On Fuzzy Derivation Of Fuzzy Near-ring

Sameer Qasim Hassan
Mahdi Saleh Nayef
Department of Mathematics
College of Education Al-Mustansirya University

Abstract

Let $\tilde{R}$ be a fuzzy subnear-ring of a ring R, $\tilde{U}$ a semifuzzy left ideal of R and d is additive map on R . The purpose of this paper is to prove if $\tilde{R}$ admits a fuzzy derivation d which is fuzzy centralizing on $\tilde{U}$, then $\tilde{V}$ is a fuzzy commutative ideal of R.

1.Introduction

There are many researchers are engaged in extending the concept of near-ring and derivation such as [6], [7]. A fuzzy near-ring with its properties and with a fuzzy ideal has been discussed in [1],[3],[5],[7],[8].

In this paper we given new results due applied a fuzzy derivation as new definition on a fuzzy near-ring .

A non-empty set R with two binary operation '+' and '.' is called a near-ring [2] if

(1) (R,+) is a group,
(2) (R,.) is a semigroup,
(3) x.(y+z)= x.y=x.z for all x,y,z $\in$ R.

We will use the word ' near- ring ' to mean a ' left near-ring'. We denote xy instead of x. y . Note that x0=0 and x(-y)=-xbut in general $0x \neq 0$ for some $x \in R$. An ideal I of near –ring R is a subset of R such that

(4) (I,+) is a normal subgroup of (R,+),
(5) RI $\subseteq$ I,
(6) (r+i)s – rs $\in$ I for any i $\in$ I and any r,s $\in$ R. Note that I is a left ideal of R if I satisfies (4) and (5), and I is a right ideal of R if I satisfies (4) and (6).

Definition(1,2)[4], [9]:

Let X be a nonempty set, a fuzzy set Ã in X is characterized by its membership function $\mu_{\tilde{A}} : X \to J$ where J is the closed unite interval [0,1], and we write a fuzzy set by the set of points $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X , 0 \leq \mu_{\tilde{A}}(x) \leq 1\}$ and where $\mu_{\tilde{A}}(x)$ is called a fuzzy relationship function.

Definition(1,3),[7],[8]:

Let R be a near-ring and $\tilde{R}$ be a fuzzy subset of R. We say $\tilde{R}$ a fuzzy subnear- ring of R if

(7) $\mu_{\tilde{R}}(x-y) \geq \min\{\mu_{\tilde{R}}(x), \mu_{\tilde{R}}(y)\}$,

(8) $\mu_{\tilde{R}}(x\ y) \geq \min\{\mu_{\tilde{R}}(x), \mu_{\tilde{R}}(y)\}$, for all $x, y \in$R. $\tilde{I}$ is called a fuzzy ideal of R if $\tilde{I}$ is a fuzzy subnear-ring of R and

(9) $\mu_{\tilde{I}}(x) = \mu_{\tilde{I}}(y+x-y)$,

(10) $\mu_{\tilde{I}}(x\ y) \geq \mu_{\tilde{I}}(y)$,

((11) $\mu_{\tilde{I}}((x+i)\ y - x\ y) \geq \mu_{\tilde{I}}(i)$, for any $x, y, i \in R$. Note that $\tilde{I}$ is a fuzzy left ideal (respect semi fuzzy left ideal) of R if it satisfies (7), (9) and (10) (resp. (7),(8) and

$\mu_{\tilde{I}}(x\ y) = \mu_{\tilde{I}}(y)$ for all $x, y \in$R) , and $\tilde{I}$ is a fuzzy right ideal of R if it satisfies (7), (8) , (9) and (11).

We give now some examples of fuzzy ideals of near-rings.

Example(1,1) ,[7]:

Let R:={ a,b,c,d} be a set with two binary operations as follows :

+	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

.	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	a
d	a	a	b	b

Then we can easily see that (R,+,.) is a (left) near-ring . Define a fuzzy subset $\mu_{\tilde{I}} : R \to [0,1]$ by $\mu_{\tilde{I}}(c) = \mu_{\tilde{I}}(d) < \mu_{\tilde{I}}(b) < \mu_{\tilde{I}}(a)$. Then $\tilde{I}$ is a fuzzy ideal of R.

Also see the following example.

Example(1,2), [7]:

Let R:={ a,b,c,d} be a set with two binary operations as follows :

+	a	b	c	d

a	*a*	*b*	*c*	*d*
b	*b*	*a*	*d*	*c*
c	*c*	*d*	*b*	*a*
d	*d*	*c*	*a*	*b*

.	*a*	*b*	*c*	*d*
a	*a*	*a*	*a*	*a*
b	*a*	*a*	*a*	*a*
c	*a*	*a*	*a*	*a*
d	*a*	*b*	*c*	*b*

Then we can easily see that (R,+,.) is a (left) near-ring . Define a fuzzy subset $\mu_{\tilde{I}} : R \to [0,1]$ by $\mu_{\tilde{I}}(c) = \mu_{\tilde{I}}(d) < \mu_{\tilde{I}}(b) < \mu_{\tilde{I}}(a)$. Then $\tilde{I}$ is a fuzzy ideal of R, but not fuzzy right ideal of R, since $\mu_{\tilde{I}}((c+b)d - cd) = \mu_{\tilde{I}}(d) < \mu_{\tilde{I}}(b)$.

We denote $\tilde{R}$ as a fuzzy near- ring and we define its fuzzy center as $C(\tilde{R}) = \{x \in C(R) : xy = yx$ for all $y \in R$ and $\mu_{\tilde{R}}(x) \in \mu_{C(\tilde{R})}\}$, also we denoted the fuzzy commutater by ($\mu_{\tilde{R}}([x,y]) = \mu_{\tilde{R}}(xy - yx)$ and
In particular if $[x,y] = 0$, implies that $\mu_{\tilde{R}}([x,y]) = \mu_{\tilde{R}}(0)$ and $\mu_{\tilde{R}}(xy) = \mu_{\tilde{R}}(yx)$ for any elements $x, y \in R$. Also
$\mu_{\tilde{R}}([x, y z]) = \mu_{\tilde{R}}(y[x, z] + [x, y]z))$ for any elements $x, y, z \in R$ . .

<u>Lemma(1,1),[7]</u> :-

If a fuzzy subset $\tilde{A}$ of R satisfies the property (7) then
$(i)\ \mu_{\tilde{A}}(0) \geq \mu_{\tilde{A}}(x)$
$(ii)\ \mu_{\tilde{A}}(-x) \geq \mu_{\tilde{A}}(x)$ for any $x, y \in R$.

<u>Lemma(1,2),[8]</u>:

Let $\tilde{G}$ be a fuzzy subgroup of a group G and $x \in G$. Then $\mu_{\tilde{G}}(xy) = \mu_{\tilde{G}}(y)$ for every $y \in G$ if and only if $\mu_{\tilde{G}}(x) = \mu_{\tilde{G}}(0)$.

2.<u>Main results</u>

<u>Definition(2,4)</u>:

$\tilde{I}$ is called fuzzy commutative ideal if it is fuzzy ideal and $\mu_{\tilde{I}}(xy) = \mu_{\tilde{I}}(yx)$ for $x, y \in R$.

<u>Definition(2,5)</u>:

Let $d : R \to R$ be additive map and $\tilde{R}$ is a fuzzy subnear-ring with membership function $\mu_{\tilde{R}}$. Then d($\tilde{R}$) is a fuzzy set with membership function defined by $\mu_d : R \to$ [,

$$\mu_d(y) = \begin{cases} \max\{\mu_R(x) : x \in d^{-1}(y)\} & if\ d^{-1}(y) \neq \phi \\ 0 & otherwise \end{cases}$$ then d is a fuzzy derivation on $\tilde{R}$ if satisfy the following

i) d(xy)=xd(y)+d(x)y.

ii) $\mu_d(d(xy)) = \max\{\min\{\mu_{\tilde{R}}(x), \mu_d(d(y))\}, \min\{\mu_d(d(x)), \mu_{\tilde{R}}(y)\}\}\ \forall x, y \in R$.

Example(2.1)

Let R={0,1,2,3} be a set with two binary operations as follows:

+	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	1	0
3	3	2	0	1

.	0	1	2	3
0	0	0	0	0
1	0	0	0	0
2	0	0	0	0
3	0	0	1	1

It is easy to see that R is near ring . Define a fuzzy subset $\mu_{\tilde{I}} : R \to [0,1]$ by $\mu(2) = \mu(3) < \mu(1) = \mu(0)$ .We can insure that $\tilde{I}$ is a fuzzy near ring .Let now define a nonzero derivation on M={$\begin{pmatrix} x & y \\ z & w \end{pmatrix}$: $x, y, z, w \in R\}$ as follows:

$d(\begin{pmatrix} x & y \\ z & w \end{pmatrix}) = \begin{pmatrix} 0 & -y \\ z & 0 \end{pmatrix}$, also μ_d :M$\to [0,1]$, for any two elements $\begin{pmatrix} x_1 & y_1 \\ z_1 & w_1 \end{pmatrix}, \begin{pmatrix} x_2 & y_2 \\ z_2 & w_2 \end{pmatrix} \in$ M , with following conditions :

$\mu_d(0) = \mu_{\tilde{I}}(x_2) = \mu_{\tilde{I}}(x_1) = \mu_{\tilde{I}}(w_2) = \mu_{\tilde{I}}(w_1)$,

$\min\{\mu_d(-y_1), \mu_{\tilde{I}}(w_2)\} > \mu_{\tilde{I}}(y_2)$, $\min\{\mu_{\tilde{I}}(x_1), \mu_d(-y_2)\} > \mu_{\tilde{I}}(y_1)$,

$\min\{\mu_d(z_1), \mu_{\tilde{I}}(x_2)\} > \mu_{\tilde{I}}(z_2)$ and $\min\{\mu_{\tilde{I}}(w_1), \mu_d(z_2)\} > \mu_{\tilde{I}}(z_1)$.

also $\mu_{\tilde{I}}(w_2) = \mu_d(w_2)$, $\mu_{\tilde{I}}(x_1) = \mu_d(x_1)$, $\mu_{\tilde{I}}(x_2) = \mu_d(x_2)$ and $\mu_{\tilde{I}}(w_1) = \mu_d(w_1)$.

we have that

$$\mu_d\left(d\left(\begin{pmatrix} x_1 & y_1 \\ z_1 & w_1 \end{pmatrix}\begin{pmatrix} x_2 & y_2 \\ z_2 & w_2 \end{pmatrix}\right)\right) = \mu_d\left(d\left(\begin{pmatrix} x_1x_2 + y_1z_2 & x_1y_2 + y_1w_2 \\ z_1x_2 + w_1z_2 & z_1y_2 + w_1w_2 \end{pmatrix}\right)\right)$$

$$= \mu_d\left(d\left(\begin{pmatrix} x_1x_2 + y_1z_2 & x_1y_2 + y_1w_2 \\ z_1x_2 + w_1z_2 & z_1y_2 + w_1w_2 \end{pmatrix}\right)\right) = \mu_d\left(\begin{pmatrix} 0 & -(x_1y_2 + y_1w_2) \\ z_1x_2 + w_1z_2 & 0 \end{pmatrix}\right)$$

$$= \begin{pmatrix} \mu_d(0) & \mu_d(-(x_1y_2 + y_1w_2)) \\ \mu_d(z_1x_2 + w_1z_2) & \mu_d(0) \end{pmatrix} =$$

$$\begin{pmatrix} \mu_d(0) & \max\{\min\{\mu_d(x_1), \mu_d(-y_2)\}, \min\{\mu_d(-y_1), \mu_d(w_2)\}\} \\ \max\{\min\{\mu_d(z_1), \mu_d(x_2)\}, \min\{\mu_d(w_1), \mu_d(z_2)\}\} & \mu_d(0) \end{pmatrix}$$

From other side , we have that

$$\mu\left(d\left(\begin{pmatrix} x_1 & y_1 \\ z_1 & w_1 \end{pmatrix}\right).\begin{pmatrix} x_2 & y_2 \\ z_2 & w_2 \end{pmatrix} + \begin{pmatrix} x_1 & y_1 \\ z_1 & w_1 \end{pmatrix}.d\left(\begin{pmatrix} x_2 & y_2 \\ z_2 & w_2 \end{pmatrix}\right)\right)$$

$$= \begin{pmatrix} \mu_d(0) & \mu_d(-y_1) \\ \mu_d(z_1) & \mu_d(0) \end{pmatrix}\begin{pmatrix} \mu_{\tilde{I}}(x_2) & \mu_{\tilde{I}}(y_2) \\ \mu_{\tilde{I}}(z_2) & \mu_{\tilde{I}}(w_2) \end{pmatrix} + \begin{pmatrix} \mu_{\tilde{I}}(x_1) & \mu_{\tilde{I}}(y_1) \\ \mu_{\tilde{I}}(z_1) & \mu_{\tilde{I}}(w_1) \end{pmatrix}\begin{pmatrix} \mu_d(0) & \mu_d(-y_2) \\ \mu_d(z_2) & \mu_d(0) \end{pmatrix}$$

$=$

$$\begin{pmatrix} \max\{\min\{\mu_d(0), \mu_{\tilde{I}}(x_2)\}, \min\{\mu_d(-y_1), \mu_{\tilde{I}}(z_2)\}\} & \max\{\min\{\mu_d(0), \mu_{\tilde{I}}(y_2)\}, \min\{\mu_d(-y_1), \mu_{\tilde{I}}(w_2)\}\} \\ \max\{\min\{\mu_d(z_1), \mu_{\tilde{I}}(x_2)\}, \min\{\mu_d(0), \mu_{\tilde{I}}(z_2)\}\} & \max\{\min\{\mu_d(z_1), \mu_{\tilde{I}}(y_2)\}, \min\{\mu_d(0), \mu_{\tilde{I}}(w_2)\}\} \end{pmatrix}$$

$$+ \begin{pmatrix} \max\{\min\{\mu_{\tilde{I}}(x_1), \mu_d(0)\}, \min\{\mu_{\tilde{i}}(y_1), \mu_d(z_2)\}\} & \max\{\min\{\mu_{\tilde{I}}(x_1), \mu_d(-y_2)\}, \min\{\mu_{\tilde{I}}(y_1), \mu_d(0)\}\} \\ \max\{\min\{\mu_{\tilde{I}}(z_1), \mu_d(0)\}, \min\{\mu_{\tilde{I}}(w_1), \mu_d(z_2)\}\} & \max\{\min\{\mu_{\tilde{I}}(z_1), \mu_d(-y_2)\}, \min\{\mu_{\tilde{I}}(w_1), \mu_d(0)\}\} \end{pmatrix}$$

Since $\mu_d(0) = \mu_{\tilde{I}}(x_2) = \mu_{\tilde{I}}(x_1) = \mu_{\tilde{I}}(w_2) = \mu_{\tilde{I}}(w_1)$, $\min\{\mu_d(-y_1), \mu_{\tilde{I}}(w_2)\} > \mu_{\tilde{I}}(y_2)$, $\min\{\mu_{\tilde{I}}(x_1), \mu_d(-y_2)\} > \mu_{\tilde{I}}(y_1)$, $\min\{\mu_d(z_1), \mu_{\tilde{I}}(x_2)\} > \mu_{\tilde{I}}(z_2)$ and $\min\{\mu_{\tilde{I}}(w_1), \mu_d(z_2)\} > \mu_{\tilde{I}}(z_1)$, we get

$$=\max\left(\begin{pmatrix} \max\{\mu_d(0), \min\{\mu_d(-y_1), \mu_{\tilde{I}}(z_2)\}\} & \min\{\mu_d(-y_1), \mu_{\tilde{I}}(w_2)\} \\ \min\{\mu_d(z_1), \mu_{\tilde{I}}(x_2)\} & \max\{\min\{\mu_d(z_1), \mu_{\tilde{I}}(y_2)\}, \mu_d(0)\} \end{pmatrix}\right.$$

$$\left., \begin{pmatrix} \max\{\mu_d(0), \min\{\mu_{\tilde{i}}(y_1), \mu_d(z_2)\}\} & \min\{\mu_{\tilde{I}}(x_1), \mu_d(-y_2)\} \\ \min\{\mu_{\tilde{I}}(w_1), \mu_d(z_2)\} & \max\{\min\{\mu_{\tilde{I}}(z_1), \mu_d(-y_2)\}, \mu_d(0)\} \end{pmatrix}\right)$$

Since $\mu_{\tilde{I}}(w_2) = \mu_d(w_2)$, $\mu_{\tilde{I}}(x_1) = \mu_d(x_1)$, $\mu_{\tilde{I}}(x_2) = \mu_d(x_2)$ and $\mu_{\tilde{I}}(w_1) = \mu_d(w_1)$.

Then the final fuzzy matrix is

$$\begin{pmatrix} \mu_d(0) & \max\{\min\{\mu_d(x_1), \mu_d(-y_2)\}, \min\{\mu_d(-y_1), \mu_d(w_2)\}\} \\ \max\{\min\{\mu_d(z_1), \mu_d(x_2)\}, \min\{\mu_d(w_1), \mu_d(z_2)\}\} & \mu_d(0) \end{pmatrix}$$

Hence,

$$\mu_d\left(d\left(\begin{pmatrix} x_1 & y_1 \\ z_1 & w_1 \end{pmatrix}\begin{pmatrix} x_2 & y_2 \\ z_2 & w_2 \end{pmatrix}\right)\right) = \mu_d\left(d\left(\begin{pmatrix} x_1 & y_1 \\ z_1 & w_1 \end{pmatrix}\right).\begin{pmatrix} x_2 & y_2 \\ z_2 & w_2 \end{pmatrix} + \begin{pmatrix} x_1 & y_1 \\ z_1 & w_1 \end{pmatrix}.d\left(\begin{pmatrix} x_2 & y_2 \\ z_2 & w_2 \end{pmatrix}\right)\right)$$

Proposition(2,1):

let $\tilde{A}$ be a fuzzy subset of R and $d: R \to R$ be additive map .

if $\mu_{\tilde{A}}([x, d(x)]) = \mu_{\tilde{A}}(0)$ then

$i)\ \mu_{\tilde{A}}(xd(x)) = \mu_{\tilde{A}}(d(x)x)$ (1)

$ii)\ \mu_{\tilde{A}}([x,[x,d(x)]]) \geq \mu_{\tilde{A}}(x)$. (2)

poof

Assume that $\mu_{\tilde{A}}([x, d(x)]) = \mu_{\tilde{A}}(0)$,for all $x \in R$.

Then

$i)\ \mu_{\tilde{A}}(xd(x)) = \mu_{\tilde{A}}(xd(x) - d(x)x + d(x)x)$ from properties (7),(8), we get

$\mu_{\tilde{A}}(xd(x)) \geq \min \{\mu_{\tilde{A}}(xd(x) - d(x)x), \mu_{\tilde{A}}(d(x)x)\}$

$= \min \{\mu_{\tilde{A}}([x, d(x)]), \mu_{\tilde{A}}(d(x)x\}$

$= \min \{\mu_{\tilde{A}}(0), \mu_{\tilde{A}}(d(x)x)\}$

$= \mu_{\tilde{A}}(d(x)x).$

Similarly, using $\mu_{\tilde{A}}(d(x)x - xd(x)) = \mu_{\tilde{A}}(xd(x) - d(x)x) = \mu_{\tilde{A}}(0),$,we get

$\mu_{\tilde{A}}(xd(x)) \geq \mu_{\tilde{A}}(d(x)x)$.

Hence $\mu_{\tilde{A}}(d(x)x) = \mu_{\tilde{A}}(xd(x))$.

$ii)\ \mu_{\tilde{A}}([[x, d(x)], x] \geq \min\{\mu_{\tilde{A}}([x, d(x)]x, \mu_{\tilde{A}}(x[x, d(x)]\}$

$\geq \min\{\min\{\mu_{\tilde{A}}([x, d(x)], \mu_{\tilde{A}}(x)\}, \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}([x, d(x)]\}\}$

$\geq \min\{\min\{\mu_{\tilde{A}}(0), \mu_{\tilde{A}}(x)\}, \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(0)\}\}$

$\geq \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(x)\}$

$= \mu_{\tilde{A}}(x)$

Hence

$\mu_{\tilde{A}}([[x, d(x)], x]) \geq \mu_{\tilde{A}}(x)$ for any $x \in R$.

Corollary(2,1):

let $\tilde{A}$ be a fuzzy subset of R and $d: R \to R$ be additive map . If $\mu_{\tilde{A}}([x, d(x)]) = 0$ then $\mu_{\tilde{A}}(xd(x)) = \mu_{\tilde{A}}(d(x)x) = 0$.

Theorem(2,1):

Let d be derivation admits on a near- ring R , and $\tilde{R}$ a fuzzy subnear-ring of R

.If $y \in d^{-1}d(x)$ for $x, y \in R$, $x \in d^{-1}d(y)$ and $\mu_d(d(x)) = \sup_{t \in d^{-1}(d(x))} \mu_{\tilde{R}}(t)$ then

$\mu_{\tilde{R}}(d(xy)) \geq \min\{\mu_{\tilde{R}}(x), \mu_{\tilde{R}}(y)\}$.

Proof

Let d be a derivation on a near- ring R and , and $\tilde{R}$ fuzzy subnear-ring of R . By definition(2.5),we have that

$\mu_d(d(xy)) = \mu_{\tilde{R}}(xd(y) + d(x)y)$, then

by definition(1.3) (7) and definition(2.5), we obtain

$$\mu_d(d(xy)) \geq \min \{\mu_{\tilde{R}}(d(x)y), \mu_{\tilde{R}}(xd(y))\}$$
$$\geq \min \{\min\{\mu_d(d(x)), \mu_{\tilde{R}}(y)\}, \min\{\mu_{\tilde{R}}(x), \mu_d(d(y))\}\}$$
$$= \min\left\{\min\left\{\sup_{t\in d^{-1}(d(x))} \mu_{\tilde{R}}(t), \mu_{\tilde{R}}(y)\right\}, \min\left\{\mu_{\tilde{R}}(x), \sup_{t\in d^{-1}(d(y))} \mu_{\tilde{R}}(t)\right\}\right\}$$

From $y \in d^{-1}d(x)$ and $x \in d^{-1}d(y)$ for $x, y \in R$, we obtain

$$\min\left\{\min\left\{\sup_{t\in d^{-1}(d(x))} \mu_{\tilde{R}}(t), \mu_{\tilde{R}}(y)\right\}, \min\left\{\mu_{\tilde{R}}(x), \sup_{t\in d^{-1}(d(y))} \mu_{\tilde{R}}(t)\right\}\right\}$$
$$\geq \min\{\min\{\mu_{\tilde{R}}(x_0), \mu_{\tilde{R}}(y)\}, \min\{\mu_{\tilde{R}}(x), \mu_{\tilde{R}}(y_0)\}\} \quad \text{for some } x_0 \text{ , } y_0 \in R$$
$$\geq \min\{\mu_{\tilde{R}}(x), \mu_{\tilde{R}}(y)\}.$$

Hence

$$\mu_d(d(xy)) \geq \min\{\mu_{\tilde{R}}(x), \mu_{\tilde{R}}(y)\}$$

Lemma(2,3):

Let $\tilde{R}$ be a fuzzy subnear ring of R and d is derivation of a near- ring R then for any $x, y \in R$

$(i)\mu_{\tilde{R}}(0) \geq \mu_{\tilde{R}}(d(x)x)$

$(ii)\mu_{\tilde{R}}(-xd(x)) \geq \mu_{\tilde{R}}(xd(x))$

$(iii)\mu_{\tilde{R}}(d(x-y)) \geq \min\{\mu_d(d(x)), \mu_d(d(y))\}$

$(v)\mu_{\tilde{R}}((d(x)x + xd(x)) \geq \min\{\mu_{\tilde{R}}(x), \mu_d(d(x))\}$

Proof

(i) we have that for any $x \in R$

$\mu_{\tilde{R}}(0) = \mu_{\tilde{R}}(d(x)x - d(x)x)$. By (7), we get

$$\mu_{\tilde{R}}(-xd(x)) = \mu_{\tilde{R}}(0 - xd(x)) \geq \min\{\mu_{\tilde{R}}(0), \mu_{\tilde{R}}(xd(x))\} = \mu_{\tilde{R}}(xd(x))$$

For all $x \in R$.since x is arbitrary ,we conclude that

$\mu_{\tilde{R}}(-xd(x)) \geq \mu_{\tilde{R}}(xd(x))$

(iii) Since d is additive map ,thus

$$\mu_{\tilde{R}}(d(x-y)) = \mu_{\tilde{R}}(d(x) - d(y)) \geq \min\{\mu_d(d(x)), \mu_d(d(y))\}$$

(v) It is clear.

Theorem(2,2):-

Let $\tilde{R}$ be a fuzzy near ring and $\tilde{U}$ is a nonzero semifuzzy left ideal .If $\tilde{R}$ admits a fuzzy derivation d which is nonzero on $\tilde{U}$ such that $\mu_{\tilde{U}}([x,d(x)]) \in \mu_{C(\tilde{R})}$, then $\tilde{V}$ is a nonzero fuzzy commutative ideal of R.

In order to prove the theorem we need the following lemma, which shows under certain conditions a fuzzy centralizing derivation is equal $\mu(0)$.

Lemma(2,4):-

Let $\tilde{R}$ b ea fuzzy near ring and $\tilde{U}$ is a semifuzzy left ideal of $\tilde{R}$, if d is a fuzzy derivation of R which is $\mu_{\tilde{U}}([x,d(x)]) \in \mu_{C(\tilde{R})}$.then $\mu_{\tilde{U}}([x,d(x)]) = \mu_{\tilde{U}}(0), \forall x, y \in U$..

proof

since d is a fuzzy derivation of $\tilde{R}$ which is centralizing on $\tilde{U}$ .Thus

$\mu_{\tilde{U}}([x,d(x)]) \in \mu_{C(\tilde{R})}$

(3)

Replacing x be x^2 in (3) , we get :

$\mu_{\tilde{U}}([x^2,d(x^2)])$

$= \mu_{\tilde{U}}([x^2,xd(x)+d(x)x])$

$= \mu_{\tilde{U}}(4x^2[x,d(x)]) \in \mu_{C(\tilde{R})}$

(4)

Commuting last equation with $d(x)$,and since (4) is in $C(\tilde{R})$, yield

$\mu_{\tilde{U}}(4[x^2[x,\tilde{d}(x)],d(x)]=0)$

(5)

Thus $\mu_{\tilde{U}}(8x[x,d(x)]^3) = \mu_{\tilde{R}}(0)$

(6)

Therefore

$\mu_{\tilde{U}}(8x[x,d(x)]^2) = \mu_{\tilde{R}}(0) = \mu_{\tilde{U}}(8x[x,d(x)][x,d(x)])$

Since $\tilde{U}$ is semifuzzy left ideal , we have

$\mu_{\tilde{U}}([x,d(x)]) = \mu_{\tilde{R}}(0)$.

Definition(2.6):-

$\tilde{I}$ is a fuzzy Maximal ideal in a near ring R if there is no fuzzy ideal $\tilde{J}$ in R such that $\mu_{\tilde{I}}(x) \le \mu_{\tilde{J}}(y)$ for $x, y \in R.$

Lemma(2,5):-

Any fuzzy near ring $\tilde{R}$ has a fuzzy family $\tilde{\Omega} = \{\tilde{P}_\alpha / \alpha \in \lambda\}$ of fuzzy maximal ideal such that $\mu_{\bigcap_{\alpha\in\lambda}\tilde{P}_\alpha}(x)=\mu_{\tilde{R}}(0)$ for any $x\in R$.

proof

Let $(a_1,\mu_{\tilde{R}}(a_1))=(a_0^2,\mu_{\tilde{R}}(a_0^2))$ be anon zero fuzzy element of the fuzzy near ring $\tilde{R}$,also $(a_2,\mu_{\tilde{R}}(a_2))=(a_1^2,\mu_{\tilde{R}}(a_1^2))$,so continuing this process, we get a countable sequence of nonzero fuzzy elements $(a_0,\mu_{\tilde{R}}(a_0)),(a_1,\mu_{\tilde{R}}(a_1)),(a_2,\mu_{\tilde{R}}(a_2))...(a_{n+1},\mu_{\tilde{R}}(a_{n+1}))=(a_n,\mu_{\tilde{R}}(a_n))$.

let $\tilde{M}$ be the set of all fuzzy ideals of $\tilde{R}$ that contain no elements of this sequence . $\tilde{M}$ is not empty since the zero fuzzy ideal is an element in $\tilde{M}$ .By Zorn' s lemma the set $\tilde{M}$ contains a maximal fuzzy element say $\tilde{p}_{a_0}$,the ideal dose not intersect the sequence $(a_0,\mu_{\tilde{R}}(a_0)),(a_1,\mu_{\tilde{R}}(a_1)),(a_2,\mu_{\tilde{R}}(a_2))..............(a_n,\mu_{\tilde{R}}(a_n))$

but any fuzzy ideal containing $\tilde{p}_a$ has a nonempty intersection with this sequence .Now since $(a,\mu_{\tilde{R}}(a))\notin\tilde{p}_a$,then $\mu_{\bigcap_{0\neq a\in R}\tilde{P}_a}(x)=\mu_{\tilde{R}}(0)$ for any $x\in R$.

Proof of theorem (2,2)

By lemma (2.4), we have that

$\mu_{\tilde{U}}([x,d(x)])=\mu_{\tilde{R}}(0)$,replacing x by $x+y$, we obtain

$$\mu_{\tilde{U}}([x+y,d(x+y)])=\mu_{\tilde{R}}(0)$$

$$\mu_{\tilde{U}}([x,d(x+y)]+[y,d(x+y)])=\mu_{\tilde{R}}(0)$$

$$\mu_{\tilde{U}}([x,d(x)]+[x,d(y)]+[y,d(x)]+[y,d(y)])=\mu_{\tilde{R}}(0)$$

From (7),we obtain

$\mu_{\tilde{R}}(0)\geq\min\{\mu_{\tilde{U}}([x,d(x)]),\mu_{\tilde{U}}([x,d(y)]),\mu_{\tilde{U}}([y,d(x)]),\mu_{\tilde{U}}([y,d(y)])\}$ for $x,y\in U$.

Also from lemma(2.4) ,we get

$$\mu_{\tilde{R}}(0)\geq\min\{\mu_{\tilde{R}}(0),\mu_{\tilde{U}}([x,d(y)]),\mu_{\tilde{U}}([y,d(x)])\}$$

From lemma(1.1),we have that

$$\mu_{\tilde{R}}(0)\geq\min\{\mu_{\tilde{U}}([x,d(y)]),\mu_{\tilde{U}}([y,d(x)])\} \quad (7)$$

Replacing y by yx in (7) ,we get

$$\mu_{\tilde{R}}(0)\geq\min\{\mu_{\tilde{U}}([x,d(yx)]),\mu([yx,d(x)])\}$$

$$\geq\min\{\mu_{\tilde{U}}([x,d(yx)]),\mu_{\tilde{U}}([xy,d(x)])\}$$

$$\geq\min\{\mu_{\tilde{U}}([x,d(y)x]+[x,yd(x)]),\mu_{\tilde{U}}(x[y,d(x)]+[x,d(x)]y)\}$$

$$\mu_{\tilde{R}}(0)\geq\min\{\mu_{\tilde{U}}([x,d(y)]x+d(y)[x,x]+[x,y]d(x)+y[x,d(x)]),\mu_{\tilde{U}}(x[y,d(x)]+[x,d(x)]y)\} \quad (8)$$

Since $\mu(y[x,d(x)])=\mu(0)$, $[x,x]=0$ and (8),become

$$\mu_{\tilde{R}}(0)\geq\min\{\mu_{\tilde{U}}([x,d(y)]x+[x,y]d(x)),\mu_{\tilde{U}}(x[y,d(x)])\}$$

$$=\min\{\min\{\mu_{\tilde{U}}([x,d(y)]x),\mu(x[y,d(x)]),\mu([x,y]d(x))\}\} \quad (9)$$

Since $\mu_{\tilde{U}}([x,d(y)]x)=\mu_{\tilde{U}}(x)$, $\mu_{\tilde{U}}(x[y,d(x)])=\mu_{\tilde{U}}([y,d(x)])$ and (9),yield

$\mu_{\tilde{R}}(0)\geq\min\left\{\min\left\{\mu_{\tilde{U}}(x),\mu_{\tilde{U}}([y,d(x)]),\mu_{\tilde{U}}([x,y]d(x))\right\}\right\}$

$\geq\min\left\{\min\left\{\mu_{\tilde{U}}(x),\min\{\mu(y),\mu_d(d(x))\},\mu_{\tilde{U}}([x,y]d(x))\right\}\right\}$, hence,

$\mu_{\tilde{R}}(0)\geq\mu_{\tilde{U}}([x,y]d(x))$

By replacing y by wy for arbitrary $w\in U$

$\mu_{\tilde{U}}([x,wy]d(x))=\mu_{\tilde{U}}(w[x,y]d(x)+[x,w]yd(x))$

From properties (7) ,(8) of semi fuzzy left ideals $\tilde{U}$, yield

$\mu_{\tilde{U}}(w[x,y]d(x)+[x,w]yd(x))\geq\mu_{\tilde{U}}([x,w]yd(x))$

Thus

$\mu_{\tilde{R}}(0)\geq\mu_{\tilde{U}}([x,w]yd(x))\quad$ *for all* $y\in U$

Since $RU\in U$,we obtain

$\mu_{\tilde{R}}(0)\geq\mu_{\tilde{U}}([x,y]Ud(x))$

From lemma (2.5), we get

$\mu_{\bigcap_{0\neq a\in R}\tilde{P}_a}(z)=\mu_{\tilde{R}}(0)\geq\mu_{\tilde{U}}([x,y]Ud(x))$ for any $z\in R$.

Therefore

$\mu_{\tilde{U}}([x,y]Ud(x))\leq\min\left\{\mu_{\tilde{P}_\alpha}(z)\right\}_{\alpha\in\lambda}$ for any $z\in R$.

a) $\mu_{\tilde{U}}([x,y])\leq\mu_{\tilde{P}_\alpha}(z)\quad\forall\alpha\in\lambda\quad z\in R$.

Or

b) $\mu_{\tilde{U}d(\tilde{U})}(x)\leq\mu_{\tilde{P}_\alpha}(z)\quad\forall\alpha\in\lambda\quad z\in R$.

Let $\tilde{P}_1$ and $\tilde{P}_2$ be respectively the intersections of all type-one and type-two fuzzy maximal ideal such that $\mu_{\tilde{P}_1\cap\tilde{P}_2}(z)=\{0\}$ $z\in R$. .

Now we investigate a typical-two maximal $\tilde{P}_2=\tilde{P}_\alpha$,from (b) and the fact that $\mu_{\tilde{U}}([x,d(x)])=\mu_{\tilde{R}}(0)$.

For all $x\in U$, we have from proposition(2.1) that $\mu_{\tilde{U}}(xd(x))=\mu_{\tilde{U}}(d(x)x)$ and $\mu_{\tilde{U}}(d(x)x)\leq\mu_{\tilde{P}_2}(z)$ for any $z\in R$..

Thus,

$\mu_{\tilde{U}}(x+y)d(x+y))\leq\mu_{\tilde{P}_2}(z)\quad\forall x,y\in U$

$\Rightarrow\mu_{\tilde{U}}((x+y)(d(x)+d(y)))\leq\mu_{\tilde{P}_2}(z)$

$=\mu_{\tilde{U}}(xd(x)+xd(y)+yd(x)+yd(y))\leq\mu_{\tilde{P}_2}(z)$

But $\mu_{\tilde{U}}(xd(x))$ and $\mu_{\tilde{U}}(yd(y)\leq\mu_{\tilde{P}_2}(z)$

Thus

$$\min\{\mu_{\tilde{U}}(xd(y)),\mu_{\tilde{U}}(yd(x))\}\leq\mu_{\tilde{P}_2}(z),\ z\in R. \qquad (10)$$

and the same way $\mu_{\tilde{U}}(d(x+y)(x+y)) \le \mu_{\tilde{P}_2}(z)$, we have that :

$$\min\{\mu_{\tilde{U}}(d(x)y), \mu_{\tilde{U}}(d(y)x)\} \le \mu_{\tilde{P}_2}(z) \qquad (11)$$

From (10) and (11) , we have that ,

$$\min\left\{\min\{\mu_{\tilde{U}}(d(x)y), \mu(d(y)x)\}, \min\{\mu(xd(y)), \mu(yd(x))\}\right\} \le \mu_{\tilde{P}_2}(z) \qquad (12)$$

Replacing x by z and y by $xy+yx$ in (12), we get

$$\min\{\min\{\mu_{\tilde{U}}(d(z)(xy+yx)), \mu_{\tilde{U}}(d(xy+yx)z)\}, \min\{\mu_{\tilde{U}}(zd(xy+yx)),$$

$$\mu_{\tilde{U}}((xy+yx)d(z))\} \le \mu_{\tilde{P}_2}(z_1), \qquad z_1 \in R.$$

So from (12),we get

$$\min\{\min\{\mu_{\tilde{U}}(xy+yx), \mu_{\tilde{U}}(z)\}, \min\{\mu_{d(\tilde{U})}(d(xy+yx), \mu_{d(\tilde{U})}(d(z)\} \le \mu_{\tilde{P}_2}(z_1) \qquad z_1 \in R ,$$

(13) implies to

$$\min\{\min\{\mu_{\tilde{U}}(y), \mu_{\tilde{U}}(x), \mu_{\tilde{U}}(z)\}, \min\{\mu_{d(\tilde{U})}(d(y), \mu_{d(\tilde{U})}(d(x)), \mu_{d(\tilde{U})}(d(z)\} \le \mu_{\tilde{P}_2}(z_1)$$

Since all above relations worked with $\forall x, y, z \in U$, thus

$$\min\{\min\{\mu_{\tilde{U}}(U), \mu_{\tilde{U}}(U), \mu_{\tilde{U}}(U)\}, \min\{\mu_{d(\tilde{U})}(d(U), \mu_{d(\tilde{U})}(d(U)), \mu_{d(\tilde{U})}(d(U))\} \le \mu_{\tilde{P}_2}(z_1)$$

Hence $\min\left\{\mu_{\tilde{U}}(U), \mu_{d(\tilde{U})}(d(U))\right\} \le \mu_{\tilde{P}_2}(z)$,which implies to the following

$a')\mu_{\tilde{P}_2}(R) \ge \mu_{d(\tilde{U})}(d(U))$

Or

$b')\mu_{\tilde{P}_2}(R) \ge \mu_{\tilde{U}}(U)$

So if (b') satisfied that, make condition (a) also satisfied and obtain contradiction of our definition of type two. Therefore

$$\mu_{d(\tilde{U})}(d(u)) \le \mu_{\tilde{P}_2}(r) \quad \text{for any} \quad u \in U, r \in R. \qquad (14)$$

From (b) ,we have $d(u)u = ud(u)$,thus $\mu_{\tilde{U}}(d(u)u) = \mu_{\tilde{U}}(ud(u)) \le \mu_{\tilde{P}_2}(z_1)$.

Since

$$\min\{\mu_{\tilde{U}}(rd(r)d(u)), \mu(u)\} \le \mu_{\tilde{U}}(rd(r)d(u)u) = \mu_{\tilde{U}}(d(u)u) = \mu_{\tilde{U}}(ud(u)) \le \mu_{\tilde{P}_2}(z_1)$$

$\min\{\min\{\mu_{\tilde{R}}(rd(r)), \mu_{d(\tilde{U})}(d(u))\}, \mu_{\tilde{U}}(u)\} \le \mu_{\tilde{P}_2}(z_1)$, there are two cases

$$\min\{\mu_{\tilde{R}}(rd(r)), \mu_{\tilde{U}}(u)\} \le \mu_{\tilde{P}_2}(z_1) \qquad (15)$$

or

$$\min\{\mu_{d(\tilde{U})}(d(u)), \mu_{\tilde{U}}(u)\} \le \mu_{\tilde{P}_2}(z_1) \qquad (16)$$

By () ,we get $\mu_{\tilde{R}}(rd(r)) \le \mu_{\tilde{P}_2}(z_1)$ or $\mu_{\tilde{U}}(u) \le \mu_{\tilde{P}_2}(z_1)$

But the second parte is not true, we have that

$\mu_{\tilde{R}}(rd(r) \le \mu_{\tilde{P}_2}(z_1)$

Consider now the fuzzy left ideal $\tilde{V}$ generated by the set $(d(R)U, \mu_{\tilde{U}}(d(R)U))$,,we shall show that $\tilde{V}$ is commutative. Atypical element of $\tilde{V}$ are of the form $(d(r)u, \mu_{\tilde{U}}(d(r)u)$ and $(sd(r)u, \mu_{\tilde{U}}(sd(r)u))$,where $r, s \in R$ and $u \in U$. So we need only show that the commutators of the form

$[d(r_1)u_1, d(r_2)u_2], [s_1 d(r_1)u_1, d(r_2)u_2]$ and $[s_1 d(r_1)u_1, s_2 d(r_2)u_2]$ are equal to zero.

By using (a), we obtain

$$\mu_{\tilde{U}}([d(r_1)u_1, d(r_2)u_2]) \le \mu_{\tilde{P}_1}(z_1),$$
$$\mu_{\tilde{U}}([s_1 d(r_1)u_1, d(r_2)u_2]) \le \mu_{\tilde{P}_1}(z_1), \qquad (17)$$
$$and \qquad \mu_{\tilde{U}}([s_1 d(r_1)u_1, s_2 d(r_2)u_2]) \le \mu_{\tilde{P}_1}(z_1)$$

Also

$$[d(r_1)u_1, d(r_2)u_2], [s_1 d(r_1)u_1, d(r_2)u_2], [s_1 d(r_1)u_1, s_2 d(r_2)u_2] \in Rd(R)$$

The form (17) ,we have that

$$\mu_{\tilde{U}}([d(r_1)u_1, d(r_2)u_2]) \le \mu_{\tilde{P}_2}(z_1),$$
$$\mu_{\tilde{U}}([s_1 d(r_1)u_1, d(r_2)u_2]) \le \mu_{\tilde{P}_2}(z_1), \qquad (18)$$
$$and \qquad \mu_{\tilde{U}}([s_1 d(r_1)u_1, s_2 d(r_2)u_2]) \le \mu_{\tilde{P}_2}(z_1)$$

From (17) and (18) ,we get

$$\mu_{\tilde{U}}([d(r_1)u_1, d(r_2)u_2]) \le \min\{(\mu_{\tilde{P}_1}(z), \mu_{\tilde{P}_2}(z)\} = \mu_{\tilde{P}_1 \cap \tilde{p}_2}(z)$$
$$\mu_{\tilde{U}}([s_1 d(r_1)u_1, d(r_2)u_2]) \le \min\{(\mu_{\tilde{P}_1}(z), \mu_{\tilde{P}_2}(z)\} = \mu_{\tilde{P}_1 \cap \tilde{p}_2}(z) \quad \text{for any } z \in R.$$
$$and \qquad \mu_{\tilde{U}}([s_1 d(r_1)u_1, s_2 d(r_2)u_2]) \le \mu_{\tilde{P}_1 \cap \tilde{p}_2}(z)$$

Hence by lemma (2,5) ,we obtain $\tilde{V}$ is fuzzy commutative ideal.

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المستخلص

ليكن $\tilde{R}$ حلقة قرب جزئية ضبابية من الحلقة R وليكن $\tilde{U}$ مثالي ايسر شبه ضبابي معرف على Rو كذلك d تطبيق أضافة على R . أن الغرض من هذا البحث هو برهنة مايلي أذا كانت $\tilde{R}$ تسمح بأشتقاق

ضبابي d الذي يكون متمركز ضبابيا على $\tilde{U}$ ، فأن Rتحوي مثالي ابدالي ضبابي $\tilde{V}$.