New Techniques of Image Denoising using Multiwavelet by Neighbor Mapping

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Abstract

The Image denoising naturally corrupted by noise is a classical problem in the field of signal or image processing. Denoising of a natural images corrupted by Gaussian noise using multi-wavelet techniques are very effective because of its ability to capture the energy of a signal in few energy transfer values. Multi-wavelet can satisfy with symmetry and asymmetry which are very important characteristics in signal processing. The better denoising result depends on the degree of the noise. Generally, its energy is distributed over low frequency band while both its noise and details are distributed over high frequency band. Corresponding hard threshold used in different scale high frequency sub-bands. In this paper proposed to indicate the suitability of different wavelet and multi-wavelet based and a size of different neighborhood on the performance of image Denoising algorithm in terms of PSNR value. Finally it's compare wavelet and multi-wavelet techniques and produced best denoised image using neighbor mapping and multiwavelet technique based on the performance of image denoising algorithm in terms of PSNR Values.
Keywords: PSNR peak signal to noise ratio.

1. Introduction

This paper investigates the suitability of different wavelet bases and the size of different neighborhood on the performance of image de-noising algorithms in terms of PSNR. Over the past decade, wavelet transforms have received a lot of attention from researchers in many different areas. Both discrete and continuous wavelet transforms have shown great promise in such diverse fields as image compression, image de noising, signal processing, computer graphics, and pattern recognition to name only a few. In de-noising, single orthogonal wavelets with a single-mother wavelet function have played an important role. De-noising of natural images corrupted by Gaussian noise using wavelet techniques is very effective because of its ability to capture the energy of a signal in few energy transform values. Crudely, it states that the wavelet transform yields a large number of small coefficients and a small number of large coefficients.

The problem of Image de-noising can be summarized as follows. Let A(i,j) be the noise-free image and B(i,j)the image corrupted with independent Gaussian noise Z(i,j),

$$B(i,j)= A(i,j)+\sigma Z(i,j) \quad \cdots (1)$$

Where Z(i,j) has normal distribution N(0,1) . The problem is to estimate the desired signal as accurately as possible according to some criteria. In the wavelet domain, if an orthogonal wavelet transform is used, the problem can be formulated as

$$Y(i,j)= W(i,j)+N(i,j) \quad \cdots (2)$$

where Y(i,j) is noisy wavelet coefficient; W(i,j) is true coefficient and N(i,j) noise, which is independent Gaussian.

In multi-wavelet aspects, the symmetry and dissymmetry of the wavelet is rather important in signal processing. But single-wavelets with orthogonal intersection and compact-supporting are not symmetric except Harr. Recently, research on multi-wavelet is an active orientation. As multi-wavelet can satisfy both symmetry and asymmetry which are very important characters in signal processing. Multi-wavelet is commonly used in image compression, image de-noising, digital watermark and other signal processing field, so it is especially appropriate to processing complex images.

There are r compact-supporting scaling functions φ=(φ1, φ2,….φ r)T and they are inter-orthogonal with the wavelet functions ψ=(ψ1,ψ2,……ψr)T φr(t)(l=1,2,…r). The orthogonal basis of L2(R) space is 2j/2 ψr(2jt-k)(j, k∈Z,l=1,2,…,r). Hk,, Gk are the N*N matrix finite response filters with orthogonal basis, then the following specific equations can be obtained:

$$\Phi(t)= \sqrt{2} \sum_{k=-\infty} H_K\, \Phi(2t-k) \quad \cdots (3)$$

$$\psi(t)= \sqrt{2} \sum_{k=-\infty} G_K\, \psi(2t-k) \quad \cdots (4)$$

<u>2. Multiwavelet Transform</u>

In multiwavelet transform, we use multiwavelet as transform basis. Multiwavelet functions are functions generated from one single function ϕ by scaling and translation:

$$\psi_{a,b}(t)= \quad \cdots (5)$$

The mother wavelet $\psi(t)$ has to be zero integral, $\int \psi_{a,b}(t)dt = 0$. From (1) we see that high frequency multiwavelet correspond to $a > 1$ or narrow width, while low frequency multiwavelet corresponds to $a < 1$ or wider width. The basic idea of wavelet transform is to represent any function f as a linear superposition of wavelets. Any such superposition decomposes f to different scale levels, where each level can be then further decomposed with a resolution adapted to that level. One general way to do this is writing f as the sum of wavelets $\psi_{m,n}(t)$over m and n . This leads to discrete wavelet transform:

$$f(t) = \sum_{m,n} \psi_{m,n}(t) \quad \cdots (6)$$

By introducing the multi-resolution analysis (MRA) idea by Mallat [3], in discrete wavelet transform we really use two functions: wavelet function $\psi(t)$ and scaling function $\varphi(t)$. If we have a scaling function $\varphi(t) \in L^2(R)$, then the sequence of subspaces spanned by its scaling and translations $\psi_{j,k}(t)=2^{j/2} \varphi(2^j t - k)$, i.e

$$Vj = span\{ \varphi j,k (t),j,k \in Z\} \quad \cdots (7)$$

Constitute a MRA for $L^2(R)$.

$\varphi(t)$ must satisfy the MRA condition:

$$\varphi(t)=\sqrt{2}\sum h(n)\, \varphi(2t-2) \quad \cdots (8)$$

For $n \in Z$. In this manner, we can span the difference between spaces V_j by wavelet functions produced from mother wavelet: $\psi_{j,k}(t)=2^{j/2} \varphi(2^j t - k)$ Then we have:

$$\psi_{j,k}(t) = \sqrt{2}\sum g(n)\, \varphi(2t-2) \quad \cdots (9)$$

For orthogonal basis we have:

$$g(n) = (-1)^n h(-n+1) \quad \cdots (10)$$

If we want to find the projection of a function $f(t) \in L^2(R)$ on this set of subspaces, we must express it in e as a linear combination of expansion functions of that subspace [4]:

$$f(t) = \sum_{n=-\infty}^{\infty} c(t)\, \varphi(t) + \sum_{n=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} d(t)\, \psi_{j,k} \quad \cdots (11)$$

Where $\varphi_k(t)$ corresponds to the space V_0 and $\psi_{j,k}(t)$ corresponds to wavelet spaces.By using the idea of MRA implementation of wavelet decomposition can be performed using filter bank constructed by a pyramidal structure of lowpass filters $h(n)$ and highpass filters $g(n)$[3, 4].

The Multi-Wavelet Transform of image signals produces a non-redundant image representation, which provides better spatial and spectral localization of image formation, compared with other multi scale representations such as Gaussian and Laplacian pyramid. Recently, Multi-Wavelet Transform has attracted more and more interest in image de-noising.

Multi-wavelet iterates on the low-frequency components generated by the first decomposition. After scalar wavelet decomposition, the low-frequency components have only one sub-band, but after multiwavelet decomposition, the low-frequency components have four small sub-bands, one low-pass sub band and three band-pass sub bands. The next iteration continued to decompose the low frequency components L= {L1L1, L1L2,L2L1, L2L2}. In this situation, a structure of 5(4*J+1) subbands can be generated after J times decomposition, as shown in figure 1. The hierarchical relationship between every sub-band is shown in figure 2. Similar to single wavelet, multi-wavelet can be decomposed to 3 to 5 layers.

The Gaussian noise will nearby be averaged out in low frequency Wavelet coefficients. Therefore only the Multi-Wavelet coefficients in the high frequency level need to hard are threshold.

L_1L_1	L_1L_2	L_1H_1	L_1H_2
L_2L_1	L_2L_2	L_2H_1	L_2H_2
H_1L_1	H_1L_2	H_1H_1	H_1H_2
H_2L_1	H_2L_2	H_2H_1	H_2H_2

Figure (1): The structure of sub-band distribution

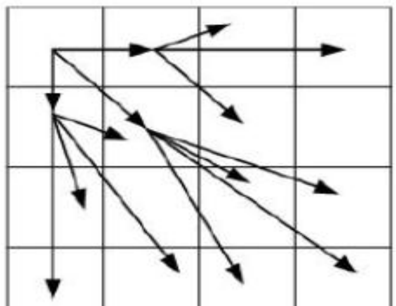


Figure (2): The hierarchical relationship between every sub-band

3. Proposed method for Image Neighbor

During experimentation, it was seen that when the noise content was high, the reconstructed image using Neighshrink contained mat like aberrations. These aberrations could be removed by wiener filtering the reconstructed image at the last stage of IDWT. The cost of additional filtering was slight reduction in sharpness of the reconstructed image. However, there was a slight improvement in the PSNR of the reconstructed image using wiener filtering. The de-noised image using Neighshrink sometimes unacceptably blurred and lost some details. The reason could be the suppression of too many detail wavelet coefficients. This problem will be avoided by reducing the value of threshold itself. So, the shrinkage factor is given by

B(i,j) = (1- (3/4)*T2/ S2 (i,j)).

The key of wavelet threshold in image de-noising is how to evaluate the coefficients. Although the methods of hard and soft threshold are used widely in practice, there are many faults in their nature. Hard threshold is to keep datum greater than the threshold, and all data less than the threshold are put to zero, the formula is as following:

$$A'_{j,k} = \begin{cases} A_{j,k} & \ldots\ldots \; |A_{j,k}| >= \\ 0 & \ldots\ldots\ldots \; |A_{j,k}| \end{cases} \qquad \cdots \; (12)$$

Where σ is threshold and Aj,k in formula (5) the wavelet coefficients. In hard threshold, Aj,k in formula (12) which are discontinuous at σ will bring some concussions and large mean-square deviation to the reconstructed signal.

4. Denoising Process for Multiwavelet

If the noised image

$$I(i,j)=X(i,j)+n(i,j)\ i,j=1,2,\ldots,N \qquad \cdots(13)$$

Where $n(i, j)$ is white Gaussian noise whose mean value is zero, σ is its variance, and $X(i,j)$ the original signal. The problem of de-noising can be thought as how to recover $X(i, j)$ from $I(i, j)$. Transform the formula (13) with multiwavelet, formula (14) is obtained

$$WI\ (i,j)= Wx(i,j)+ Wn\ (i,j) \qquad \cdots(14)$$

It is known from multi-wavelet transformation that, the multi-wavelet transformation of Gaussian noise is also Gaussian distributed, there are components at different scales, but energy distributes evenly in high frequency area, and the specific signal of the image has projecting section in every high frequency components.

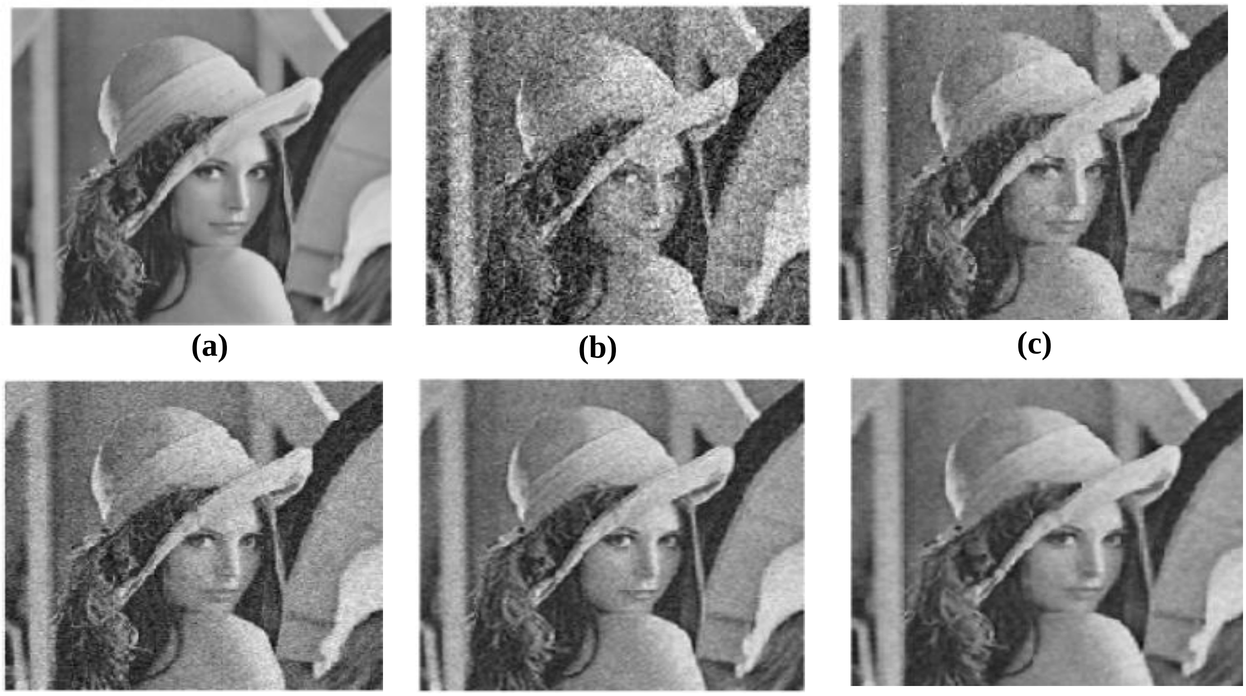
So image de-noising can be performed in high frequency area of multi-wavelet transformation.

5. Evaluation Criteria for Wavelet and Multiwavelet

The above said methods are evaluated using the quality measure Peak Signal to Noise ratio which is calculated using the formulae, PSNR= 10log 10 (255) 2/MSE (db) where MSE is the mean squared error between the original image and the reconstructed de-noised image. It is used to evaluate the different de-noising scheme like Wiener filter, Visushrink, Neighshrink, Modified Neighshrink and multi-wavelet.

6. Results and Discussion

For the above mentioned Wavelet and Multi-Wavelet methods, image de-noising is performed using wavelets from the second level to fourth level decomposition and the results are shown in figure (3) and table if formulated for second level decomposition for different noise variance as follows. It was found that three level decomposition and fourth level decomposition gave optimum results. However, third and fourth level decomposition resulted in more blurring. The experiments were done using a window size of 3X3, 5X5 and 7X7 for Multi-Wavelet. The neighborhood window of 3X3 and 5X5 are good choices.



(a) (b) (c)

(d) (e) (f)

Figure (3): Results of Various Image Denoising Methods
(a) Original Image,(b) Noisy Image,(c) Denoising image by Visushrink (d) Denoising Image by Neighshrink (e) Denoising Image by Mod.Nei (f) Denoising Image using Multiwavelet

Table (1): PSNR Values Comparison for Wavelet and Multiwavelet

Wavelet	Variance	0.02	0.04	0.06	0.08	0.02	0.04	0.06	0.08	0.02	0.04	0.06	0.08
	Noisy	16.86	14.10	12.64	11.67	16.83	14.09	12.67	11.68	16.84	14.10	12.64	11.65
	Wiener	24.05	21.34	19.94	19.02	26.41	24.1466	22.89	21.98	26.63	24.82	23.73	22.90
Harr	Visushrink	22.29	19.77	18.37	17.38	22.27	19.7681	18.37	17.431	22.28	19.80	18.33	17.40
	Neighshrink	24.57	23.30	22.29	21.54	24.58	23.2459	22.37	21.55	24.55	23.25	22.28	21.57
	Mod.Ne	25.96	25.01	24.12	23.40	25.96	24.9922	24.20	23.43	25.95	24.98	24.09	23.38
	Multiwavelet	26.87	26.04	25.24	24.96	27.18	26.73	26.12	25.11	27.34	26.877	25.87	25.00
db 16	Visushrink	22.62	20.00	18.45	17.53	22.61	19.97	18.47	17.50	22.61	19.97	18.50	17.53
	Neighshrink	23.36	22.38	21.59	21.01	23.35	22.41	21.61	21.04	23.36	22.35	21.62	21.02
	Mod.Ne	24.33	23.70	23.08	22.59	24.31	23.76	23.14	22.627	24.33	23.68	23.12	22.59
	Multiwavelet	25.41	24.94	24.60	24.01	25.45	24.94	23.14	22.627	24.33	23.68	23.12	22.59
Sym 8	Visushrink	22.60	19.97	18.50	17.47	22.56	19.95	18.51	17.517	22.60	19.98	18.45	17.49
	Neighshrink	23.42	22.50	21.65	21.11	23.46	22.48	21.73	21.05	23.41	22.48	21.62	21.04
	Mod.Ne	24.38	23.87	23.20	22.73	24.42	23.82	23.27	22.68	24.36	23.83	23.15	22.66
	Multiwavelet	25.13	25.14	24.47	23.94	25.41	24.94	24.96	25.13	25.26	24.97	24.56	23.98
Coif 5	Visushrink	22.56	19.93	18.50	17.50	22.61	19.98	18.45	17.49	22.61	19.91	18.48	17.49

	Neighshrink	26.07	24.27	23.18	22.22	26.03	24.32	23.08	22.28	26.06	24.27	23.12	22.26
	Mod.Ne	27.27	26.00	25.01	25.01	27.27	26.01	24.92	24.16	27.29	25.98	24.99	24.15

7. Conclusion

In this paper, the image de-noising using Discrete Wavelet Transform and Multi-Wavelet transform is analyzed. The experiments were conduced to study the suitability of different wavelet and multi-wavelet bases and also different window sizes. Experimental Results also show that multi-wavelet with hard threshold gives better result than Modified Neighshrink, Neighshrink, Weiner filter and Visushrink.

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الخلاصة:

عادةً معالجة الصورة الطبيعية التي أفسدت بسبب نوع من انواع التشويه و التي تدعى بضوضاء الصادر عن المصدر والتي تسخدم تقنية ناقل المويجات تكون فعالة جداً بسبب قدرة الطريقة لأَسْر طاقةِ الاشارة، خاصتةً من خلال نقل طاقة الاشارة عن طريق ناقل متعدد المويجات المتقطع والذي ممكن ان يحدث من خلال التناظر او عدم التناظر و التي تعتبر من الخصائص المهمة جدا في معالجة الاشارات الرقمية. تعتمد طريقة الغاء الضوضاء على الطاقة الموزعة في الجزء الذي يكون ذات التردد الواطيء بينما تفاصيل الصورة تكون موزعة او في جزء ذات التردد العالي. العتبة الصعبة المطابقة استعملت في تذبذب المقياس العالي المختلف في الاجزاء المختلفة من الصورة. في هذا البحث تم اقتراح طريقة جديدة لاستعمال ناقل متعدد المويجات لمعالجة الاشارات مختلفة استندت على الحجم الحي في عملية الغاء الضوضاء و الخوارزمية اعتمدت على قيمة الاشارة النقية بالنسبة الى الضوضاء. اخيراً يتم مقارنة ناقل متعدد المويجات و تقنية ناقل المويجات و التي تم الاستناد على الغاء الضوضاء في الصورة من خلال مقارنة قيم ال بي إس إن آر وهي نسبة الاشارة النقية في الصورة الى الاشارة المشوهه.