0N Quasi and pseudo prime module

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Abstract

We shall study the new relation between two the quasi-prime module and pseudo-prime module .The purpose of this paper is to study and gave the new relation between the two different prime module

Introduction:

We shall study the new relation between two different prime modules.

A module is called Quasi-prime module if $ann_R N$ is a prime idal for every N be a submodule of a module M[2].And M is called pseudo-prime Module if $ann_R N$ is primary ideal for every N be asubmodule of A module M [3],[4],where where $ann_R N=\{r\in R;rx=0 \text{ for each } x\in N\}$.

The primary goal of this paper to give some cond ition to show That there is some relation between the Quasi-prime modules and pseudo Prime modules.In the start we show that every Quasi-prime modules is Pseudo-prime modules but the converse is not true in general unless M be cyclic .And we showing that if N is pseudo-prime submodule of M then the R-homomorphism $\varphi^{-1}(N)$ is also pseudo-prime Submodule of M .And we prove if N is a pseudo-prime submodule of M and φ is an epimorphism such that ker $\varphi \subseteq N$ then $\varphi(N)$ is a pseudo prime submodule of M

Theorm(1):

Every Quasi-prime module is pseudo-prime module

Proof:

Let M be Quasi-prime module so $ann_R N$ is a prime ideal For every N be a submodule of a module M.But every prime Ideal is primary ideal so $ann_R N$ is a primary ideal for every N be a submodule of a module M.Which mean that M is a pseudo Prime module (by definition)

But the converse of the above theorem is not true.

We can show that by the following example:

Let $M=Z_8$,N={[o],[4]} is pseudo prime

Since $ann_R N=\{4Z\}$ is primary ideal since $4\in\{4Z\}$

i.e $2.2\in\{4Z\}$ but $2\notin\{4Z\}$ where $4\in\{4Z\}$,but M is not Quasi-prime module

since $ann_R N=\{4Z\}$ isnot prime ideal

Theorem (2):

If M be cyclic module and pseudo prime then M is Quasi-prime module.

Proof:

If $ab \in ann_R N$ to prove $a \in ann_R N$ or $b \in ann_R N$.
Since M be cyclic so every submodule is cyclic ,[1].
Let N be submodule of a module M and $N=(x);x \in M$.
Since $ab \in ann_R N$ so $abN=0$.
$\therefore$ $abx=0$,if $ax \neq 0$ then $(bx)^n=0$ (since $ann_R N$ is Primary ideal).So $bx=0$ so $b \in ann_R N$
$\therefore$ $ann_R N$ is prime ideal for each N submodule of M.

Remark (1):

The Z-module Z_n is pseudo prime iff n is prime ideal.

Proof: Trivial.

Every non-zero submodule of pseudo brime module is pseudo prime module.

Proof:

Let M be pseudo prime module and let N be Submodule of M.To prove N be pseudo prime module.
Let $U \leq N \leq M$ .Since M be pseudo prime module so $Ann_R N$ is primary ideal.So N is pseudo prime module

Remark (3):

It is clear that if M is pseudo prime module Then $ann_R M$ is primary ideal.

Definition (1):

Let N be a submodule of a module M then N is Called pseudo prime submodule of M if $xym \in N$ for $X,y \in R$ and $m \in M$ then either $x^n m \in N$ or $Y^n m \in N$ For some $n \in Z_+$,[3].

Definition (2):

A proper submodule N of a module M is said to be Quasi-prime submodule if whenever $r_1,r_2\, m \in N$ for $r_1,r_2 \in R, m \in M$,then either $r_1 m \in N$ or $r_2 m \in N$ [2].

Remark (4):

Every quasi-prime submodule is pseudo prime Submodule.

Proof:

Trivial,but the converse isnot true for example
(6Z) is pseudo prime submodule of Z since $2.3.2 \in (6Z)$
But $2^n.2 \notin 6Z$ whenever $3^n.2 \in (6Z)$.But (6Z) is not quasi-prime submdule.[2]

Definition (3):

A proper submodule N of an R-module M is Semi-prime if $r^k x \in N$ for $r \in R, x \in M$ and K be Positive integer,then $r x \in N$.

Remark (4):

Every quasi-prime submodule is semi-prime Submodule. [2]

Remark (5):

Every semi-prime submodule is pseudo prime Submodule.

Proof:

If $r^k x \epsilon N$ then $rx \epsilon N$ for $k \epsilon Z_+$, $x \epsilon M, r \epsilon R$, (by def. of semi-prime Submodule) so $r^{k-1} rx \epsilon N$ so either $r^{k-1} x \epsilon N$ or $rx \epsilon N$ so N is pseudo prime submodule.

Theorem (3):

Let M and M_1 be two R-modules and
Let $\emptyset : M \to M_1$ be an R-homomorphism then

1- if N is pseudo prime submodule of M_1 then $\emptyset^{-1}(N)$ is also pseudo prime submodule of M.

2- if N is a pseudo prime submodule of M and $\emptyset$ Is an epimorphism such that $\ker \emptyset \subseteq N$ then $\phi(N)$ is a pseudo prime submodule of M_1 .

Proof (1):

Let $r_1, r_2 \epsilon R$ and $m \epsilon M_1$. If $r_1 .r_2.m\ _1\epsilon\phi^{-1}$ (N)
So $r\ _{1,}r_1\phi$ (m)ϵN,which is pseudo prime submodule
So either $r_1^n \phi(m) \epsilon N$ or $r_2^n \phi(m) \epsilon N$.
So $r_1^n m \epsilon \phi^{-1}(N)$ or $r_2^n m \epsilon \phi^{-1}(N)$ therefore $\phi^{-1}(N)$ is pseudo prime submodule of M.

Proof (2):

Let $r_1, r_2 \epsilon R$ and $m_1 \epsilon M$ such that $r_1, r_2\ m_1 \epsilon \phi(N)$
Hence,there exist $n \epsilon N$ such that $r_1.r_2 m\ \phi(n)$.
But $m_1 = \phi(m)$ some $m \epsilon M$ so $r_1.r_2\ \phi(m) = \phi(n)$.
So $r_1.r_2(m-n)\ \epsilon \ker \phi \subseteq N$ so $r_1.r_2(m-n) = n_1$ for Some $n_1 \epsilon N$ so $r_1.r_2 m \epsilon N$ but N is pseudo prime Submodule of M so that thereexist $r_1{}^k m \epsilon \phi(N)$ Or $r_2{}^k m \epsilon \phi(N)$,which mean that $\phi(N)$ is a pseudo Prime submodule of M_1.

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