On Some Open Mapping

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Abstract

In this work we give new theorems to definition of [5], and we prove Some properties of these mappings.

1 . Introduction:

The concept of open mapping is very important generalization of open mapping.[3] In this work ,we study in details inductively semi open mapping , which are generalization of inductively open mapping By a space X, we mean a topologicaly fal space X By a mapping , we mean Continuous function , the symbol f : X $\longrightarrow$ Y means that f is onto.

2 . Basic definitions and remarks

In this Section , we recall the basic definitions needed in this work

2 .1 . Definition . [2]

Let f : X $\longrightarrow$ Y be a function , we Say that f is open if and only if the image of every open set in X is open in Y.

2 .2 Remarks and examples

1 . f : R $\longrightarrow$ R where f (x) = x^2 is not open , because if w= (-1,1) then f (w)=[0,1) is not open in R .

2 . if f : X $\longrightarrow$ Y is an open function and also continuous then we say that f is an open mapping (abbreviated O . M) for example , f : [2,5] $\longrightarrow$ R , where f (X)= X^2 . is O.M

Next , we recall the definition of inductively open mapping

2 . 3 Definition . [3]

Let f : X $\longrightarrow$ Y be a mapping from X into Y , we say that f is inductively open mapping (abbreviated I . O.M) if and only if there exists a subset X_1 , of X such that $f(X_1) = f(X)$ and $f\backslash_{X_1} : X_1 \longrightarrow f(X_1)$ is open mapping.

2 . 4 Remarks and examples

1 . f : R $\longrightarrow$ R , where f(x) = x^2 is I.O.M because , let X_1= [0,∞) then $f\backslash_{X_1} : X_1 \longrightarrow$ R is open .

2 . Every open mapping is an inductively open mapping.

Before we introduce the concept of inductively semi-open mapping, we recall the following definition.

2.5 Definition .[1]

Let $A \subseteq X$, we say that A is semi – open in X if and only if there exist an open set G in X such that $G^{\circ} \subseteq A \subseteq G$.

For example , [0,1) is semi open in (R,T_U)

2.6 Definition [1]

Let $f : X \longrightarrow Y$ be a mapping , we say that f is a semi-open mapping (abbreviated S.O.M) if the image of every open set in X is semi-open in Y .

2.7 Definition [5]

Let $f : X \rightarrow Y$ be a mapping ,we say that f is inductively semi open mapping (abbreviated I.S.O.M) if and only if there exist $X_1 \subseteq X$ Such that $f\backslash_{X_1} : X_1 \longrightarrow f(X_1)$ is a semi open mapping and $f(X_1)=f(X)$.

2.8 .Remarks and examples

1- Every inductively open mapping is an inductively semi open mapping

2- Every semi open mapping is an inductivity semi open mapping.

3- X_1 in the above definition is Called an inductive set .

3. Inductive Semi-Open Mapping

In this Section ,we shall prove several theorems Concerning inductively semi open mapping :

3.1 Theorem :

Let f: X →Y be an I.S.O.M and let $A \subseteq X$ be an inverse set(that is $A= f^{-1}(f(A))$ such that f(A) is an open in f (X)) then $f\backslash_A : A \to Y$ is also an I.S.O.M

Proof : since $f : X \to Y$ is an I.S.O.M , then $\exists\ X_1 \subseteq X$,

$f(X_1)= f(X)$ and $f\backslash_{X_1} : X_1 \to f(X_1)$ is a S.O.M,

let $A_1 = A \cap X_1$, then we have ,

$f(A_1) = f(A \cap X_1)$

$=f(f^{-1}(f(A)) \cap X_1)$

$= f(A) \cap f(X_1)$

$= f(A) \cap f(X)$

$= f(A)$

Now ,let W be open in A , then $\exists\ W^*$ open in X_1

$W = W^* \cap A_1$

Then we have ,

$f(W) = f(W^* \cap A_1)$

$= f(W^* \cap (A \cap X_1))$

$= f(W^* \cap A)$

$=f(W^* \cap f^{-1}(f(A))$

$= f(W^*) \cap f(A)$

Now , f(W*) is semi open in f(X) , but f(A) is open in f(X) then f(W*) ∩ f (A) is semi open in f(A) , So $f \backslash_{A_1}$: A_1 →f (A) is a S.O.M therefore $f \backslash_A$ is an I.S.O.M

3.2 Theorem:

Let f : X → Y be an I.S.O.M and let T⊆Y , define $f_T : f^{-1}(T) \to T$ as follows: $f_T(x) = f(x)$ for every $x \in f^{-1}(T)$ then f_T is also an I.S.O.M if f is onto.

Proof: since f : X → Y is an I.S.O.M , then ∃ $X_1 \subseteq X$ $f(X_1) = f(X) = Y$ and $f \backslash_{X_1}$: X1 →Y is a S.O.M,

let $T^* = X_1 \cap f^{-1}(T)$, then we have ,

$f(T^*) = f(X_1 \cap f^{-1}(T))$

$= f(X_1) \cap T$

$= f(X) \cap T$

$= Y \cap T$

$= T$

Now , let w* be an open in T*, then ∃ w

Open in X_1 ,W* = w ∩T* , then

$f(W^*) = f(w \cap T^*)$

$= f(W \cap X_1 \cap f^{-1}(T))$

$= f(w \cap f^{-1}(T))$

$= f(W) \cap T$

Now , f(W) semi open in Y and T open in Y , then f(W) ∩ T is semi open in T, So $f \backslash_{T^*}$: T* →T is a S.O.M therefore $f_T : f^{-1}(T) \to T$ is an I.S.O.M

3.3 .Theorem:

Let f : X→Y is I.S.O.M and g: Y →Z is I.S.O.M then $g \circ f$: X→Z is I.S.O.M

Proof: since f: X →Y is I.S.O.M, then ∃ $X_1 \subseteq X$, $f(X_1) = f(X)$ and $f \backslash_{X_1}$: $X_1 \to f(X_1)$ is S.O.M and since g: Y →Z is S.O.M

Then ∃ $Y_1 \subseteq Y$, g (Y_1) = g (Y) and g$\backslash_{Y_1}$: Y_1 →g(Y_1) is S.O.M

T.P g∘f : X→Z is I.S.O.M

Let $X_1 \subseteq X$; g∘f (X_1) = g∘f(X) and let w= w* ∩X_1

g∘f (X_1)= g(f(X_1))

=g(f(X))

= g∘f(X)

Let w open in X and w* open in X_1 T.p g∘f(w) is S.O.M

g∘f (w) = g∘f (w*∩X_1)

=g (f(w*∩X_1))

= g (f(w*)∩f(X_1))

but f(w*) and f(X_1) are two S.O.M so g∘f(w)∩g∘f (X_1) are two S.O.M so g∘f(w) is S.O.M so g∘f $\backslash_{X_1}$ is I.S.O.M

3.4 Remark :

The identity mapping is not I.S.O.M

3.5 Theorem :

Let f: X →Z is I.S.O.M then
g: X →X/ Z is I.S.O.M

Proof: since f : X→Z is I.S.O.M so ∃ X_1 ⊆X ; f (X_1) = f(X)

and f $\backslash_{X_1}$: X_1 → f(X_1) is S.O.M

Let w be open in X_1 ; w = X∩ X_1

g (w) = g(X∩X_1)

= g(X)∩g(X_1)

=(X+Z) ∩(X_1 +Z)

=(X∩X_1) +Z

=X_1 +Z

=g(X_1)

Let w* be in w, then $\exists$ k open in X_1 ; $w^* = k \cap w$

$g(w^*) = g(k \cap w)$

$= g(k) \cap g(w)$

$g(w^*) = g(k) \cap g(X \cap X_1)$

$= g(k) \cap g(X_1)$

$= g(x_1)$ is S.O.M

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