(T, L) Semi α- Homotopy and (T, L) semi α- Isotopy

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Abstract

In this work we present the definitions of (T,L) semi α– homotopy (T,L), strongly semi - α homotopy and we give the relation among them and (T,L) homotopy , (T,L) α- homotopy (T,L) semi homotopy , (T,L) strongly α - homotopy , also we present the definition of (T,L) semi α - isotropy , (T,L) strongly semi α - isotopy and we study the relation awong them and (T,L) isotopy , (T,L) α - isotopy, (T,L) strongly α - isotopy and (T,L) semi isotopy .

Several Characterization of these concepts.

1-Introduction

In 1979 Kasahara [1] introduced the concept of an operator associated to a topogolgy . In 1991 ogata [3] introduced the T-open set and study (T,L) continuous function while Rosas in 2002 study the concept of T- semi open set.

In 1965 Mjasted introduce the concept of α - set as follow a set A is called α- set if $A \subseteq A^{o - o}$ where (o) and (-) are interior and closure respectively . In 1983 Moshhovr and other in [6] present the concept of α - continuous . In 2002 Maiahy [2] introduce the concept of strongly α- continuous. Monsour [7] introduce the concept of T- α- operator. Mushtt [8] introduce the concept of (T,L) semi α- operater, (T,L) semi α- continuous. Yousef [10] gave the concept of(T,L) α - homotopy and(T,L) α- isotopy (T,L) .To get more properties of T- operater see [4,5,9]. In this paper we present the new concepts which is the definitions of (T,L) semi α – homotopy and (T,L) strongly semi α - homotopy. Adding we define (T,L) semi α isotopy, (T,L) strongly semi α – isotopy with some properties of these concepts. In §2 we define (T,L) semi α – homotopy alse we give the relationship among them and (T,L) homotopy, (T,L) α- homotopy , (T,L) strongly α – homotopy , (T,L) Semi homotopy

In §3 we present the definition of (T,L) semi α– deformation and (T,L) strongly semi α- deformation and we study the relation among them and (T,L) deformation , (T,L) α - defamation , (T,L) strongly α- deformation, (T,L) semi deformation . In §4 we define (T,L) semi α– constraction , (T,L) strongly semi

α- constraction also we study the relation among them and (T,L) constraction, (T,L) **α**- constraction , (T,L) strongly **α**– constraction and (T,L) semi constraction.

In **§5** we define (T,L) semi **α**– imbedding , (T,L) strongly semi **α**- imbedding also we study the relation among them and (T,L) imbedding , (T,L) **α**-imbedding , (T,L) strongly **α**–imbedding and (T,L) semi imbedding.

Finally in **§6** we present the definition of (T,L) semi **α** – isotopy. (T,L) strongly semi **α**– isotopy and study the relationship among them and (T,L) isotopy , (T,L) **α**- isotopy (T,L) strongly **α**- isotopy and (T,L) semi isotopy also we give some properties of (T,L) semi **α**- isotopy , (T,L) strongly semi **α** -isotopy.

2- (T, L) Semi α - Homotopy.

In this section we define (T,L) semi **α**- homotopy, (T, L) strongly semi **α**-homotopy.

2-1 Definition : Let (**X**,Γ) and (Y,δ) be two topological spaces and T,L are operators associated with Γ and δ respectively . The map B: X × I → Y is said to be (T,L) homotopy between f, g : X → Y iff B (X , i), ∀x ∈ X ,i∈I is (T,L) continuous s.t B(x, 0) = f(x) ,B(x,1) = g(x) [10].

2-2 Definition: Let (X,Γ) and (Y,δ) be two topological spaces and T,L are **α**-operators associated with Γ and δ respectively. The map B: X × I → Y is said to be (T, L) α– homotopy between f,g : X → Y iff B (x, i) ∀ x∈X , i ∈ I is (T, L) **α** - continuous s.t B (x,0) = f(x) , B (x,1) = g(x) [10].

In [10] gave the relation between above definition as follow

Theorem (1) : Every (T,L) homotopy is (T,L) **α** - homotopy [10].

2-3 Definition: Let (X,Γ) and (Y, δ) be two topological spaces and T,L are two α-operators associated with Γ and δ respectively .The map B: X × I→ Y is said to be (T,L) strongly α– homotopy between f,g : X → Y iff B (x, i) ∀ x ∈X , i ∈ I is (T, L) strongly **α** - continuons s.t B (x,0) = f(x) , B (x,1) = g(x) .

Theorem (2) : Every (T,L) strongly α – homotopy is (T,L) α-homotopy [10].

2-4 Definition: let (X,Γ) and (Y,δ) be two topological spaces and (T,L) are two semi operators associated with Γ and δ receptivity. The map B: X × I → Y is said to be
(T,L) semi-homotopy between f, g iff B(x,i) ∀x ∈X , i ∈I is (T,L) semi continuous s.t B(x,0) = f(x),B(x,1) = g(x).

2-5 Definition: Let (X ,Γ) and (Y,δ) be two topological spaces and T , L are two semi α- operators associated with Γ and δ respectively.

The map B : X × I → Y is said to be (T,L) semi **α**- homotopy between f,g : X → Y iff B(x,i) ∀ x ∈ X , i ∈ I is (T,L) semi **α**- continuous s.t B(x, 0) = f(x) , B(x.1) = g(x)

Theorem (3) : Every (T,L) homotopy is (T,L) semi **α**– homotopy

proof: Let (X, Γ) and (Y, δ) be two topological spaces and T,L are two operators associated with Γ and δ respectively and let B : X × I →Y be(T,L) homotopy map.

That means B(x,i) ∀ x∈, X , i∈ I is (T,L) continuous. Sine [every (T, L) continuous is (T,L) semi α- continuous] [8]. Therefore B(x,i) is (T,L) is semi α- continuous ∀ x ∈ X , i∈ I. Then B(x,i) is (T,L) semi **α**- homotopy.

Theorem (4) : Every (T,L) **α** -homotopy is (T,L) semi **α**- homotopy.

proof: Let (X.Γ) and (Y, δ) be two topological spaces and T,L are two operators associated with Γ and δ respectively .Let B: X × I → Y be (T,L) **α**–homotopy. That is B (x,i) ∀ x ∈ X, i ∈ I is (T,L) **α**- continuous . Since [Every (T,L) α- continents is (T,L) semi **α**- continuous] [8].Therefore B(x, i) is (T, L) is semi α- continuous. Hence B(x,i) is (T,L) semi **α**- homotopy.

Theorem (5) : Every (T,L) strongly **α** – homotopy is (T,L) semi **α** – homotopy.

proof: (X, Γ) and (Y,δ) be two topological spaces and T,L are two **α**- operators associated with Γ and δ respectively. Let B: X×I → Y be (T,L) strongly **α**-homotopy. That is B (x,i) is (T,L) strongly α- continuous, since [Every (T,L) strongly **α**- continuous is (T, L) semi α- continuous] [8]. Therefore B (x,i) ∀ x ∈ X , i∈I is (T,L) semi α- continuous. Hence B (x,i) is (T, L) semi α-homotopy.

Theorem (6): Every (T,L) semi homotopy is (T,L) semi α- homotopy

proof: Suppose that (X ,Γ) and (Y, δ) be two topological and T,L are two semi operators with Γ and δ respectively. Since B : X×I→ Y be (T,L) semi homotopy . Then B(x, i) is (T,L) semi continuous. Since [Every (T,L) semi continuous is (T,L) semi α-continues]. Therefore B (x,i) is (T,L) semi α- continues ∀ x ∈X , i∈ I. Hence B (x,i) is (T,L) semi α – homotopy.

2-6 Definition: Let (X, Γ) and (Y,δ) be two topological spaces and T,L are two semi α – operators associated with Γ and δ receptivity. The map B: X × I→ Y is called (T,L) strongly semi α- homotopy between f,g : X→ Y iff B (x, 0) = f (x), B (x, 1) = g(x) ∀ x ∈ X.

Theorem (7): Every (T,L) strongly semi α –homotopy is (T,L) semi α-homotopy.

Proof :- Suppose that (X ,Γ) and (Y, δ) be two topological spaces and B: X× I → Y is (T,L) strongly semi α- homotopy. We get B(x,i) ∀ x ∈X , i∈I, is (T,L) strongly semi α – continuous. Since [Every (T,L) strongly semi α – continuous is (T, L) semi α-continuous] [8]. Then B (x ,i) is (T, L) semi α - continuous and B(x,0) = f(x) , B(x, 1) = g(x) ∀ x ∈ X . Therefore B(x,i) is (T,L) strongly semi α–homotopy.

Theorem (8): (T,L) semi α–homotopy relation between maps X into Y is an equivalence relation.

Proof: We must to prove (T,L) semi α- homotopy is reflexive, symmetric and transitive.

Let (X,Γ) and (Y,δ) be two topological spaces and T,L are semi α–operaters associated with Γ and δ respectively.

First to show (T,L) semi α- homotopy is reflexive.

Define (T,L) semi α- homotopy B: X× I → Y s.t B(x,i) = f(x) ∀ x∈X, i∈I

We get B(x,0) = f(x) , B(x,1) = f(x)

Thus B(x, i) is veflexive.

Then (T, L) semi α- homotopy is reflexive.

Next to show (T, L) semi α– homotopy is symmetric.

Suppose f is (T, L) semi α- homotopic to g.

Then there exists (T,L) semi α –homotopy B: X × I → Y s.t B(x,0) = f(x) , B(x,i)= g(x)

We Define (T,L) semi α-homotopy K: X × I →Y such that K(x,i) = B(x ,1-i) ∀ x ∈X, i∈I.

We get K(x,0) = B(x, 1) = g(x) , K(x ,1) = B(x ,0) = f(x)

Then g is (T, L) semi α- homotopic to f.

Thus (T, L) semi α- homotopy is symmetric.

Finally show that (T,L) semi α- homotopy is transitive.

Suppose f is (T,L) semi α- homotopic to g and is (T,L) semi α- homotopic to h.

Then there exist (T,L) semi α – homoptopy B: X × I → Y , K: X xI → Y

Such that B(x,0)= f(x) , B(x,1) = g(x) , K(x,0)= g(x) , K(x,1) = h(x)

Now we define (T,L) semi α- homotopy H:XxI → Y s.t

$$H(x,i) : \begin{cases} B(x,2i) & 0 \le i \le 1/2 \\ K(x,2i-1) & 1/2 \le i \le 1 \end{cases}$$

We get H(x,0) = B(x, 0) = f(x) and H (x ,1) = K (x, 1) = h (x)

Therefore f is (T,L) semi α - homotopic to h.

Hence H (x.i) is (T,L) semi α- homotopy

Thus (T, L) semi α- homotopy is transitive.

Theorem (9): 1. The (T,L) homotopy is an equivalence relation
2. The (T, L) α- homtopy is an equivalence relation.

3. The (T,L) strongly α – homotopy is an equivalence relation.
4. The (T,L) semi – homotopy is an equivalence relation.
5. The (T,L) strongly semi α- homotopy is an equivalence relation.

Proof **1**. Let B: X × I → Y be (T,L) homotopy. Therefore by theorem (3) B is (T,L) semi α- homotopy and by theorem (8) we get (T,L) homotopy is an equivalence relation.

2. Suppose that B: X × I → Y is (T,L) α- homotopy. By theorem (4) B is (T, L semi α- homotopy and by theorem (8) we get (T,L) α- homotopy is an equivalence relation.

3. Suppose that B X × I → Y be (T,L) strongly α- homotopy then by theorem (5) B is (T,L) semi α- homotopy and to theorem (8), we get (T,L) strongly α-homotopy is an equivalence relation.

4. Let B : X ×I → Y be (T,L) semi homotopy. Then by theorem (6) B is (T,L) semi α- homotopy .Therefore by theorem (8) , (T,L) semi homotopy is an equivalence relation.

5. Suppose that B: X× I → Y be (T,L) strongly semi α- homotopy then by theorem (7) B is (T,L) semi α- homotopy and by theorem (8) (T, L) strongly semi α - homotopy is an equivalence relation.

3. (T,L) Semi α- Deformation

In this section we present the definition of (T,L) semi α- deformation and strongly (T, L) semi α-deformation.

3-1 Definition: The (T,L) α-homotopy B : X × I → Y is said to be (T,L) α-deformation iff B(x,0) is inclusion map .[10]

3-2 Definition: The (T,L) semi α- homotopy B: X × I → Y is called (T,L) semi α- deformation iff B(x,0) is inclusion map.

Theorem (10): Every (T,L) deformation is (T,L) semi α- deformation .

Poof : Suppose that B : X × I → Y is (T, L) deformation.

By theorem (3) we get B(x,i) ∀x ∈X , i∈I is (T,L) semi α-homotopy and B (x,0) is inclusion map. Thus B is (T,L) semi α- deformation.

Theorem (11): Every (T,L) α-deformation is (T,L) semi α- deformation.

Proof : Suppose that B : X × I→ Y is (T, L) α- deformation. That mean B is (T,L) α- homotopy and B(x,0) is inclusion map. By theorem (4) we get B(x,i) ∀x ∈X, i ∈I is (T,L) semi α- homotopy and B(x,0) is inclusion map . Thus B is (T,L) semi α- deformation.

3-3 Definition: The (T,L) strongly α-homotopy B: X×I → Y is called (T,L) strongly α- deformation iff B(x,0) is inclusion map.

Theorem (12) : Every (T,L)strongly α-deformation is(T,L) semi α-deformation.

Proof : Suppose that B(x,i) ∀x ∈X, i ∈I is (T,L) strongly α- deformation. That is B is (T,L) strongly α-homotopy and B(x,0) is inclusion map .By theorem(5) we get B(x,i) is (T,L) semi α- homotopy and B(x,0) is inclusion map. Therefore B is (T,L) semi α- deformation.

Theorem (13): Every (T,L) semi deformation is (T,L) semi α- deformation.

Proof : Suppose B : X ×I → Y is (T,L) semi α-deformation. Then B is (T,L) semi homotopy and B(x,0) is inclusion map. By theorem (6) we get B is (T,L) semi α-homotopy and B(x,0) is inclusion map. Therefore B is (T,L) semi α-deformation

3-4 Definition: The (T,L) strongly semi α-homotopy B : X × I → Y is called (T,L) strongly semi α-deformation iff B(x,0) is inclusion map.

Theorem (14): Every (T,L) strongly semi α- deformation is (T,L) semi α-deformation.

Proof : Suppose B(x,i) ∀x ∈X, i ∈I is (T,L) strongly semi α-deformation. That is B is (T,L) strongly semi α- homopoy and B(x,0) is inclusion map. By theorem(7). we get B(x,i) is (T,L) semi α-homotopy and B(x,0) is inclusion map . Then B is (T,L) semi α- deformation.

4- (T, L)Semi α- Constraction

In this section we use the results of previous section into define the (T,L) semi α - constraction , (T,L) strongly semi α - constraction also we study the relation between them.

4-1 Definition: Let (X,Γ) and (Y,δ) be two topological spaces and T,L are α-operators associated with Γ and δ respectively. The (T,L) semi α- deformation B : X × I→ Y is called (T, L) semi α- constraction iff B (x,l) is constant map.

Theorem (15): Every (T,L) constraction is (T,L) semi α-constraction.

Proof : Suppose B:X × I → Y is (T,L) constraction. That means B is (T,L) deformation and B (x,l) is constant map. Then by theorem (10) we get B (x,i) is (T,L)sem α- deformation. and B (x,l) is constant map. Thus B(x,i) is (T,L) semi α- constraction.

Theorem (16): Every (T,L) α- constraction is (T,L) semi α- constraction .

Proof : let B: X× I →Y is (T,L) α- constraction .We get B(x,i) is (T, L) α-deformation and B (x,1) is constant map. By theorem (11) B(x,i) ∀ x∈ X, i∈I is

(T,L) semi α-deformation and B (x,1) is constant map. Therefore B is (T,L) semi α- constraction.

Theorem (17): Every (T,L) strongly α- constraction is (T, L) semi α-constraction.

Proof : Let B (x,i) is (T, L) strongly α- constraction. That mean B is (T, L) strongly α- deformation and B (x,l) is constant map . By theorem (12) B is (T,L) semi α- deformation and B(x,l) is constant map. Thus B is (T, L) semi α-constraction.

Theorem (18): Every (T,L) semi constraction is (T,L) semi α- constraction.

Proof : Suppose that B : X× I → Y is (T,L) semi constraction . That mean B is (T,L) semi- deformation and B (x,l) is constant map. And by theorem (13) B(x,i) is (T,L) semi α- deformation and B(x,l) is constaut map. Therefore B is (T,L) semi α- constraction.

4-2 Definition: Let (X,Γ) and (Y,δ) be two topological spaces and T,L are Two semi α- operators associated with Γ and δ respectively. The(T,L) strongly semi α- deformation B: X × I → Y called (T,L) strongly semi α – constraction iff B (x,l) is constant map.

Theorem(19): Every (T,L) strongly semi α – constraction is (T,L) semi α-constraction.

Proof: Let B: X×I → Y is (T,L) strongly semi α- constraction. We get B is (T,L) strongly semi α- deformation and B(x,l)is constant map. And theorem (14) B(x,i) is (T,L) semi α- deformation and B(x,l) is constant map. Therefore B is (T,L) is semi α- constraction.

5. (T, L) Semi α- Imbedding

We present and study in this section the definition of (T, L) semi α- Imbedding and (T, L) strongly semi α- Imbedding.

5-1 Definition : Let (X , Γ) and (Y, δ) be two topological spaces and T,L are semi α-operators associated with Γ and δ respectively, a one to one map
f :X → Y is called (T,L) semi α- imbedding iff there exists (T,L) semi α-homeomorphism from X onto f(X).

Theorem (20): Every (T,L) imbedding is (T,L) semi α- imbedding.

Proof : Let (X,Γ) and (Y,δ) be two topological spaces and (T,L) are semi α-operators associated with Γ and δ respectively and let f: X →Y be (T,L) imbedding. That means there exists (T,L) homeomorphism g: X → f(X) since [Every (T,L) homeomorphism is (T,L) semi α – homeomorphism][8]. Then g is (T,L) semi – α- homeomorphism . Therefore f is (T,L) Semi α- imbedding.

Theorem (21): Every (T,L) α- imbedding is (T,L) semi α- imbedding.

Proof : Let (X ,Γ) and (Y,δ) be two topological spaces and T,L are two semi α- operator associated with Γ and δ respectively. Let f: X →Y be (T,L) α- imbedding Then there exists (T,L) α- homeomorphism g:X → f (X). Since [Every (T,L) α- homeomorphism is (T,L) semi α - homeomorphism] [8]. Then g is (T, L) is semi α- homeomorphism. Therefore f is (T,L). semi α- imbedding.

Theorem (22) : Every (T,L) strongly α- imbedding is (T,L) semi α – imbedding.

Proof : Suppose that (X,Γ) and (Y,δ) be two topological spaces and T,L are two α- operators associated with Γ and δ respectively and f: X → Y be (T, L) strongly α- imbedding. Then there exists (T, L) strongly α- homeomorphism g: X → f (X)

Since [Every (T,L) strongly α- homeomorphism is (T,L) semi α- homeomorphism][8].Therefore g is (T,L) semi α- homeomorphism. Hence f is(T,L) semi α – imbedding.

Theorem (23): Every (T,L) semi imbedding is (T,L) semi α- imbedding.

Proof : let (X, Γ) and (Y,δ) be two topological spaces and T,L are two semi operators associated with Γ and δ respectively and f: X → Y be (T,L) semi imbedding. Then ∃ (T,L) semi homeomorphism. g: X → f (X) Since[Every (T,I) semi homeomorphism is (T,L) semi α- homeomorphism].

Then g is (T,L) semi α- homeomorphism. Hence f is (T, L) semi α- imbedding.

5-2 Definition : Let (x, Γ) and (Y,δ) be two topological spaces and T,L are two semi α- operators associated with Γ and δ respectively. A one to one map f : X → Y is called (T,L) strongly semi α–imbedding iff there exists a (T,L) strongly semi α- homeomorphism from X on to f (X).

Theorem (24): Every (T,L) strongly semi α- imbedding is (T,L) semi α- imbedding.

Proof: Let (X, Γ) and (Y,δ) be two topological spaces and T,L are two semi α- operators associated with Γ and δ respectively. Let f: X→ Y be (T,L) strongly semi α- imbedding. Then there exists (T,L) strongly semi α- homeomorphism g : X → f(X) [Every (Y,L) strongly semi α - homeomorphism is (T,L) semi α – homeomorphism][8].

Therefore g is (T,L) semi α- homeomorphism . Hence f is (T,L) semi α- imbedding.

6 . (T,L) Semi α- Isotopy

In the section we define and study (T,L) semi α - isotopy , (T,L) strongly semi α - isotopy.

6-1 Definition: let (X,Γ) and (Y,δ) two topological spaces and T,L are semi α- operators associated with Γ and δ respectively . A map B : X × I → Y, I = [0,1]

is said to be (T,L) semi α- isotopy between f , g: X → Y iff B(x,i) ∀ x ∈ X, i∈ I is (T,L) semi α-imbedding s.t B (x,0) = f (x), B (X, l) = g (x)

Theorem (25): Every (T,L) isotropy is (T,L) semi α- isotopy

Proof :- Let (X,Γ) and (Y,δ) be two topological spaces and T,L are semi α-operators associated with Γ and δ respectively. Let B : X × I → Y be (T, L) isotopy map between f, g X → Y. That mean B (x,i) is (T,L) imbedding s.t B (x,0) = f (x), B(x,1) = g (x) then by theorem(15) B (x,i) ∀ x ∈ X, i ∈ I is (T,L) semi α- imbedding. Thus B (x, i) is (T,L) semi α- isotopy.

Theorem (26) : Every (T, L) α- isotopy is (T,L) semi α- isotopy.

Proof : Let (X ,Γ) and (Y,δ) be two topological spaces and T,L are α -operators associated with Γ and δ respectively. Suppose B : X × I → Y is (T,L) α- isotopy. Therefore B(x,i) , x ∈ X, i ∈ I is (T,L) α- imbedding s.t B (x,0) = f(x), B (x,l) = g(x) then by theorem (16) B(x,i) ∀ x ∈ X , i ∈ I is (T,L) semi α- imbedding . Therefore B (x,i) is (T,L) semi α –isotopy.

Theorem(27) : Every (T,L) strongly α- istopoy is (T,L) α- isotopy.

Proof : Let (X,Γ) and (Y,δ) be two topological spaces and T,L are α- operators associated with Γ and δ respectively.

Suppose that B : X × I → Y is (T,L) strongly α- isotopy. Therefore B (x,i) ∀ x ∈X , i∈I is (T,L) strongly α – imbedding s.t B (x, 0) = f (x), B(x,l) = g (x) then by theorem (17) we get B(x,i) is (T,L) semi α- imbedding s.t B(x,o) = f(x) , B (x,l) = g(x) Thus B is (T,L) semi α – isotopy.

Theorem(28): Every (T,L) semi isotopy is (T,L) semi α– isotopy.

Proof : Let (X,Γ) and (Y,δ) be two topological spaces and T,L are semi – operator associated with Γ and δ respectively. Suppose that B : X × I → Y is (T,L) semi isotopy , therefore B(x,i) is (T,L) semi imbedding s.t B (x, l) = f(x) , B(x,o) = g(x) by theorem (18) we get B (x,i) is (T,L) semi α- imbedding s.t B(x,0) = f(x), B(x,l) = g(x). Hence B is (T, L) semi α- isotopy.

6-2 Definition: Let (X,Γ) and (Y,δ) two topological spaces and T,L are two semi α- opeteros assoiated with Γ and δ respectively. A map B : X×I → y , I = [0 ,1] is said to be (T,L) strongly semi α- istotopy between f ,g : X→ Y iff B (x,i) ∀ x ∈ X, i∈ I (T,L) strongly semi α- imbedding s.t B(x,0) = f(x) , B (x,i) = g (x).

Theorem (29): Every (T,L) strongly semi α- istopoy is (T,L) semi α- isotopy.

Proof : Suppose that (X,Γ) and (Y,δ) be two topological spaces and T,L are semi α- operators associated with Γ and δ respectively. Let B: X×I → Y be (T,L) strongly semi α- istopoy. We get B (x,i) , ∀ x ∈ X, i∈ I is (T,L) strongly semi α- imbedding s.t B(x,0) = f(x) , B (x,i) = g(x) then by theorem (19) we get

B(x,i) , ∀ x ∈ X , i ∈ I is (T,L) semi α- imbedding s.t B (x,o) = f(x) , B(x,1) = g(x).Thus B(x,i) is (T, L) semi α- isotopy.

Theorem (30): The (T,L) semi α- isotopy relation between maps X into Y is an equivalence relation.

Proof : We must to prove (T,L) semi α- isotopy is reflexive, symmetric and transitive.

Let (X,Γ) and)(Y, δ) be two topological spaces and T,L are semi α- operators associated with Γ and δ respectively. First to show (T,L) semi α- isotopy is reflexive.

Let f : X → Y be any map.

Define (T,L) semi α- isotopy B : X× I → Y by B(x,i) = f(x) ∀ x∈ X,i ∈ I , we get B (x,o) = f(x),B (x, l) = f(x).

Therefore (T, L) semi α isotopy is reflexive.

Next to show (T,L) semi α – isotopy is symmetric.

Suppose f : X →Y s.t f is (T, L) semi α– isotopic to g then ∃ B : X×I → Y s.t B (x, 0) = f(x) , B (x,1) = g(x).

Define (T,L)semi α- isotopy K : X× I → Y s.t K (x, i) =B (x, l- i)∀ x ∈ X, i ∈ I.

We get K (x,0) = B (x,1) = g(x), K (x,1) = B (x,0) = f(x).

Then g is (T,L) semi α – isotopic to f . Thus (T,L) semi α- isotopy is symmetric.

Finally suppose f is (T,L) semi α – isotopic to f and is (T,L) semi α – isotopic to h, then there exist (T,L) semi α – isotopy B: X× I →Y , K : X× I → Y s.t B(x,0) = f(x) , B (x,1) = g(x), K (x,0) = g(x) , K(x,l) =h(x)

We define (T, L) semi α- isotopy H : X × I → Y s.t

$$H(x,i) = \begin{cases} B(x,2i) & 0 \le i \le 1/2 \\ K(x, 2i-1) & 1/2 \le i \le 1 \end{cases}$$

We get H(x,o) = B (x, o) = f(x) , H(x,1) = K (x,1) =h(x)

Therefore f is (T,L) semi α- isotopic to h

Hence H (x,i) is (T,L) semi α- isotopy.

Thus (T, L) semi α- isotopy is transitive.

Theorem (31):

1- The (T,L) isotopy is an equivalenc relation

2- The (T,L) α – isotopy is an equivalenc relation

3- The (T,L) strongly α- isotopy is an equivalence relation

4- The (T,L) semi is isotopy is an is an equivalence relation

5- The (T,L) strongly semi α- isotopy is an equivalence relation

proof 1. Let B : X × I → Y be (T,L) isotopy. Therefore by theorem (25) B is (T,L) semi α- isotopy and theorem (30) we get (T,L) isotopy is an equivalence relation.

2. Suppose that B: X× I → Y is (T,L) α- istopoy . By theorem (26) B is (T,L) semi α- isotopy. And by theorem (30) we get (T,L) α- isotopy in an equivalence relation.

3. Suppose that B : X×I → Y be (T,L) strongly α- isotopy then by theorem (27) B is (T,L) semi α – isotopy and by theorem (30) we get (T,L) strongly α – isotopy is an equivalence relation.

4. Let B : X×I → Y be (T,L) semi isotopy then by th theorem (28) B is (T,L) semi α - isotopy. By theorem (30) we get (T, L) semi isotopy is an equivalence relation.

5. Let B : X× I → y be (T,L) strongly semi α- isotopy , then by theorem (29) we get B is (T,L) semi α – isotopy . By theorem (30) we get (T, L) strongly semi α- isotopy is an equivalence relation

References

1. S.K. Kasahara, " Operation – Compact spaces" Mathematica Japonica ,Vol. (21), pp97-105 , 1979.
2. K.A Al – Miahi , " Studies in α- space " , M. Sc thesis , Al- Bayet Univ. Jardan (2002).
3. H. Ogata ," Operation on topological spaces and associated topology", Math. Japonica 36 , No .1, pp 175-184,1991.
4. E . Rosas , J. Vielma and C. Carpintero ," α- semi connected and locally α – semi connected properties in topology spaces", scientiae mathematica Japonicae online , Vol. 6 pp465-472, 2002.
5. O.Njasted " On some class of Nearly open sets, " pacific J. of Math. , Vol. (15), No.3, pp961-970 , 1965.
6. A.S.Mashhour , A , Hasanein and I, A. El .Deeb, " α- continuous and α – open mapping ", A cta, Math , 4(3-4) , pp. 213-218, 1983.
7. N.G . Mansour , A.M Ibrahim " T- α – operator " J . of colloge of education No. 3, 2006.
8. I.Z .Mushtt " T – semi α- operatar, " College of education , to apper.
9. A.A. Qassim "The operater T and new types of open sets and spaces " , M.Sc .thesis .Mutah University 2004..
10. S.M . Yousif ," On the (T,L) α- homotopy and (T,L) α- isotopy" colloge of education , to opper.

الخلاصة

في هذا البحث قدمنا تعريف (T,L) شبه α هموتوبي ، (T,L) شبه α هموتوبي بقوة ودرسنا العلاقة بينهم وبين (T,L) هموتوبي ،(T,L) α – هموتوبي و (T,L) شبه هموتوبي ، (T,L) α هموتوبي بقوة كذلك قدمنا تعريف (T,L) شبه α – ايزوتوبي و (T,L) شبه α ايزوتوبي بقوة ودرسنا العلاقة بينهم وبين (T,L) ايزوتوبي و (T,L) α ايزوتوبي ، (T,L) شبه ايزوتوبي ، و(T,L) α – ايزوتوبي بقوة واعطينا عدة تمييزات لهذه المفاهيم.