Notes On Weakly Separation Properties

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ABSTRACT

In this work , we obtain the definitions to α-$T_{1/4}$ and α-$T_{1/2}$, also we study the relationship between α-T_0 , α-$T_{1/4}$, α-$T_{1/2}$ and α-T_1, and give the relation between them and the ordinary separation axioms with examples .

1-Introduction

<u>1-1 Definition :</u> Let (X,T) be a topological space and A $\subseteq$ X , A is called α – open set in X if and only if A $\subseteq A^{o-o}$. the family of all α –set in X denoted by $\overset{\alpha}{T}$; $\overset{\alpha}{T}$ ={ A : A $\subseteq$ X ; A $\subseteq A^{o-o}$ } where (o) is interior and (-) is closure **.** [3]

<u>1-2 Definition :</u>

1- A space X is called a $T_{1/4}$ space ,if for every finite subset F of X, and every y$\not\subset$ F, there exist a set A containing F and disjoint from y , such that A either open or closed .[1]

2- A space X is called a $T_{1/2}$ space , if for every singleton set in X is either open or closed .[1]

<u>1-3 Example</u> :

Let X = {a,b} , T = { X,ϕ, {a} } , then $\overset{\alpha}{T}$ = { X,ϕ, {a} } , this space its called a Seprenki space, and its clear is a $T_{1/2}$, also (X, $\overset{\alpha}{T}$) its $T_{1/2}$.

<u>1-4 Definition :</u> Let (X,T) be a topological space ,and A $\subseteq$ X , then A is called α –closed set in X iff (X – A) is α – open set. [3]

1-5 Definition :

1- A space X is said to be an α-T_1 space ,if for each pair of disjoint points x and y , of X , there exist α – open sets U and V in X containing x and y respectively , such that y$\notin$U and x$\notin$V .[4]

2- A space X is said to be an α-T_0 space ,iff for each pair of disjoint points in X , there exist α – open set in X containing one of them but not both .[4]

1-6 Proposition : Let (X,T) be a topological space , and if (X,T) is T_i space ; i=0,1 , then (X, $\overset{\alpha}{T}$) is T_i space respectively .[5]

2)Weakly α-Separation Axioms Properties :

2-1 Definition : A space X is called an α–$T_{1/4}$ space ,if for every finite subset F of X, and every y$\notin$ F, there is exists a set A containing F and disjoint from y, such that A either α –open or α –closed .

2-2 Proposition : Let (X,T) be a topological space , then (X,T) is α–$T_{1/4}$ space iff (X,$\overset{\alpha}{T}$) is $T_{1/4}$.

Proof : let (X,T) be an α–$T_{1/4}$ space , to prove (X, $\overset{\alpha}{T}$) is $T_{1/4}$ space.

Let F be a finite set in X and p is a point in X , such that p $\notin$ F .

Since (X,T) is α–$T_{1/4}$ space , there is V either α–open or α–closed containing F but not p .

So V is open or closed in (X, $\overset{\alpha}{T}$) , then we have V is open or closed in (X, $\overset{\alpha}{T}$) containing F not p , and hence (X, $\overset{\alpha}{T}$) is $T_{1/4}$ space .

On the other hand : let F be a finite set in X and p is a point in X , such that p $\notin$ F.

Since (X, $\overset{\alpha}{T}$) is $T_{1/4}$, there is W is open or closed set in (X, $\overset{\alpha}{T}$) containing F not p.

So W is α–open or α–closed in (X,T) containing F not p ,and hence (X,T) is α–$T_{1/4}$ space.

<u>2-3 Example</u> : Not every α–$T_{1/4}$ space is $T_{1/4}$ space and as follows :

Let X = {a , b, c} , T = { X , ϕ , {a} } , then $\overset{\alpha}{T}$ = { X , ϕ , {a} , {a,b} , {a,c}}, and let F be a set of all α–closed sets ; F={X , ϕ, {b,c}, {c}, {b}} .

Its clear that (X,T) is α–$T_{1/4}$, but its not $T_{1/4}$:

let K be a set of all closed sets in T ; K = { X , ϕ , {b,c} } , and let {c} be a set , then its clear {c} is finite and b $\notin$ {c} , but there is not exists open set or closed set containing {c} and not contain b .

<u>2-4 Definition</u> : A space X is said to be an α–$T_{1/2}$ space , if every singleton set in X is α–open or α–closed. [4]

<u>2-5 Proposition :</u> Let (X,T) be a topological space , then (X,T) is α–$T_{1/2}$ space iff (X,$\overset{\alpha}{T}$) is $T_{1/2}$ space .[4]

<u>2-6 Lemma :</u> If (X,T) is α–$T_{1/4}$ space , then (X,T) is α– T_o space.

<u>Proof :</u> let $p \neq q$ and F = {p} , since (X,T) is α–$T_{1/4}$ space ,then $\exists$ V α–open or α–closed set , such that F $\subseteq$ V and q $\notin$ V .

Case I : if V is α–open set ,then p $\in$ V and q $\notin$ V .

Case II : if V is α–closed set ,then W = X – V $\in$ $\overset{\alpha}{T}$, and hence W containing q not p, so we have (X,T) is α– T_o space .

<u>2-7 Lemma :</u> If (X,T) is α–$T_{1/2}$ space , then (X,T) is α–$T_{1/4}$ space .

<u>Proof :</u> let F be any finite set in X , and since X is α–$T_{1/2}$ space , then {p} is either α–open or α–closed set .

Case I : suppose {p} is α–open set, and let W = X – {p} , then W is α–closed set , and F $\subseteq$ W , and hence p $\notin$ W .

Case II : suppose {p} is α–closed set, and let W = X – {p} , then W is α–open set , and F ⊆ W , and hence p ∉ W .

<u>2-8 Lemma :</u> If (X,T) is α–T_1 space , then (X,T) is α– $T_{1/2}$ space.

<u>Proof :</u> let p ∈ X , to prove {p} either α–open or α–closed set.

If X = {p} , then {p} is either α–open or α–closed set .

If X ≠ {p} , then X – {p} ≠ ϕ , let q be any point in X – {p}, then q ≠ p .

Since X is α–T_1 space , then ∃ U ∈ $\overset{\alpha}{T}$, such that U containing q not p , So U ∩ {p} = ϕ , and hence q ∉ {p}' ∀ U ≠ p , so {p} is α–closed set.

<u>2-9 Example</u> : Not every α– $T_{1/2}$ space is α–T_1 space as follows :

Let X ={a , b, c}, T ={X , ϕ, {a}} , then $\overset{\alpha}{T}$ = { X , ϕ , {a} , {a,b},{a,c}} , and let F be a set of all α–closed sets ; F = { X , ϕ , {b,c} , {c} , {b} } .

Its clear the space X is α– $T_{1/2}$ **,** because every singleton sets either α–open or α–closed set.

A space X is not α–T_1 space because :

There is no α–open set containing b and not contain a .

A space X is not $T_{1/2}$ space because :

Let A = { X , ϕ , {b,c} } be the family of all closed sets in T , then the singleton sets {b} and {c} are not open or closed in T .

<u>2-10 Example</u> : Not every α–T_0 space is T_0 space as follows :

Let X = {a , b, c, d} , T = { X , ϕ , {a} , {a,b} } ,

then $\overset{\alpha}{T}$ = { X,ϕ,{a},{a,b},{a,c},{a,b,c},{a,b,d},{a,c,d},{a,d} } , the set of all closed sets in T is F = { X , ϕ , {b,c,d} , {c,d} } .

Its clear that the space (X,T) is α–T_0 , but not T_0 , because d ≠ c and we can not disjoints between them by a sets in T .

<u>2-11 Example</u> : Not every α–$T_{1/4}$ space is α–$T_{1/2}$ space as follows :

Let $X = [o,1)$, we define $r_a = \{ x : x < a \}$, such that $a \in (0,1)$,
Then $T = \{ r_a : a \in (0,1) \} \cup \{ \phi \} \cup \{ X \}$, be a topology on X .
Let $r_a \subseteq A \subseteq r_a^{-o} = [0,1)^o = X$, then $A = r_a \cup A_a$, such that $A_a \in [a,1)$, and hence $\overset{\alpha}{T} = \{ r_a \cup A_a : a \in (0,1) \}$, so A_a is α–open set and $o \in A$.

1)) The topological space (X,T) is not α–$T_{1/2}$ space :

Case I : $\{0\}$ is singleton set , and its clear $\{0\} \notin \overset{\alpha}{T}$.

Case II : To prove $\{0\}$ is not α–closed set .

Let $\{0\}$ be α–closed set , then $X - \{0\} \in \overset{\alpha}{T}$,
since $X - \{0\}$ is α–open set , then $X - \{0\} = r_a \cup A_a$, where $a \in (0,1)$, so $a \in r_a$, i.e. $0 \in r_a \cup A_a$, then $0 \in X - \{0\}$, this is contradiction !!
i.e. $X - \{0\}$ is not α–open set, and hence $\{0\}$ is not α–closed set , and its not α–open set .

2)) (X,T) is α–$T_{1/4}$ space :

Let $H = \{ a_1 < a_2 < a_3 < \ldots < a_n \}$ be a subset of X , and $p \notin H$, then there exist many cases :

Case I :

If $p = 0$ and $p \notin H$, then $\exists\ t \in X$, such that $0 < t < a_1$,
let $A = r_t \in \overset{\alpha}{T}$, then $X - A = [t,1)$ is α–closed set containing H not p .

Case II :

Let $p \neq 0$, and $0 \notin H$, in this case we have many probabilities for position of p :

1)) if $p > a_n$, then $\exists\ t \in X$, such that $a_n < t < p$, take r_t , so $r_t \in \overset{\alpha}{T}$ containing H not p .

2)) if $a_i < p < a_{i+1}$, then $\exists\ t$, such that $a_i < t < p$, let $V = r_t \cup \{a_{i+1}, \ldots, a_n\}$, we can see $V \in \overset{\alpha}{T}$ containing H not p .

3)) if $0 < p < a_1$, then ∃ t , such that $p < t < a_1$, let $W = X - r_t$,then W is α–closed set containing H not p .

Case III :

If $a_1 = 0$, then there is many probabilities for position of p :

1)) if $p > a_n$,then ∃ t , such that $a_n < t < p$,so r_t is α–open set containing H not p.

2)) if $a_i < p < a_{i+1}$, then ∃ t , such that $a_i < t < p$, then $A= r_t \cup \{a_{i+1} , \dots , a_n\}$, and A is α–open containing H not p .

3)) if $0 < p < a_2$, then ∃ t , such that $0 < t < p$, then $A= r_t \cup \{a_2 ,a_3 , \dots , a_n\}$, and A is α–open containing H not p .

From above cases , we have (X,T) is α–$T_{1/4}$ not α–$T_{1/2}$ space.

REFRENCE

1) Levine,N. , **Generalized Closed Set in Topology** , Rend Girc.Math. Palermo,19(2) , pp.89-96 ,1970 .

2) Naser, A. , **On Separation Properties** , M.Sc. Thesis Baghdad university , Iraq 1989.

3) Njasted,O. , **On Some Class of Nearly Open Set** , pacific J. of Math., Vol.15 ,No.3 ,PP. 961-970, 1965.

4) Maki H.,Devi R. and Balochandran K. , **Generalized α-Closed sets in Topology** , Bull Fukuoka Uni.Ed. Part III ,42,pp.13-21,1993.

5)Alobaidi, A., **On Some Maps and Space** , M.Sc. Thesis Baghdad university , Iraq 1989.

6) Aggarwal,R.S. , **A Text Book on Topology** , S. Hand and Company Ltd.,1996.