tilde{a}_i B_i(x)$

Where

$w(x) = (1-x)e^x - \sqrt{1-\alpha}$,

Which satisfies $w(0) = 1-\sqrt{1-\alpha}$, $w(1) = 0-\sqrt{1-\alpha}$

i.e., satisfies the non-homogeneous boundary condition. The functions B_i ,∀ i = 1, 2, 3; which satisfy the homogeneous boundary conditions y (0) = 0, y (1) = 0 ,y′(0)=0, y′(1)=0 may be Chosen as:

$$B_1(x) = x^2(x-1)$$
$$B_2(x) = x^3(x-1)$$
$$B_3(x) = x^4(x-1)$$

And $y(\underline{a}_1, \underline{a}_2, \underline{a}_3; x)$ Will takes the form:

$\underline{y}(\underline{a}_1, \underline{a}_2, \underline{a}_3; x) = (1-x)e^x - \sqrt{1-\alpha} + x^2(x-1)(\underline{a}_1 + \underline{a}_2 x + \underline{a}_3 x^2)$

$=(1-x)e^x - \sqrt{1-\alpha} + \underline{a}_1(x^3-x^2) + \underline{a}_2(x^4-x^3) + \underline{a}_3(x^5-x^4)$

And upon substituting in eq.(7), yields

$(1-x)e^x - 4e^x + \underline{a}_2(24) + \underline{a}_3(120x-24) - (1-x)e^x - 2e^x + \underline{a}_1(6x-2) + \underline{a}_2(12x^2-6x) + \underline{a}_3(20x^3-12x^2) - (1-x)e^x - \sqrt{1-\alpha} + \underline{a}_1(x^3-x^2) + \underline{a}_2(x^4-x^3) + \underline{a}_3(x^5-x^4) = e^x\alpha(x-3)$

Or equivalently:

:$\underline{a}_1(x^3-x^2+6x-2) + \underline{a}_2(x^4-x^3+12x^2-6x+24) + \underline{a}_3(x^5-x^4+20x^3-12x^2+120x-24) = e^x\alpha(x-3) + 6e^x + (1-x)e^x + \sqrt{1-\alpha}$ …… (8)

Now, evaluate eq.(8) at x1 = 0, x2 = 1/2, x3 = 1; which will yield to the following linear system of algebraic equations:

$[-2\ 24\ -24\ 0.875\ 23.9375\ 35.46875\ 3\ 40\ 104\,][\underline{a}_1\ \underline{a}_2\ \underline{a}_3\,]$=$[-3\alpha+6+\sqrt{1-\alpha}\ -4.12175\ \alpha+9.71655+\sqrt{1-\alpha}\ -5.4366\ \ \alpha+15.3098+\sqrt{1-\alpha}\,]$
Solving this system where $\alpha=1$, yields:
$\underline{a}_1$ = 2.3042, $\underline{a}_2$ = 0.2495 , $\underline{a}_3$ = -0.0675

Therefore: $\underline{y}(x)=(1-x)e^x-\sqrt{1-\alpha}$ + 2.3042(x^3-x^2) + 0.2495 (x^4-x^3) + -0.0675 (x^5-x^4)
Similarly, for the upper solution $\underline{y}$, consider the problem:
$\underline{y}^{(4)}(x)-\underline{y}''(x)-\underline{y}(x)=\frac{e^x(x-3)}{\alpha}$……………………(9)
With upper bound of boundary conditions:
$\underline{y}(0)$ = 1+ $\sqrt{1-\alpha}$ $\underline{y}'$ (0)=0+$\sqrt{1-\alpha}$,$\underline{y}$(1)= 0+$\sqrt{1-\alpha}$, $\underline{y}'$ (1) = $-e+\sqrt{1-\alpha}$,α∈(0,1]
Now, let:
$\underline{y}(x)$ = w(x)+$\sum_{i=1}^{3}$ $\tilde{a}_i B_i(x)$
Where:
$w(x)=(1-x)e^x+\sqrt{1-\alpha}$
Which satisfies $w(0)=1+\sqrt{1-\alpha}\,,$ $w(1)=0+\sqrt{1-\alpha}$
And letting also:

$$B_1(x)=x^2(x-1)$$
$$B_2(x)=x^3(x-1)$$
$$B_3(x)=x^4(x-1)$$

Therefore, $\underline{y}(x)$ will take the form:

$$\underline{y}(\underline{a}_1,\underline{a}_2,\underline{a}_3;x)=(1-x)e^x+\sqrt{1-\alpha}+x^2(x-1)(a_1+a_2x+a_3x^2)$$
$$=(1-x)e^x+\sqrt{1-\alpha}+a_1(x^3-x^2)+a_2(x^4-x^3)\ +\ a_3(x^5-x^4)$$

And upon substituting in eq.(9), yields:

$$\underline{a}_1(x^3-x^2+6x-2)\ +\underline{a}_2(x^4-x^3+12x^2-6x+24)$$
$$+\underline{a}_3(x^5-x^4+20x^3-12x^2+120x-24)$$
$$=\frac{e^x(x-3)}{\alpha}+6e^x+(1-x)e^x-\sqrt{1-\alpha}\ldots\ldots(10)$$

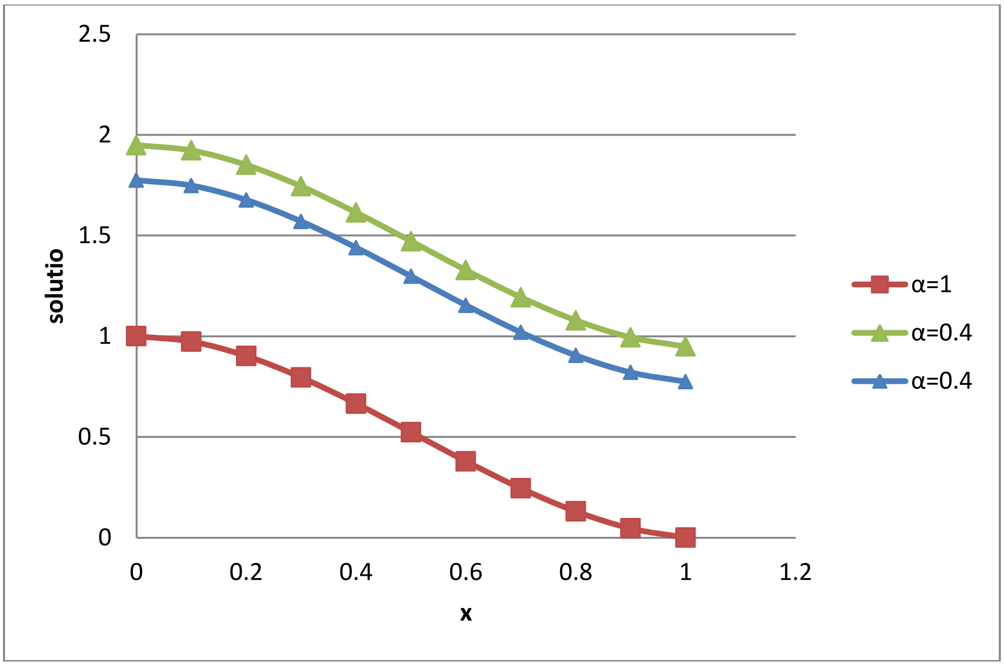
Hence, evaluating eq.(10) at x1 = 0, x2 = 1/2, x3 = 1, which will yield to the following linear system of algebraic equations:
$[-2\ 24\ -24\ 0.875\ 23.9375\ 35.46875\ 3\ 40\ 104\,][\underline{a}_1\ \underline{a}_2\ \underline{a}_3\,]$=$[\frac{-3}{\alpha}+6-\sqrt{1-\alpha}\ \frac{-4.12175}{\alpha}+9.71655-\sqrt{1-\alpha}\ \frac{-5.4366}{\alpha}+15.3098-\sqrt{1-\alpha}\,]$

Solving this system where $\alpha = 1$, yields:
$\underline{a}_1$ = 2.3042, $\underline{a}_2$ = 0.2495 ,$\underline{a}_3$ = -0.0675
Therefore:
$\underline{y}(x) = (1 - x)e^x + \sqrt{1-\alpha} +$ 2.3042 $(x^3 - x^2)$ + 0.2495 $(x^4 - x^3)$
+ - 0.067 $(x^5 - x^4)$

Combining $\underline{y}$ and $\underline{y}$ yields the fuzzy solution of the fuzzy boundary value problem (6) as y_α(x) = [$\underline{y}$ (x), $\underline{y}$ (x)], ∀ α ∈ (0, 1], x ∈ [0, 1]. In addition, it is clear that for α = 1, we get $\underline{y}$ (x) = $\underline{y}$ (x), which is the same as the
Crisp solution of the related nonfuzzy boundary value problem.
The results of the calculations are given in table(1):

x	$\underline{y}$	$\underline{y}$	**Crisp solution**
0	**1**	**1**	**1**
0.1	**0.973697551**	**0.973697551**	**0.994654**
0.2	**0.901877407**	**0.901877407**	**0.977122**
0.3	**0.79540374**	**0.79540374**	**0.944901**
0.4	**0.665347619**	**0.665347619**	**0.895095**
0.5	**0.52285126**	**0.52285126**	**0.824361**
0.6	**0.37898512**	**0.37898512**	**0.728848**
0.7	**0.244596887**	**0.244596887**	**0.604126**
0.8	**0.130151386**	**0.130151386**	**0.445108**
0.9	**0.045560236**	**0.045560236**	**0.24596**
1	**0**	**0**	**0**

Also, the fuzzy solution $\tilde{y}$ in terms of the lower bound of solution $\underline{y}$ and upper bound of solution $\underline{y}$ and for different α-levels (where α ∈ (0, 1]) are presented in Fig.(1):



5-References

[1] Pal S. K. and Majumder D.K Fuzzy Mathematical Approach to Pattern Recognition , John Wiley and sons, Inc., New York, 1986.

[2] Dubois, D. and Prade, H., Fuzzy Sets and Systems; Theory and Applications , Academic Press, Inc., 1988.

[3] Pearson, D. W.: “A Property of Linear Fuzzy Differential Equations”, Appl. Math. Lett., Vol.10, No.3, 1997, pp.99-103.

[4] Zimmerman, H. J., "Fuzzy Set Theory and Its Applications", Kluwer-Nijhcff Publishing, USA, 1988.

[5] Kandel A., Fuzzy Mathematical Techniques with Applications , Addison Wesley Publishing Company, Inc., 1986.

[6] Al-Saedy A. J. M., "Solution of Boundary Fuzzy Differential Equations", M. Sc. Thesis, College of Science, Al-Nahrain University, 2006.

[7] Yan, J., Ryan, M. and Power, J., Using Fuzzy Logic: Towards Intelligent Systems , Prentice Hall, Inc., 1994.

الخلاصة:

في هذا البحث، تم دراسة طريقة الحشد (the collocation method) لحل المعادلات التفاضلية الحدودية الضبابية غيرمتجانسة من الدرجة الرابعة حيث كانت الضبابية في الشروط الحدودية، والطرف الغير متجانس في المعادلة التفاضلية،اعتمدت طريقة الحل على تحويل المعادلات التفاضلية الحدودية الضبابية الى مسألة مكافئة غيرضبابية (crisp problem) باستخدام مبدأ مجموعات مستويات القطع(α-level sets).