On #RG-Compact spaces

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Abstract:

In this paper, we give a new type of compact spaces, and study some of their properties, namely #RG- compact space in topological spaces upon via the # rg-open set with modify some theorem of compact spaces.

Keywords

rg- open sets , rw-closed sets , # rg-open set and # rg-compact spaces

1- Introduction:

In 1906, Freched used for the first time the term of compactness in topological space . Asha Mathur [1] described compactness and weaker forms through a table containing 72 properties . M.E.Abd EL- Monsef and Kozae [3] introduced a property $P_{\alpha\beta_8}$ for generalized 1920 types of compactness and closeness. M.E.Abd EL-Monsef , A.E.Radwan , F.A.Ibrahem andA.I.Nasir [4] used the property $P_{\alpha\beta_8}$to generalized 15456 types of compactness and closeness .While, the concepts (#rg- closed sets , #rg-open sets, #RG- continuous functions and #RG- irresolute functions)were discussed and introduced by (S.A. Fathima and Mariasingam ,2012 , in [6],[7]) .

In this paper , we introduced a new type of compactness on topological spaces , namely #RG- compact , and we study some of their properties .
Throughout this paper (X,τ) and (Y,σ) (or simply X and Y) represent topological spaces and the family of all #rg-open (resp .#rg-closed) sets of a space (X,τ) denoted by # RGO(X,τ) (resp . # RGC(X,τ)) . For a subset A of a space X. cl(A), int (A) and A^c denoted the closure of A, the interior of A and the complement of A in X respectively.

2-Preliminaries:

Basic concepts and some definitions of this paper have been given in this section and we start by the following definitions.

Definition 2.1:

A subset A of a topological space (X,τ) is said to be :
1- regular open [3] if A = int(cl(A)) and regular closed if A = cl(int(A)) .
2- regular semi open set [2] if there is a regular open set U in (X,τ) such that

U⊆ A ⊆ cl(A).

Definition 2.2:

A subset A of a topological space (X,τ) is said to:

1- rw- closed set [8] if cl(A)⊆ U whenever A⊆ U and U is regular semi-open set in (X,τ) .The complement of rw –closed sets is called rw –open sets.

2- #rg- closed set [6] if cl(A)⊆ U whenever A⊆U and U is rw- open set in (X, τ) .

The complement of #rg -closed sets is called #rg-open sets.

Remark 2.3 [6]:

Every open (respectively closed) set in a topological space (X, τ) is #rg- closed (respectively # rg-open) set).

Example 2-4 [6] : Consider the topological space (X,τ),where X={a,b,c,d} andτ={X,ϕ,{a},{b},{c},{a,b},{a,b,c}}.Then**#RGO**(X,τ),τ={X,ϕ,{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c}}, the set A= {a,c} is #rg open set which is not open in (X,τ) and it is complements {b,d}is #rg –closed set which is not closed**.**

Definition 2.5, [7]:

A function f : (X ,τ) → (Y ,σ) is said to be

1- #rg- continuous if the inverse image of every closed set in(Y,σ) is #rg-closed set in (X,τ).

2- #rg- irresolute if the inverse image of every #rg-closed set in(Y,σ) is #rg-closed set in (X,τ).

3- On #RG- Compact Spaces :

In this section we give and study the concept of #RG- compactness. As we know. A family C= $\{V_\alpha : \alpha \in \Lambda\}$of subsets of X is said to be cover of X , if X ⊆ $\bigcup\{V_\alpha : \alpha \in \Lambda\}$,where V_α are open subsets of X for every $\alpha \in \Lambda$ and . Then C is called an open cover of X [1] .

Here we generalized this definition to get a new cover of X is called # rg- open cover of X and defined by:

Definition 3.1:

A family G_n of # rg- open subsets of a topological space (X,τ) is said to be # rg-open cover of X , if X ⊆ $\{G_n$: n∈Λ } .

Now, we used this cover to give the definition of # RG- compact space .

Definition 3.2:

A topological space (X,τ) is said to be #RG -compact space if every #rg –open cover of X has a finite subcover.

Remark 3.3:

It is clear that every #RG-compact spaces is compact , but the converse is not necessary to be true .

Example 3.4 :

Let X be the set of natural numbers and τ be indiscrete topology define on X .Then X is compact , but is not #rg-compact , Since {{n} : n∈ X} is #rg-open cover of X which has no finite subcover .

We know , a closed subset of compact space (X,τ) is compact, [4]. We modify this on # RG- compact space by the following theorem.

Theorem 3.5:

Let (X,τ) be a #RG-compact space, a # rg- closed subset of (X,τ) is # RG- compact set .

Proof: Let (X,τ) be a #RG - compact space , A⊆ X be a # rg-closed set and $\{G_n\}$ n∈∧ ,be # rg- open cover of A . Thus, A ⊆ ∪ G_n , also since A^c is # rg-open set in X, then X ⊆ ∪ G_n ∪ A^c . This means ∪ G_n ∪ A^c is a # rg – open cover of X, but X is a # RG- compact space. Hence, X has finite subcover, such that X ⊆ $G_1 \cup G_2 \cup . . . \cup G_n \cup A^c$ and $A \cap A^c = \phi$, so A ⊆ $G_1 \cup G_2 \cup . . . \cup G_n$ .Therefore, A is #RG - compact space.

Theorem 3.6:

If f : (X ,τ) →(Y ,σ) is a # rg-continuous function from #RG -compact space (X,τ) onto topological space (Y,σ) , then (Y,σ) is compact space .

Proof:

Let $\{A_\alpha : \in I\}$ be any open cover of (Y,σ) . Since f is a # rg-continuous function, $\{ f^{-1}(A_\alpha) : \alpha \in I \}$ is a # rg- open cover of X . By hypotheses X is # rg- compact space, thus X has finite sub cover $\{f^{-1}(A_{\alpha_1}), f^{-1}(A_{\alpha_2}) ,. . . f^{-1}(A_{\alpha_n}) \}$Which means there exists $\alpha_1, \alpha_2, . . . \alpha_n$, such that X =$\cup\{f^{-1}((A_{\alpha_i})): i = 1,2, \dots n\}$. Also, since f is onto, hence Y= f (X) =$\cup\{ f(f^{-1}((A_{\alpha_i})): i = 1,2, \dots n \}$ ==$\cup\{A_{\alpha_i} : i = 1,2, \dots n\}$. Therefore, Y is compact space.

Theorem 3.7:

If f : (X ,τ) →(Y ,σ) is a # rg-irresolute function from #RG -compact space (X,τ) onto topological space (Y,σ) , then (Y,σ) is #RG - compact space .

Proof:

Let $\{A_\alpha : \in I\}$ be # rg- open cover of (Y,σ) . Since f is a # rg-irresolute function, then $\{f^{-1} (A_\alpha) : \alpha \in I \}$ is a # rg- open cover of X . By hypotheses X is # rg- compact space , thus X has finite sub cover$\{f^{-1}(A_{\alpha_1}), f^{-1}(A_{\alpha_2}) ,. . . f^{-1}(A_{\alpha_n}) \}$. Which means there exists α_1, $\alpha_2, . . . \alpha_n$ such that X =$\cup\{f^{-1}((A_{\alpha_i})): i = 1,2, \dots n\}$ Also, since f is onto ,hence Y= f (X) = $\cup\{\ f(f^{-1}((A_{\alpha_i})): i = 1,2,\dots n\}$ = $\cup\{A_{\alpha_i} : i = 1,2,\dots n\}$. Therefore , Y is #RG - compact space .

Corollary 3.8 :

If f : (X ,τ) →(Y ,σ) is a # rg-irresolute function from #RG -compact space (X,τ) onto topological space (Y,σ) , then (Y,σ) is compact space .

Proof: Clear by using theorem (3-6) and Remark (3-3) .

Theorem 3.9 : If f : (X ,τ) →(Y ,σ) is a # rg-irresolute function and Fis a subset of (X, τ) . If F is a # rg- compact relative to X , then the image *f*(F) is a #RG - compact relative to Y.

Proof:

Let $\{A_\alpha : \in I\}$ be a collection of # rg- open sets in (Y,σ) , such that f (F) $\subseteq\cup\{A_\alpha : \alpha \in I\}$. Then F$\subseteq\{f^{-1}(A_\alpha) : \alpha \in \mathrm{I}\}$,where $f^{-1}(A_\alpha)$ is an # rg-open set in (X,τ) for each α∈I .By hypotheses F is a #RG - compact relative to X , then there exists$\{\mathrm{A}_1, A_2, \dots A_n\}$ such that F $\subseteq\{f^{-1}(A_\alpha) : \alpha = 1,2,\dots,\mathrm{n}\}$ .Thus *f*(F) $\subseteq\{ff^{-1}(A_\alpha) : \alpha = 1,2,\dots,\mathrm{n}\}$. Hence , *f*(F) $\subseteq\{(A_\alpha) : \alpha = 1,2,\dots,\mathrm{n}\}$. Therefore , f(F) is #RG-compact space relative to Y .

References :-

1-A. Mathur , Some Weaker Forms of Compactness,General Topolgy and it is Relations to Modern Analysis and Algebra ,. Proc . Kanpur Topological Conf.,(1968) , 181-191.

2-D.E. Cameron , Properties of S-Closed Spaces , .Proc. Amer. Math. Soc.72(1978) , 581-586.

3-M.E. Abd EL-Monsef and A. M.Kozae , Some Generalized Forms of Compactness and Closeness , Delta J.Sci., 9(2) (1985) , 257-269.

4-M.E. Abd EL- Monsef . A.E. Radwan , F.A. Ibrahem and A.I. Nasir , Some Generalized Forms of Compactness , International Mathematical Forum , Vol.7,2012,no.56,2767-2782.

5-M.H. Stone , Application Of The Theory Of Boolean Ring to General Topology, Trans, Amer. Math Soc.,41(1937),375-381.

6- S. A. Fathima , S and M. Mariasingam , On #regular Generalized Open Sets in Toploogical Spaces , Int . J . Comput . Appl ., 42(7), 2012,37-41 .

7- S. A. Fathima , S and M. Mariasingam , On #RG-Continuous and #RG-Irresolute Functions , Journal of Advanced Studies , Vol.3,No(4),2012, 28-33 .

8-S.S. Benchalli and R.S Wali , rw-Closed Sets in Topological Spaces , Bull. Malays .Math . Sci. Soc. (2)30(2), 2007,99-110 .

الفضاءات المتراصة من النمط #RG

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المستخلص:

في هذا البحث سنقدم نوعا جديدا من الفضاءات المتراصة , تسمى الفضاءات المتراصة من النمط -#RG في الفضاءات التبولوجية بالاعتماد على المجموعات المفتوحة-#rg مع تعميم بعض مبرهنات الفضاءات المتراصة.