On Not Canonical Equivalence Between Diffeomorphism Subgroupoid And Fundemental Fibre Bundle

Mohammed Jabbar Hussein
Department of Mathematics
College of Basic Education-Al-Mustansiriya University

Abstract

The main purpose of this work is to find not canonical equivalence to category and morphism of diffeomorphism from smooth manifold category with Fundemental Fibre Bundle denoted by DG_H groupoids morphism and denoted P.

Key Words: category, diffeomorphism, smooth manifold , groupoids morphism , fundamental Fibre Bundle .

Introducation

The term of groupoid category is one of the genuine meanings which explained the group applications in the twentieth century. The work in these groups is more general and more powerful than working in groups in terms of structure , it is the groupoid as a category of everything which defined as a category and these categories are identical if we can find stocks among its kits.

The first people who paid attention to the theory of groupoid categories are Brandt in the twenties of this century before Earsman makes the term of groupoid category in the center of his work in differential Geometry. Earsman is the first one who presented the term of groupoid categories in differential Geometry in the Fifties of this century throughout his work on the principal fiber Bundle, where he resolved most of questions which were presented through Algebraic theory of grouoid categories after it was known as differential structure.

The main objective of the research is to prove the existence of equal not canonical category between Lie groupoid categories and the principal Fibre Bundle categories. Thus, in the first section , we have started the study of the categories and some of the important definitions in our research , and then we moved to the study of the groupoid categories in Algebraic way and all that is related to be the main category of the groupoid categories , while the second section includes definitions of Lie groups, group action in differential condition and principle Fibre Bundle, as well as examples, various issues, and some evidences.

The third section includes definition of the differential groupoid category and other definitions, and the purpose is to configure a groupoid of Lie groupoid,

and then we proved that if there is a principal Fibre Bundle then DG_H categories

P is Lie groupoid category, and vice verse,finally, the fourth section includes proofing the not canonical equivalence among Lie groupoid categories and the principal Fibre Bundle.

Groupoids

In this section we give the fundamental concepts releated to this work:

Definition(1.1)[1]:

A category C to be conditions:

1- A collect from object.

2- For all order binary from object (X,Y), then a set hom(X,Y) to be represent of $.f : X \to Y$ all arrows that the domain X and codomain Y,if hom(X,Y) to write Such that $id_y : Y \to Y$,there exists $f : X \to Y$

3- for all $hid_y = h$ then $h : Y \to Z$ if $id_y f = f$ id_y is called identity element from Y.

Definition(1.2)[2]:

Let A,B be categories, F is said to be Functor from B into A if F to $g : X \to Y$ determine for all object X in object B in A and to determine for all arrow such that $F(g) : F(X) \to F(Y)$ in B arrow.

1- if $F(id_X) = id_{F(X)}$ is identity element in B then $id_X : X \to X$

2- if $F(gf) = F(g)F(f)$ are arrows in B then $g : Y \to Z$ and $f : X \to Y$.

Example(1.3)[2]:

Functor there exsits whose category T from category C for all topology space in T a set denoted on that space in C. and called Forgetful Functor because is called topological structure.

Definition(1.4)[3]:

Let $\varphi : F_2 \to F_1$ be a natural transformation then φ is called Natural equivalence iff there exsists a natural transformation $\varphi' : F_2 \to F_1$ such that $\varphi' \circ \varphi = id_{F_1}$ and $\varphi \circ \varphi' = id_{F_2}$, from that is said to be Functors F_1, F_2 is Natural equivalence $(F_1 \cong F_2)$.

Definition(1.5)[3]:

Let φ be a natural transformation from F_1 into F_2 then φ is called a not canonical equivalence if there exists a natural transformation φ' from F_2 into F_1 such that $\varphi \circ \varphi' \cong id_{F_2}, \varphi' \circ \varphi \cong id_{F_1}$.

Definition(1.6)[4]:

Let G be a group and P be a set, to say that G to have action on P if there exists an applying $\psi : G \times P \to P$ denoted by for all $x \in P, s \in G$ then $\psi(s,x) = s.x$ if :

1- e the identity element in G then $\psi(e,x) = e.x = x$ for all $x \in P$.

2- For all $s,t \in G$ then $\psi(s,\psi(t,x)) = \psi(st,x)$, for all $x \in P$.

Remarks(1.7)[4]:

1- for all $x \in P$ then a subset $G.x = \{s.x / s \in G\}$ from P define by action G on $\{x\} \subset P$ is called orbit x by G.

2- G is called to be Free action on a set P (on left) if the element e in G is unique element if $\psi(e,x) = e.x = x$ for all $x \in P$.

3- The group action G on a set P is denoted by relation on P if for all $(x,y) \in P \times P$ then x equivalence (x~y)y if there exists an element $s \in G$ such that $\psi(s,x) = s.x = y$.

Example(1.8)[5]:

let (R,+) be a real number group with addition operation, and let S^1 be a unit sphere such that $S^1 = \{(x,y) \in x^2 + y^2 = 1\}$ then R has action on S^1 define by $\varphi : R \times S \to S^1$ such that $\varphi(x,s) = e^{2\pi ix}$ for all $x \in R, s \in S^1, \varphi$ to represent action R on S^1 .

1- let $0 \in R$ is identity element then $\varphi(0,s) = e^{2\pi i0}.s = s.$

2- let $x,y \in R$ and $s \in S^1$ then $\varphi(x,\varphi(y,s)) = e^{2\pi ix}.e^{2\pi iy}.s = e^{2\pi i(x+y)}.s = \varphi((x+y),s)$

Definition(1.9)[5]:

Let X,Y,Z be a sets , and let $f : X \to Z$, $g : Y \to Z$ be mappings in e. a Fibre product $X \times_Z Y$ for mappings f,g is a subset from a set $X \times Y$ to be form for all order pair (x,y) in $X \times Y$ so that f(x)=f(y), for which the following diagram commutes:

$$\begin{tikzcd} X\times_Z Y \arrow[r, "g^*"] \arrow[d, "f"'] & X \arrow[d, "f^*"] \\ Y \arrow[r, "g"'] & Z \end{tikzcd}$$

if W is a set and $h: W \to X$, $h^*: W \to Y$ be mappings in C such that $f \circ h = g \circ h^*$ then there exists a unique mapping $\theta: W \to X \times Y$

$$\begin{tikzcd} W \arrow[dr] \arrow[drr] \arrow[ddr] & & \\ & X\times Y \arrow[r, "g^*"] \arrow[d, "f"'] & X \arrow[d, "f^*"] \\ & Y \arrow[r, "g"'] & Z \end{tikzcd}$$

is commutative in C.

A mapping θ is defined by $\theta(a) = (h(a), h^*(a))$ for all $a \in W$.

2- Lie group

Definition(2.1)[6]:

A Lie group is a set G defined on a two structure:

1- G a group structure.

2- G is a C^∞-manifold.

Such that a mapping $\mu: G \times G \to G$ defined by $\mu(x, y) = xy$ for all $(x, y) \in G \times G$ and a mapping $\upsilon: G \to G$ defined by $\upsilon(x) = x^{-1}$ for all $x \in G$, are smooth mapping.

Example(2.2)[6]:

R is a Lie group because R is smooth manifold , and (R,+) is commutative group. A mapping $\mu: R \times R \to R$ defined by $\mu(x, y) = x + y$ for all $(x, y) \in R \times R$ is a smooth mapping ,so that the mapping $\upsilon: R \to R$ defined form $\upsilon(x) = x^{-1}$ is smooth mapping from R represent Lie group.

Definition(2.3)[7]:

Let G , G' be a two Lie group then the mapping $J: G \to G$ is called Lie group homomorphism if:

1- J be represent group homomorphism .

2- J a smooth mapping between two smooth manifold .

Example(2.4)[7]:

A determinative mapping $\det : GL(n,R) \to R^*$ defined by $\det((aij)) = |(aij)|$ for all $(aij) \in GL(n,R)$ represent Lie group homomorphism , so that mapping is continuous . and mapping Kernal det that $\det^{-1}(1)$ is a subgroup no closed variable from $GL(n,R)$ and denoted by $SL(n)$ and is called special Linear group and represent all square matrices that determinative equal 1 and also represents Lie group.

Definition(2.5)[8]:

Let $J : G \to G$ be a Lie group morphism then J is called Lie group correspondence if J be a group correspondence and so diffeomorphism equivalence between two smooth manifold.

Definition(2.6)[9]:

Let G be a Lie group and P a smooth manifold. That is called G has smooth action on P (on left) if a mapping $\psi : G \times P \to P$ is smooth mapping between two smooth manifold.

Remark(2.7)[9]:

If G be a Lie group to smooth action on smooth manifold P then a set P/G is called orbit space but P/G is not nessecary to smooth manifold.

Definition(2.8)[11]:

Let X,Y,Z be a smooth manifold, and $F' : Y \to Z$ be a smooth mapping, $F : X \to Z$ be a submersion mapping, then a subset $X \times_Z Y$ from $X \times Y$ defined on subdiffeomorphism manifold structure from $X \times Y$, is called a smooth Fibre Product.

Proposition(2.9)[10]:

In the commutative diagram:

$$\begin{tikzcd}
X \arrow[r, "F'_1"] \arrow[d, "F(A)"] & Y \arrow[r, "F'_2"] \arrow[d, "F'(B)"] & Z \arrow[d, "F''"] \\
X' \arrow[r, "F''_1"] & Y' \arrow[r, "F''_2"] & Z'
\end{tikzcd}$$

1- a square (BA) be a smooth Fibre product if (A) and (B) be a smooth Fibre product.

2- a square (B) be a smooth Fibre product if (BA) and (A) be a smooth Fibre product.

3- a square (A) be a smooth Fibre product if from all (BA) and (B) be a smooth Fibre product.

3-Principal Fibre Bundle

Definition(3.1)[12]:

A $G(P,\pi,B)$ is a principal Fibre Bundle so that from all B,P be a smooth manifold and G is a Lie group to free and smooth action on P(on right) and $\pi : P \to B$ be a bijection submersion mapping such that:

1-π be Fibres equal orbits P, $(\pi(x) = \pi(y))$ equivalence(there exist g in G so that xg=y) for all $x, y \in P$.

2-there exists open cover $\{P_i\}$ into B and a smooth mapping $S_i : P_i \to P$ so that $\pi \circ S_i = id_{P_i}$, P is called completely space and B is base.

Remark(3.2)[11]:

The principal Fibre Bundle is to be condition Local triviality that is $G(P,\pi,B)$ be a principal Fibre Bundle, then for all point $b \in B$ there exists open neighborhood U belongs to b, that $U \times G$ a diffeomorphism of $\pi^{'}(U)$ and the following diagram:

$$\begin{tikzcd} U \times G \arrow[r] \arrow[dr] & \pi^{-1}(U) \arrow[d, "\pi / \pi^{-1}(U)"] \\ & U \end{tikzcd}$$

Is commutative in D.

Example(3.3)[6]:

Let H be a Lie subgroup from Lie group G, then H is action on G(on right) such that G/H on define smooth manifold structure and projective mapping $\pi : G \to G$ is a bijective projective mapping, a quadrangular $H(G,\pi,G/H)$ represented principal Fibre Bundle.

Definition(3.4)[6]:

Let $\eta = G(P,\pi,B)$ and $\eta^{'} = G^{'}(P^{'},\pi^{'},B^{'})$ be a principal Fibre Bundle, a morphism from λ into $\lambda^{'}$ to be mapping from $J : G \to G^{'}$, $F : P \to P^{'}$, $h : B \to B^{'}$ so that from all F and h be a smooth mapping , J a Lie group morphism. And the following diagram is commutative in D.

$$\begin{tikzcd} P \arrow[r, "F"] \arrow[d, "\pi"] & P^{'} \arrow[d, "\pi^{'}"] \\ B \arrow[r, "h"'] & B^{'} \end{tikzcd}$$

So, for all $x \in P, r \in G$ then F(x,r)=F(x)J(r).

Definition(3.5)[6]:

A groupoid (G,B) is called diffeomorphism groupoid if G and B are a smooth manifold and $\varepsilon \in G, \delta \in G, \alpha \in G$ (so that $\varepsilon : B \to G, \delta : AG \to G, \alpha : G \to B$).

Remarks(3.6):

1- $\alpha \in S$ to be the following diagram:

$$\begin{tikzcd} AG \arrow[r, "\delta"] \arrow[d, "\alpha"'] & G \arrow[d, "\delta"] \\ G \arrow[r, "\alpha"'] & B \end{tikzcd}$$

Is a smooth Fibre product and AG define on regular subdiffeomorphism manifold structure from $G \times G$.

2- A mapping $\theta : G \xrightarrow{\Delta} G \times G \xrightarrow{\alpha \times id_G} B \times G \xrightarrow{\varepsilon \times id_G} G \times G$ defined by $\theta(g) = (\alpha(g), g)$ in D either valuble in AG, and $(\delta \circ \theta)(g) = \delta(\theta(g)) = \delta(\alpha(g), g) = \alpha(g)g^{-1} = g^{-1}$ so that $\delta \circ \theta = \sigma$, for all $\sigma \in D$.

3- $\beta \in S$ since $\beta = \alpha \circ \sigma$.

Example(3.7)[6]:

1- A smooth manifold B is a diffeomorphism Groupoid on itself.

2- If G be a Lie group and B a smooth manifold then $B \times G \times B$ is a diffeomorphism Groupoid on a base B.

Definition(3.8)[5]:

A diffeomorphism Groupoid (H,B) are a diffeomorphism subgroupoid from diffeomorphism Groupoid(G,B) if :

1- (H,B) are a subgroupoid from (G,B).

2- for all $H, B^{'}$ are a subdiffeomorphism manifold from G and B.

i.e. $i_H : H \to G$ and $i_B : B^{'} \to B$ is injective submersion mapping.

Definition(3.9)[5]:

A diffeomorphism Groupoid (G,B) is said to be Local trivial if there exists open cover u of base B, for all $U \in U$ there exists a Lie group G such that a diffeomorphism groupoid on U, and let $T^{-1}(U \times U) =_U G_U$ is corresponding with trivial diffeomorphism groupoid $U \times G \times U$ and the following diagram:

$$\begin{tikzcd} U\times G\times U \arrow[r, "U"'] \arrow[dr, "T"'] & G_U \arrow[d, "T/T^{-1}(U\times U)"] \\ & U\times U \end{tikzcd}$$

is commutative in G.
from the bove diagram, we find that $T:G\to B\times B$ is to be submersion mapping.

Definition(3.10)[5]:

A diffeomorphism groupoid (G,B) is said to be Lie groupoid if Localy trivial and G transformation $(T:G\to B\times B)$ is to be a surjective mapping (if $T:G\to B\times B$ is to be a surjective submersion mapping).

Theorem(3.11)[6]:

Let (H,B) be a subgroupoid from diffeomorphism groupoid (G,B) such that:

1- H is a subdiffeomorphism manifold from G.
2- $\alpha_H:H\to B$ is to be a surjective submersion mapping then H be a diffeomorphism groupoid.

Example (3.12)[6]:

Let (G, B) be a Lie Groupoid Submentsion mapping between smooth manifold then the ordered Groupoid G on f_0

Let $G' f_0^*(G)$ is Lie groupoid on basis

$B'\ f: G' \to G$ Arrow in DG_1

S0, then the Kernal f is a groupoid on that the relation equivalence Graph by defined f_0 .

Example (3.13)[6]:

Let $f: M \to N$ be surjective submersion. The associated groupoid denoted by $C(f)$ is the Lie groupoid with

$C(f)_0 = M, C(f)_1 = M \times_N M,$

$S(x, y) = y, t(x, y) = x, U(x) = (x, x)$

$i(x, y) = (y, x), (x, y) \circ (y, z) = (x, z)$

When f is of the form $M \to Pt$, we get the pair groupoid P(M)

When f is the identity map $M \to M$, we get the trivial groupoid on M denoted by M For any $f: M \to N$, there is a Lie gropuoid functor $c(f): C(f) \to N$ induced by f.

4-Not Canonical Equivalence Between DG_H and P:

Proposition(4.1):

A Functor $F : P \to DG_H$ and a point fixing in groupoid bases then there exists a Functor R $: DG_H \to P$.

Proof:

Let (K, g, g_0) be a homomorphism from principal Fibre Bundle $\eta = G(P, \pi, B)$ to principal Fibre Bundle $\eta' = G'(P', \pi', B')$ so that $K : G \to G'$ a Lie group homomorphism and the following diagram:

$$\begin{tikzcd} P \arrow[r, "g"] \arrow[d, "\pi"] & P' \arrow[d, "\pi'"] \\ B \arrow[r, "g_0"'] & B' \end{tikzcd}$$

Is commutative in DG_H.

That is

$\mathcal{F}(\eta) = (P \times P/\mathfrak{G}, B)$ and $\mathcal{F}(\eta') = (P' \times P'/\mathfrak{G}', B')$ are objects in $\mathcal{D}\mathcal{G}_{\mathcal{X}}$ since $g : P \to P'$ is a smooth mapping, then $g \times g : P \times P \to P' \times P'$ is a smooth mapping and to obtain the following diagram:

$$\begin{tikzcd} P \times P \arrow[r, "g \times g"] \arrow[d, "\mathfrak{G}"'] & P' \times P' \arrow[d, "\mathfrak{G}'"] \\ P \times P/\mathfrak{G} \arrow[r, "g \times g/\mathfrak{G} \times \mathfrak{G}'"'] & P' \times P'/\mathfrak{G}' \end{tikzcd}$$

Is commutative in $\mathcal{D}$.

$g \times g/\mathfrak{G} \times \mathfrak{G}' : P \times P/\mathfrak{G} \longrightarrow P' \times P'/\mathfrak{G}'$ is a smooth mapping. Now, let $G = P \times P/\mathfrak{G}$ and $G' = P' \times P'/\mathfrak{G}'$, to prove that $g \times g/\mathfrak{G} \times \mathfrak{G}' : P \times P/\mathfrak{G} \longrightarrow P' \times P'/\mathfrak{G}'$ represents groupoid homomorphism on $g_0 : B \to B'$

$$\begin{tikzcd} \mathfrak{G} \arrow[r, "g"] \arrow[d, "T"'] & \mathfrak{G}' \arrow[d, "T'"] \\ B \times B \arrow[r, "g_0 \times g_0"'] & B' \times B' \end{tikzcd}$$

so that the following diagram:

Is commutative in $\mathcal{DG}_{\mathcal{H}}$.

But, (g, g_0) represents arrow in $\mathcal{DG}_{\mathcal{H}}$, therefore $\mathcal{F}(K, g, g_0) = (g, g_0)$, to that arrow image in $\mathcal{P}$ by $\mathcal{F}$ represents arrow in $\mathcal{DG}_{\mathcal{H}}$, $\mathcal{F}$ is functor if:

1- Let $\eta = \mathfrak{G}(P, \pi, B)$ is object in $\mathcal{P}$. And let $id_\eta = (id_{\mathfrak{G}}, id_P, id_B)$ arrow in $\mathcal{P}$, by definition $\mathcal{F}$ to get $\mathcal{F}(id_{\mathfrak{G}}, id_P, id_B) = \left(id_{P\times P/\mathfrak{G}}, id_B\right)$ and $\eta = (P \times P/\mathfrak{G}, B)$ be object in $\mathcal{DG}_{\mathcal{H}}$ and arrow in $\mathcal{DG}_{\mathcal{H}}$ such that

$id_{P\times P/\mathfrak{G}}: P \times P/\mathfrak{G} \to P \times P/\mathfrak{G}, id_B: B \to B$, but,

$id_{\mathcal{F}}(P \times P/\mathfrak{G}, B): \mathcal{F}(P \times P/\mathfrak{G}, B) \to \mathcal{F}(P \times P/\mathfrak{G}, B)$,

The arrow represents in $\mathcal{DG}_{\mathcal{H}}$ since the identity element is unique, for all objects in $\mathcal{DG}_{\mathcal{H}}$.

2- Let (K_1, g_1, g_0) arrow from $\eta = (P, \pi, B)$ to $\eta' = (P', \pi', B')$ in $\mathcal{P}$, and let (K_2, g_2, g_0) arrow from $\eta' = (P', \pi', B')$ to $\eta'' = (P'', \pi'', B'')$

$$\begin{array}{ccccc} \mathcal{P} & \xrightarrow{g_1} & \mathcal{P}' & \xrightarrow{g_2} & \mathcal{P}'' \\ \pi \downarrow & & \downarrow \pi' & & \downarrow \pi'' \\ B & \xrightarrow[g_0]{} & B' & \xrightarrow[g_0']{} & B'' \end{array}$$

in $\mathcal{P}$, so that the following diagram:

Is commutative in $\mathcal{D}$, and $K_1: \mathfrak{G} \to \mathfrak{G}', K_2: \mathfrak{G}'' \to \mathfrak{G}''$ is a Lie group

$$\begin{array}{ccc} \mathcal{P} & \xrightarrow{L} & \mathcal{P}'' \\ \pi \downarrow & & \downarrow \pi'' \\ B & \xrightarrow[L_0]{} & B'' \end{array}$$

homomorphisms, and let (K, L, L_0) arrow from η to η'' in $\mathcal{P}$ so that the following diagram:

Is commutative in $\mathcal{D}$ and $K: \mathfrak{G} \to \mathfrak{G}''$is a Lie group homomorphism.

Now, $\mathcal{F}(\eta) = (P \times P/\mathfrak{G}, B), \mathcal{F}(\eta) = (P' \times P'/\mathfrak{G}', B')$and

$\mathcal{F}(\eta'') = (P'' \times P''/\mathfrak{G}'', B'')$ are objects in $\mathcal{DG}_{\mathcal{H}}$ and

$\mathcal{F}(K, L, L_0) = (L', L_0), \mathcal{F}(K_2, g_2, g_0') = (g_2', g_0')$ and $, \mathcal{F}(K_1, g_1, g_0) =$

$$\begin{tikzcd} P \times P/\mathfrak{G} \arrow[r, "g_1'"] \arrow[d, "T"'] & P' \times P'/\mathfrak{G}' \arrow[r, "g_2'"] \arrow[d, "T'"] & P'' \times P''/\mathfrak{G}'' \arrow[d, "T''"] \\ B \times B \arrow[r] & B' \times B' \arrow[r] & B'' \times B'' \end{tikzcd}$$

(g_1', g_0) is arrow in $\mathcal{DG}_{\mathcal{H}}$, so that the following diagrams:

$$\begin{tikzcd} P \times P/\mathfrak{G} \arrow[r, "L'"] \arrow[d, "T"'] & P'' \times P''/\mathfrak{G}'' \arrow[d, "T''"] \\ B \times B \arrow[r] & B'' \times B'' \end{tikzcd}$$

are commutative in $\mathcal{DG}_{\mathcal{H}}$, but,

$\mathcal{F}(K_2, g_2, g_0') \circ \mathcal{F}(K_1, g_1, g_0) = (g_2' \circ g_1', g_0', g_0)$

In $\mathcal{DG}_{\mathcal{H}}$ thus $\mathcal{F}: \mathcal{P} \to \mathcal{DG}_{\mathcal{H}}$ is functor.

Theorem(4.2) :-

Not canonical equivalence exists between $\mathcal{DG}_{\mathcal{H}}$ and $\mathcal{P}$ if a fixed point in Groupoid bases in $\mathcal{DG}_{\mathcal{H}}$ and a point in principal Fibre Bundle completely space in $\mathcal{P}$.

Proof:-

To prove that not canonical equivalence between $'\mathcal{DG}_{\mathcal{H}}$ and $\mathcal{P}$ then not canonical equivalence between $\mathcal{F} \circ \mathcal{Z}$ and $id_{\mathcal{DG}_{\mathcal{H}}}$, therefore not canonical equivalence between $\mathcal{F} \circ \mathcal{Z}$ and id_P.

There exist natural transformation $\phi: \mathcal{F} \circ \mathcal{Z} \to id_{\mathcal{DG}_{\mathcal{H}}}$ and natural transformation $\phi: id_{\mathcal{DG}_{\mathcal{H}}} \to \mathcal{F} \circ \mathcal{Z}$ such that $\phi \circ \phi' \cong id_{\mathcal{F} \circ \mathcal{Z}}$, $\phi' \circ \phi \cong id_{\mathcal{DG}_{\mathcal{H}}}$

Now, let $\mathcal{Z}: G \to G'$ be groupoid homomorphism on $\mathcal{Z}_0: B \to B'$ is arrow represent in $\mathcal{DG}_{\mathcal{H}}$ and $(G, B)', (G', B')$ are objects in $\mathcal{DG}_{\mathcal{H}}$ so that the following diagram:

$$\begin{tikzcd}
G \arrow[r, "\mathcal{Z}"] \arrow[d, "T"'] & G' \arrow[d, "T'"] \\
B \times B \arrow[r, "\mathcal{Z}_0 \times \mathcal{Z}_0"'] & B' \times B'
\end{tikzcd}$$

Is commutative in $\mathcal{DG}_{\mathcal{H}}$.

$$\begin{tikzcd}
& G \arrow[rr, "\mathcal{Z}"] \arrow[dl, "\mathcal{F} \circ \mathcal{Z}"'] \arrow[dd, "T"] & & G' \arrow[dl] \arrow[dd, "T'"] \\
G_1 \arrow[rr] \arrow[dd, "T_1"'] & & G_1' \arrow[dd, "T_1'"] & \\
& B \times B \arrow[rr, "\mathcal{Z}_0 \times \mathcal{Z}_0"] \arrow[dl] & & B' \times B' \arrow[dl, "T_1'"] \\
B \times B \arrow[rr, "\mathcal{Z}_0 \times \mathcal{Z}_0"'] & & B' \times B' &
\end{tikzcd}$$

From proposition (4.1) and (4.2) to obtain the following diagram:

Is commutative in $\mathcal{DG}_{\mathcal{H}}$.

Such that $G \cong G_1$ and $G' \cong G_1', \mathcal{F} \circ \mathcal{Z}(g, g_0) = (g, g_0)$.

Let is a natural transformation, and

$\phi(G, B): \mathcal{F} \circ \mathcal{Z}\big((G, B)\big) \to id_{\mathcal{DG}_{\mathcal{H}}}\big((G, B)\big)$ is arrow in , for all object (G, B) in $\mathcal{DG}_{\mathcal{H}}$ such that the following diagram:

$$
\begin{tikzcd}
\mathcal{F} \circ \mathcal{Z}\big((G,B)\big) \arrow[r, "\phi(G,B)"] \arrow[d, "T_1"'] & id_{\mathcal{DG}_{\mathcal{H}}}\big((G,B)\big) \arrow[d, "T"] \\
B_1 \times B_1 \arrow[r, "g_0 \times g_0"'] & B \times B
\end{tikzcd}
$$

Is commutative in $\mathcal{DG}_{\mathcal{H}}$. Now if $g: G \to G'$ be a groupoid homomorphism on $g_0: B \to B'$ is arrow represent in $\mathcal{DG}_{\mathcal{H}}$.

Is commutative in $\mathcal{DG}_{\mathcal{H}}$, then ϕ is natural transformation from $\mathcal{F} \circ \mathcal{Z}$ to $id_{\mathcal{DG}_{\mathcal{H}}}$ by proposition (4.2), we get, $\phi \circ \phi' \cong id_{\mathcal{DG}_{\mathcal{H}}}$ and $\phi' \circ \phi \cong id_{\mathcal{F}\circ\mathcal{Z}}$is not canonical equivalence between $\mathcal{F} \circ \mathcal{Z}$ and $id_{\mathcal{DG}_{\mathcal{H}}}$.

$$
\begin{tikzcd}
 & \mathcal{F} \circ \mathcal{Z}(G,B) \arrow[rr, "\mathcal{F} \circ \mathcal{Z}(g)"] \arrow[dl] \arrow[dd] & & \mathcal{F} \circ \mathcal{Z}(G',B') \arrow[dl] \arrow[dd] \\
id_{\mathcal{DG}_{\mathcal{H}}}(G,B) \arrow[rr] \arrow[dd] & & id_{\mathcal{DG}_{\mathcal{H}}}(G',B') \arrow[dd] & \\
 & B_1 \times B_1 \arrow[rr] \arrow[dl] & & B_1' \times B_1' \arrow[dl] \\
B \times B \arrow[rr] & & B' \times B' &
\end{tikzcd}
$$

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حول التكافؤ غير القانوني بين الفئات الزمروية الجزئية التفاضلية والحزم الليفية الاساسية

م.م محمد جبار حسين
قسم الرياضيات-كلية التربية الاساسية – الجامعة المستنصرية

المستخلص

الهدف الرئيسي من العمل هو ايجاد تكافؤ غير قانوني لفئة وتشاكلات الزمرة التفاضلية الناتجة عن فئة وتشاكل المنطوي الاملس ونرمز لها بالرمز DG_H والحزم الليفية الاساسية ونرمز لها بالرمز P .